

# Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.2-Cosine/85-4.2.12-e-x<sup>-m-a+b-cos-c+d-x<sup>n-p</sup></sup>

Nasser M. Abbasi

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# Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
<b>2</b>	<b>detailed summary tables of results</b>	<b>19</b>
<b>3</b>	<b>Listing of integrals</b>	<b>49</b>
<b>4</b>	<b>Appendix</b>	<b>439</b>

# Chapter 1

## Introduction

### Local contents

1.1	Listing of CAS systems tested . . . . .	4
1.2	Results . . . . .	5
1.3	Time and leaf size Performance . . . . .	9
1.4	list of integrals that has no closed form antiderivative . . . . .	11
1.5	List of integrals solved by CAS but has no known antiderivative . . . . .	12
1.6	list of integrals solved by CAS but failed verification . . . . .	13
1.7	Timing . . . . .	13
1.8	Verification . . . . .	14
1.9	Important notes about some of the results . . . . .	14
1.10	Design of the test system . . . . .	17

This report gives the result of running the computer algebra independent integration test. The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 99 ]. This is test number [ 85 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 ( 99 )	0.00 ( 0 )
Mathematica	100.00 ( 99 )	0.00 ( 0 )
Fricas	91.92 ( 91 )	8.08 ( 8 )
Maple	87.88 ( 87 )	12.12 ( 12 )
Maxima	81.82 ( 81 )	18.18 ( 18 )
Giac	52.53 ( 52 )	47.47 ( 47 )
Sympy	33.33 ( 33 )	66.67 ( 66 )
Mupad	30.30 ( 30 )	69.70 ( 69 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

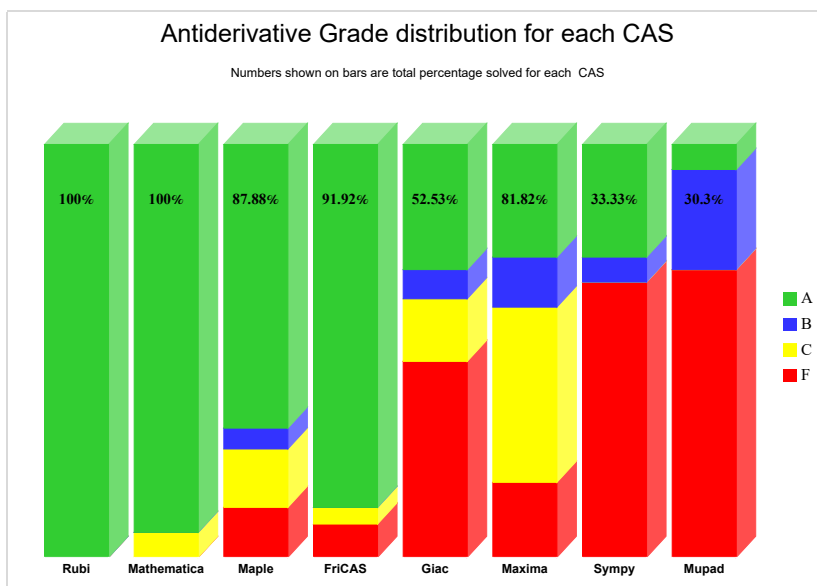
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

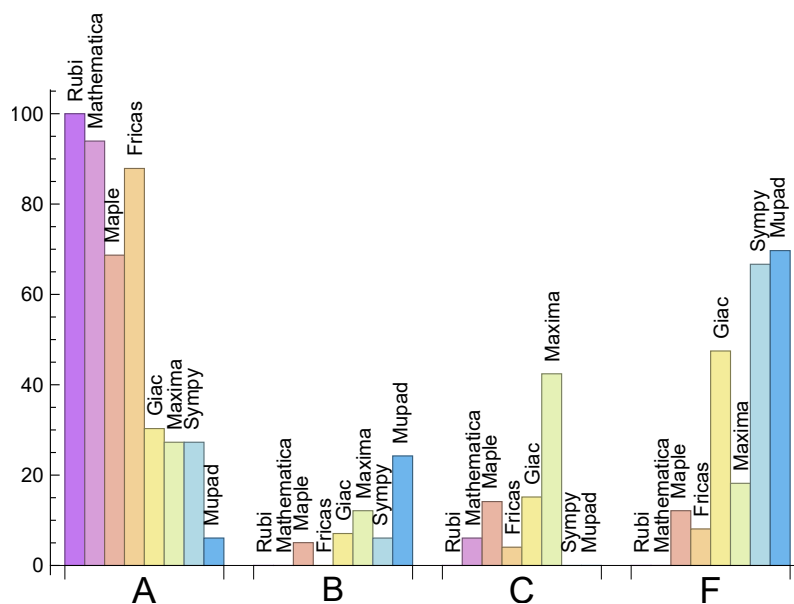
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	93.94	0.00	6.06	0.00
Fricas	87.88	0.00	4.04	8.08
Maple	68.69	5.05	14.14	12.12
Giac	30.30	7.07	15.15	47.47
Maxima	27.27	12.12	42.42	18.18
Sympy	27.27	6.06	0.00	66.67
Mupad	N/A	24.24	0.00	69.70

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**. The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	12	100.00 %	0.00 %	0.00 %
Fricas	8	100.00 %	0.00 %	0.00 %
Giac	47	100.00 %	0.00 %	0.00 %
Maxima	18	100.00 %	0.00 %	0.00 %
Sympy	66	89.39 %	4.55 %	6.06 %
Mupad	69	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS



## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

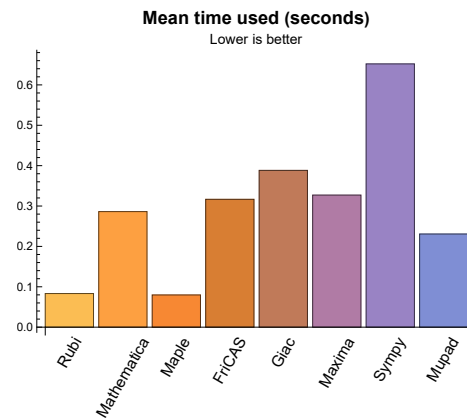
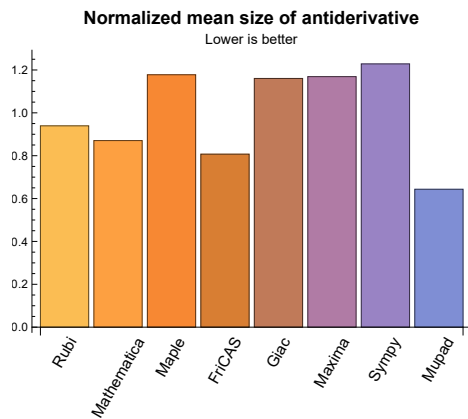
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	102.90	0.94	85.00	1.00
Mathematica	0.29	89.83	0.87	81.00	0.95
Maple	0.08	144.61	1.18	65.00	0.88
Maxima	0.33	114.21	1.17	83.00	1.00
Fricas	0.32	76.38	0.81	62.00	0.79
Sympy	0.65	86.76	1.23	46.00	1.06
Giac	0.39	112.19	1.16	65.50	1.14
Mupad	0.23	28.03	0.64	25.00	0.76

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



## **1.4 list of integrals that has no closed form antiderivative**

{63, 64, 66, 68, 88, 89}

## 1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

### 1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

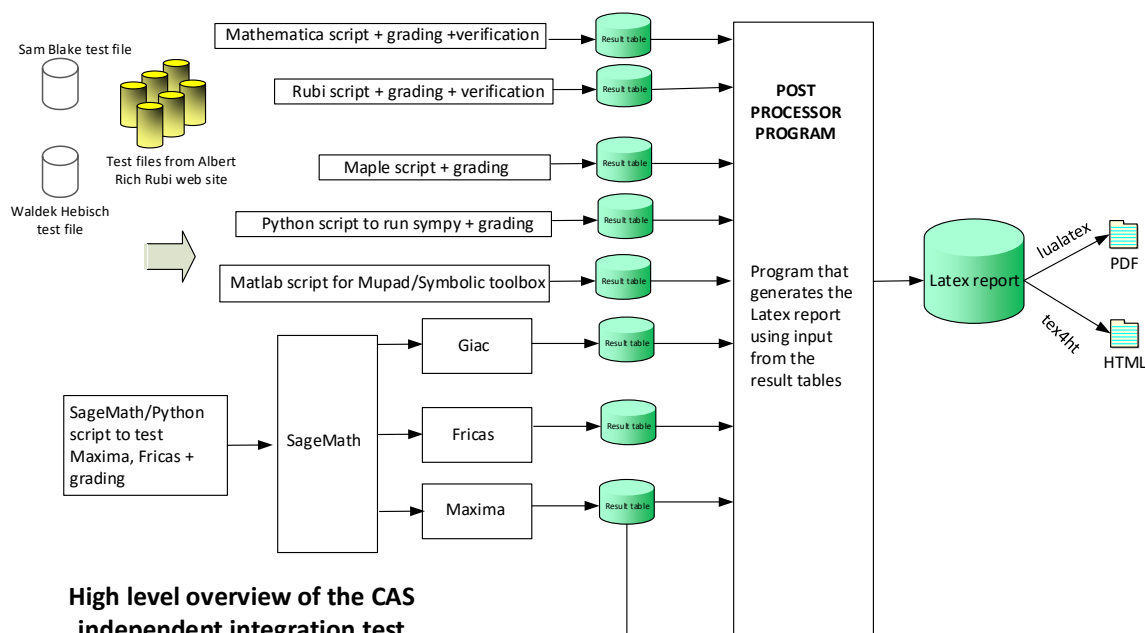
```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)
```

Which gives  $\sin(x)^2/2$



## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



### High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in Rubi Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax



# Chapter 2

## detailed summary tables of results

### Local contents

2.1	List of integrals sorted by grade for each CAS . . . . .	20
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	23
2.3	Detailed conclusion table specific for Rubi results . . . . .	44

## 2.1 List of integrals sorted by grade for each CAS

### Local contents

2.1.1	Rubi . . . . .	21
2.1.2	Mathematica . . . . .	21
2.1.3	Maple . . . . .	21
2.1.4	Maxima . . . . .	21
2.1.5	FriCAS . . . . .	22
2.1.6	Sympy . . . . .	22
2.1.7	Giac . . . . .	22
2.1.8	Mupad . . . . .	22

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 96, 97 }

B grade: { }

C grade: { 90, 93, 94, 95, 98, 99 }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18, 20, 22, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 68, 69, 70, 71, 72, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 97 }

B grade: { 90, 93, 94, 95, 96 }

C grade: { 12, 14, 19, 21, 23, 24, 25, 26, 27, 28, 73, 76, 98, 99 }

F grade: { 29, 30, 31, 32, 33, 34, 65, 67, 74, 75, 77, 78 }

### 2.1.4 Maxima

A grade: { 1, 3, 8, 10, 15, 17, 22, 29, 33, 34, 37, 43, 45, 46, 47, 48, 61, 62, 63, 64, 66, 68, 88, 89, 91, 92, 97 }

B grade: { 23, 24, 25, 26, 27, 28, 30, 31, 32, 90, 95, 96 }

C grade: { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 35, 36, 38, 39, 40, 41, 42, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 69, 70, 71, 72, 85, 86, 87 }

F grade: { 65, 67, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 93, 94, 98, 99 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 68, 69, 70, 71, 72, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 97 }

B grade: { }

C grade: { 93, 94, 98, 99 }

F grade: { 65, 67, 73, 74, 75, 76, 77, 78 }

### 2.1.6 Sympy

A grade: { 1, 3, 4, 8, 11, 15, 17, 18, 22, 36, 37, 38, 39, 43, 46, 47, 48, 62, 63, 64, 66, 88, 89, 90, 91, 92, 97 }

B grade: { 2, 9, 10, 16, 45, 61 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 40, 41, 42, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 93, 94, 95, 96, 98, 99 }

### 2.1.7 Giac

A grade: { 1, 3, 5, 8, 10, 12, 15, 17, 19, 22, 37, 38, 43, 45, 46, 47, 48, 61, 62, 63, 64, 66, 68, 88, 89, 90, 91, 92, 96, 97 }

B grade: { 7, 14, 21, 35, 36, 39, 95 }

C grade: { 2, 4, 9, 11, 16, 18, 49, 50, 51, 55, 56, 57, 85, 86, 87 }

F grade: { 6, 13, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 44, 52, 53, 54, 58, 59, 60, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 93, 94, 98, 99 }

### 2.1.8 Mupad

A grade: { 63, 64, 66, 68, 88, 89 }

B grade: { 1, 3, 4, 8, 10, 15, 17, 22, 37, 38, 39, 42, 43, 45, 46, 47, 48, 61, 62, 85, 86, 87, 92, 97 }

C grade: { }

F grade: { 2, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 93, 94, 95, 96, 98, 99 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To help make the table fit, **Mathematica** was abbrev-

	Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
viated to MMA.	grade	A	A	A	A	A	A	A	A	B
	verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
	size	34	34	29	31	27	27	36	45	27
	N.S.	1	1.00	0.85	0.91	0.79	0.79	1.06	1.32	0.79
	time (sec)	N/A	0.022	0.035	0.056	0.296	0.357	0.177	0.445	0.101

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	58	67	72	209	135	-1
N.S.	1	1.00	0.90	0.64	0.74	0.79	2.30	1.48	-0.01
time (sec)	N/A	0.049	0.098	0.106	0.299	0.368	1.224	0.435	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	19	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.27	0.87	0.87
time (sec)	N/A	0.010	0.003	0.037	0.284	0.366	0.071	0.437	0.060

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	57	44	48	61	61	95	51
N.S.	1	1.00	0.81	0.63	0.69	0.87	0.87	1.36	0.73
time (sec)	N/A	0.014	0.063	0.048	0.295	0.400	0.240	0.457	0.357

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	22	43	29	0	21	-1
N.S.	1	1.00	0.96	0.88	1.72	1.16	0.00	0.84	-0.04
time (sec)	N/A	0.020	0.029	0.075	0.366	0.365	0.000	0.417	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	57	73	70	0	0	-1
N.S.	1	1.00	1.01	0.71	0.91	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.123	0.066	0.557	0.361	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	39	48	57	0	87	-1
N.S.	1	1.00	1.00	0.93	1.14	1.36	0.00	2.07	-0.02
time (sec)	N/A	0.061	0.046	0.087	0.369	0.375	0.000	0.414	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	42	42	45	78	76	41
N.S.	1	1.00	0.78	0.82	0.82	0.88	1.53	1.49	0.80
time (sec)	N/A	0.035	0.085	0.079	0.290	0.373	0.258	0.439	0.148



Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	87	63	90	84	201	118	-1
N.S.	1	1.00	0.96	0.69	0.99	0.92	2.21	1.30	-0.01
time (sec)	N/A	0.072	0.111	0.115	0.516	0.389	1.612	0.490	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	34	23	28	60	26	22
N.S.	1	1.00	0.87	1.10	0.74	0.90	1.94	0.84	0.71
time (sec)	N/A	0.020	0.031	0.052	0.291	0.386	0.116	0.402	0.272

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	45	70	59	56	82	-1
N.S.	1	1.00	0.96	0.64	1.00	0.84	0.80	1.17	-0.01
time (sec)	N/A	0.031	0.039	0.069	0.526	0.371	0.474	0.409	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	68	51	39	0	35	-1
N.S.	1	1.00	0.92	1.84	1.38	1.05	0.00	0.95	-0.03
time (sec)	N/A	0.036	0.038	0.130	0.368	0.369	0.000	0.442	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	62	83	66	0	0	-1
N.S.	1	1.00	1.00	0.82	1.09	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.112	0.089	0.564	0.375	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	89	61	65	0	107	-1
N.S.	1	1.00	0.88	1.56	1.07	1.14	0.00	1.88	-0.02
time (sec)	N/A	0.082	0.082	0.125	0.344	0.368	0.000	0.427	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	55	66	58	58	92	92	66
N.S.	1	1.00	0.70	0.84	0.73	0.73	1.16	1.16	0.84
time (sec)	N/A	0.050	0.103	0.118	0.287	0.360	0.411	0.414	0.410

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	160	130	143	148	439	259	-1
N.S.	1	1.00	0.85	0.69	0.76	0.79	2.34	1.38	-0.01
time (sec)	N/A	0.141	0.287	0.151	0.503	0.398	2.702	0.424	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	27	25	44	26	28
N.S.	1	1.00	1.00	0.79	0.82	0.76	1.33	0.79	0.85
time (sec)	N/A	0.021	0.012	0.063	0.294	0.372	0.179	0.447	0.289

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	116	101	112	121	129	185	-1
N.S.	1	1.00	0.76	0.66	0.73	0.79	0.84	1.21	-0.01
time (sec)	N/A	0.066	0.153	0.088	0.506	0.350	0.707	0.452	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	125	89	63	0	47	-1
N.S.	1	1.00	0.91	2.27	1.62	1.15	0.00	0.85	-0.02
time (sec)	N/A	0.056	0.056	0.199	0.380	0.350	0.000	0.417	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	166	128	152	136	0	0	-1
N.S.	1	1.00	0.99	0.76	0.90	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.428	0.128	0.625	0.390	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	90	162	98	108	0	185	-1
N.S.	1	1.00	0.99	1.78	1.08	1.19	0.00	2.03	-0.01
time (sec)	N/A	0.144	0.116	0.191	0.392	0.387	0.000	0.418	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	50	55	51	94	52	55
N.S.	1	1.00	1.00	0.75	0.82	0.76	1.40	0.78	0.82
time (sec)	N/A	0.031	0.035	0.082	0.298	0.379	0.895	0.435	0.745

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	229	153	58	0	0	-1
N.S.	1	1.00	1.02	2.06	1.38	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.133	0.066	0.118	0.118	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	111	290	158	56	0	0	-1
N.S.	1	1.00	1.00	2.61	1.42	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.106	0.062	0.117	0.110	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	290	138	44	0	0	-1
N.S.	1	1.00	1.10	3.58	1.70	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.061	0.053	0.128	0.104	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	338	135	44	0	0	-1
N.S.	1	1.00	1.10	4.17	1.67	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.048	0.059	0.105	0.107	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	114	338	133	56	0	0	-1
N.S.	1	1.00	1.16	3.45	1.36	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.120	0.054	0.119	0.117	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	117	358	133	61	0	0	-1
N.S.	1	1.00	1.12	3.44	1.28	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.107	0.054	0.125	0.121	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	142	0	171	78	0	0	-1
N.S.	1	1.00	1.08	0.00	1.30	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.272	0.117	0.122	0.129	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	142	0	183	77	0	0	-1
N.S.	1	1.00	1.08	0.00	1.39	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.222	0.097	0.118	0.113	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	122	0	154	50	0	0	-1
N.S.	1	1.00	1.22	0.00	1.54	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.147	0.093	0.120	0.118	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	120	0	159	50	0	0	-1
N.S.	1	1.00	1.25	0.00	1.66	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.151	0.107	0.106	0.118	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	137	0	143	59	0	0	-1
N.S.	1	1.00	1.17	0.00	1.22	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.234	0.091	0.128	0.115	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	137	0	146	63	0	0	-1
N.S.	1	1.00	1.18	0.00	1.26	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.241	0.092	0.126	0.114	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	39	57	45	0	132	-1
N.S.	1	1.00	1.00	1.26	1.84	1.45	0.00	4.26	-0.03
time (sec)	N/A	0.047	0.024	0.118	0.319	0.398	0.000	0.406	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	43	28	15	41	-1
N.S.	1	1.00	1.00	1.05	2.15	1.40	0.75	2.05	-0.05
time (sec)	N/A	0.021	0.026	0.074	0.330	0.379	0.557	0.448	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	15	15	13
N.S.	1	1.00	1.00	1.08	1.00	1.15	1.15	1.15	1.00
time (sec)	N/A	0.010	0.004	0.043	0.293	0.387	0.339	0.406	0.264

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	42	51	33	31	49	30
N.S.	1	1.00	0.97	1.40	1.70	1.10	1.03	1.63	1.00
time (sec)	N/A	0.017	0.034	0.074	0.317	0.338	0.468	0.453	0.279

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	92	50	43	46	107	47
N.S.	1	1.00	1.00	2.00	1.09	0.93	1.00	2.33	1.02
time (sec)	N/A	0.030	0.004	0.086	0.306	0.346	0.662	0.425	0.372

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	80	57	127	73	0	0	-1
N.S.	1	1.00	1.01	0.72	1.61	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.094	0.109	0.345	0.354	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	22	43	29	0	0	-1
N.S.	1	1.00	0.96	0.88	1.72	1.16	0.00	0.00	-0.04
time (sec)	N/A	0.019	0.026	0.072	0.312	0.374	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	62	48	98	65	0	0	55
N.S.	1	1.00	0.84	0.65	1.32	0.88	0.00	0.00	0.74
time (sec)	N/A	0.023	0.074	0.071	0.328	0.363	0.000	0.000	0.406

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	22	17	13
N.S.	1	1.00	1.00	0.93	0.87	1.13	1.47	1.13	0.87
time (sec)	N/A	0.011	0.003	0.043	0.277	0.345	0.668	0.449	0.268

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	64	74	84	0	0	-1
N.S.	1	1.00	0.91	0.66	0.76	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.111	0.088	0.337	0.369	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	14	12	13	39	12	12
N.S.	1	1.00	0.95	0.74	0.63	0.68	2.05	0.63	0.63
time (sec)	N/A	0.015	0.022	0.049	0.280	0.386	0.085	0.402	0.367

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.006	0.003	0.023	0.277	0.420	0.093	0.432	0.033

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.009	0.012	0.025	0.278	0.376	0.091	0.493	0.278

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	34	23	24	51	23	23
N.S.	1	1.00	0.86	0.94	0.64	0.67	1.42	0.64	0.64
time (sec)	N/A	0.014	0.020	0.028	0.298	0.373	0.085	0.417	0.345



Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	165	196	136	145	0	241	-1
N.S.	1	1.00	0.70	0.83	0.58	0.62	0.00	1.03	-0.00
time (sec)	N/A	0.266	0.390	0.084	0.298	0.362	0.000	0.444	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	141	131	112	118	0	193	-1
N.S.	1	1.00	0.83	0.78	0.66	0.70	0.00	1.14	-0.01
time (sec)	N/A	0.148	0.244	0.055	0.303	0.382	0.000	0.437	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	94	64	73	78	0	143	-1
N.S.	1	1.00	0.95	0.65	0.74	0.79	0.00	1.44	-0.01
time (sec)	N/A	0.086	0.107	0.048	0.291	0.384	0.000	0.491	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	78	74	96	0	0	-1
N.S.	1	1.00	1.00	0.71	0.67	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.169	0.054	0.577	0.364	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	180	129	76	134	0	0	-1
N.S.	1	1.00	0.98	0.70	0.41	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.163	0.061	0.585	0.379	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	238	180	76	164	0	0	-1
N.S.	1	1.00	0.95	0.72	0.30	0.66	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.220	0.065	0.575	0.373	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	174	219	161	184	0	224	-1
N.S.	1	1.00	0.56	0.71	0.52	0.59	0.00	0.72	-0.00
time (sec)	N/A	0.249	0.432	0.079	0.516	0.387	0.000	0.422	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	148	145	137	144	0	176	-1
N.S.	1	1.00	0.68	0.67	0.63	0.66	0.00	0.81	-0.00
time (sec)	N/A	0.172	0.292	0.069	0.514	0.375	0.000	0.461	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	103	67	96	90	0	124	-1
N.S.	1	1.00	1.01	0.66	0.94	0.88	0.00	1.22	-0.01
time (sec)	N/A	0.118	0.125	0.072	0.504	0.379	0.000	0.437	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	116	87	88	100	0	0	-1
N.S.	1	1.00	1.00	0.75	0.76	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.169	0.069	0.572	0.384	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	185	146	90	154	0	0	-1
N.S.	1	1.00	0.81	0.64	0.39	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.184	0.069	0.578	0.374	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	249	207	90	192	0	0	-1
N.S.	1	1.00	0.76	0.63	0.27	0.59	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.249	0.069	0.601	0.378	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	66	58	47	48	513	47	62
N.S.	1	1.00	0.77	0.67	0.55	0.56	5.97	0.55	0.72
time (sec)	N/A	0.045	0.067	0.044	0.297	0.349	0.970	0.415	0.496

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.007	0.003	0.024	0.277	0.378	7.558	0.418	0.427

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.012	0.649	0.067	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.014	0.879	0.066	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	89	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.121	0.065	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.033	0.599	0.063	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	149	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.300	0.172	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	1.254	0.068	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	25	90	35	0	0	-1
N.S.	1	1.00	0.92	0.96	3.46	1.35	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.059	0.120	0.405	0.354	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	40	99	48	0	0	-1
N.S.	1	1.00	0.86	0.93	2.30	1.12	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.092	0.092	0.404	0.365	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	52	180	74	0	0	-1
N.S.	1	1.00	0.79	0.78	2.69	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.118	0.102	0.511	0.365	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	66	189	87	0	0	-1
N.S.	1	1.00	0.84	0.84	2.39	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.132	0.102	0.521	0.383	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	92	75	0	0	0	0	-1
N.S.	1	1.00	1.11	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.090	0.079	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	94	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.275	0.068	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	173	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.274	0.083	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	115	111	0	0	0	0	-1
N.S.	1	1.00	1.10	1.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.218	0.137	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	129	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.580	0.065	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	221	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.145	0.639	0.106	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	45	0	62	0	0	-1
N.S.	1	1.00	0.96	0.96	0.00	1.32	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.101	0.138	0.000	0.379	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	53	65	0	72	0	0	-1
N.S.	1	1.00	0.77	0.94	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.200	0.112	0.000	0.381	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	95	101	0	117	0	0	-1
N.S.	1	1.00	0.84	0.89	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.249	0.137	0.000	0.406	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	65	0	90	0	0	-1
N.S.	1	1.00	0.90	0.83	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.145	0.130	0.000	0.374	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	82	89	0	106	0	0	-1
N.S.	1	1.00	0.86	0.94	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.235	0.118	0.000	0.381	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	141	144	0	167	0	0	-1
N.S.	1	1.00	0.85	0.87	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.378	0.157	0.000	0.412	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	76	131	258	112	0	159	80
N.S.	1	1.00	0.77	1.32	2.61	1.13	0.00	1.61	0.81
time (sec)	N/A	0.051	0.287	0.173	0.779	0.365	0.000	0.465	0.146

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	63	199	63	0	119	39
N.S.	1	1.00	0.89	1.34	4.23	1.34	0.00	2.53	0.83
time (sec)	N/A	0.024	0.052	0.063	0.650	0.365	0.000	0.570	0.071

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	36	84	40	0	55	32
N.S.	1	1.00	1.00	1.24	2.90	1.38	0.00	1.90	1.10
time (sec)	N/A	0.004	0.013	0.056	0.506	0.370	0.000	0.443	0.062

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.006	2.696	0.046	0.000	0.000	0.000	0.000	0.000



Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.006	4.940	0.048	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	224	825	672	103	269	480	-1
N.S.	1	1.00	0.65	2.38	1.94	0.30	0.78	1.39	-0.00
time (sec)	N/A	0.217	0.770	0.066	0.310	0.355	0.322	0.427	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	71	299	263	67	151	166	-1
N.S.	1	1.00	0.43	1.79	1.57	0.40	0.90	0.99	-0.01
time (sec)	N/A	0.096	0.258	0.043	0.289	0.361	0.178	0.410	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	61	60	42	65	42	42
N.S.	1	1.00	0.89	1.13	1.11	0.78	1.20	0.78	0.78
time (sec)	N/A	0.019	0.085	0.040	0.307	0.382	0.132	0.495	0.344

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	145	271	0	149	0	0	-1
N.S.	1	1.00	1.15	2.15	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.698	0.070	0.000	0.366	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	240	714	0	210	0	0	-1
N.S.	1	1.00	1.30	3.88	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.249	1.226	0.075	0.000	0.398	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	537	537	382	1809	1349	182	0	1104	-1
N.S.	1	1.00	0.71	3.37	2.51	0.34	0.00	2.06	-0.00
time (sec)	N/A	0.357	1.041	0.065	0.336	0.385	0.000	0.418	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	117	655	523	110	0	370	-1
N.S.	1	1.00	0.45	2.51	2.00	0.42	0.00	1.42	-0.00
time (sec)	N/A	0.161	0.397	0.045	0.326	0.376	0.000	0.479	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	131	118	57	94	81	68
N.S.	1	1.00	0.76	1.54	1.39	0.67	1.11	0.95	0.80
time (sec)	N/A	0.038	0.127	0.041	0.279	0.415	0.239	0.408	0.382

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	243	279	0	287	0	0	-1
N.S.	1	1.00	1.04	1.19	0.00	1.23	0.00	0.00	-0.00
time (sec)	N/A	0.332	0.383	0.064	0.000	0.416	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	138	931	0	406	0	0	-1
N.S.	1	1.00	0.42	2.80	0.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.530	0.580	0.090	0.000	0.410	0.000	0.000	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [67] had the largest ratio of [22]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	12	0.250
2	A	4	4	1.00	12	0.333
3	A	2	2	1.00	10	0.200
4	A	3	3	1.00	8	0.375
5	A	3	3	1.00	12	0.250
6	A	4	4	1.00	12	0.333
7	A	5	5	1.00	12	0.417
8	A	3	3	1.00	14	0.214
9	A	6	5	1.00	14	0.357
10	A	3	3	1.00	12	0.250
11	A	5	4	1.00	10	0.400
12	A	5	4	1.00	14	0.286
13	A	6	6	1.00	14	0.429
14	A	7	6	1.00	14	0.429
15	A	4	4	1.00	14	0.286
16	A	10	5	1.00	14	0.357
17	A	3	2	1.00	12	0.167
18	A	8	4	1.00	10	0.400
19	A	8	4	1.00	14	0.286
20	A	9	5	1.00	14	0.357
21	A	12	6	1.00	14	0.429
22	A	3	2	1.00	12	0.167
23	A	4	3	1.00	14	0.214
24	A	4	3	1.00	14	0.214
25	A	3	2	1.00	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	2	1.00	14	0.143
27	A	4	3	1.00	14	0.214
28	A	4	3	1.00	14	0.214
29	A	7	5	1.00	16	0.312
30	A	7	5	1.00	16	0.312
31	A	6	4	1.00	16	0.250
32	A	6	4	1.00	16	0.250
33	A	7	5	1.00	16	0.312
34	A	7	6	1.00	16	0.375
35	A	5	5	1.00	8	0.625
36	A	3	3	1.00	12	0.250
37	A	2	2	1.00	12	0.167
38	A	3	3	1.00	12	0.250
39	A	4	3	1.00	12	0.250
40	A	5	5	1.00	8	0.625
41	A	3	3	1.00	12	0.250
42	A	4	4	1.00	12	0.333
43	A	2	2	1.00	12	0.167
44	A	5	5	1.00	12	0.417
45	A	3	3	1.00	14	0.214
46	A	2	2	1.00	12	0.167
47	A	3	3	1.00	6	0.500
48	A	3	3	1.00	8	0.375
49	A	13	7	1.00	16	0.438
50	A	10	7	1.00	16	0.438
51	A	7	7	1.00	16	0.438
52	A	8	7	1.00	16	0.438
53	A	11	7	1.00	16	0.438
54	A	14	7	1.00	16	0.438
55	A	15	10	1.00	18	0.556
56	A	12	9	1.00	18	0.500
57	A	9	8	1.00	18	0.444
58	A	10	9	1.00	18	0.500
59	A	12	10	1.00	18	0.556
60	A	16	9	1.00	18	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	5	1.00	8	0.625
62	A	2	2	1.00	12	0.167
63	A	0	0	0.00	0	0.000
64	A	0	0	0.00	0	0.000
65	A	3	3	1.00	20	0.150
66	A	0	0	0.00	0	0.000
67	A	5	5	1.00	22	0.227
68	A	0	0	0.00	0	0.000
69	A	3	3	1.00	12	0.250
70	A	5	4	1.00	14	0.286
71	A	8	4	1.00	14	0.286
72	A	8	4	1.00	14	0.286
73	A	3	2	1.00	8	0.250
74	A	5	3	1.00	10	0.300
75	A	8	3	1.00	10	0.300
76	A	3	2	1.00	12	0.167
77	A	5	3	1.00	14	0.214
78	A	8	3	1.00	14	0.214
79	A	5	5	1.00	16	0.312
80	A	7	6	1.00	18	0.333
81	A	12	6	1.00	18	0.333
82	A	6	5	1.00	16	0.312
83	A	8	6	1.00	18	0.333
84	A	14	6	1.00	18	0.333
85	A	7	6	1.00	12	0.500
86	A	5	4	1.00	10	0.400
87	A	1	1	1.00	8	0.125
88	A	0	0	0.00	0	0.000
89	A	0	0	0.00	0	0.000
90	A	14	3	1.00	18	0.167
91	A	8	3	1.00	16	0.188
92	A	3	3	1.00	14	0.214
93	A	8	4	1.00	18	0.222
94	A	10	6	1.00	18	0.333
95	A	20	4	1.00	18	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	11	4	1.00	16	0.250
97	A	4	3	1.00	14	0.214
98	A	11	4	1.00	18	0.222
99	A	13	6	1.00	18	0.333





# Chapter 3

## Listing of integrals

### Local contents

3.1	$\int x^3 \cos(a + bx^2) dx$	50
3.2	$\int x^2 \cos(a + bx^2) dx$	53
3.3	$\int x \cos(a + bx^2) dx$	57
3.4	$\int \cos(a + bx^2) dx$	60
3.5	$\int \frac{\cos(a+bx^2)}{x} dx$	64
3.6	$\int \frac{\cos(a+bx^2)}{x^2} dx$	67
3.7	$\int \frac{\cos(a+bx^2)}{x^3} dx$	71
3.8	$\int x^3 \cos^2(a + bx^2) dx$	75
3.9	$\int x^2 \cos^2(a + bx^2) dx$	78
3.10	$\int x \cos^2(a + bx^2) dx$	82
3.11	$\int \cos^2(a + bx^2) dx$	85
3.12	$\int \frac{\cos^2(a+bx^2)}{x} dx$	89
3.13	$\int \frac{\cos^2(a+bx^2)}{x^2} dx$	92
3.14	$\int \frac{\cos^2(a+bx^2)}{x^3} dx$	96
3.15	$\int x^3 \cos^3(a + bx^2) dx$	100
3.16	$\int x^2 \cos^3(a + bx^2) dx$	104
3.17	$\int x \cos^3(a + bx^2) dx$	108
3.18	$\int \cos^3(a + bx^2) dx$	111
3.19	$\int \frac{\cos^3(a+bx^2)}{x} dx$	115
3.20	$\int \frac{\cos^3(a+bx^2)}{x^2} dx$	118
3.21	$\int \frac{\cos^3(a+bx^2)}{x^3} dx$	122
3.22	$\int x \cos^7(a + bx^2) dx$	126
3.23	$\int x^{5/2} \cos(a + bx^2) dx$	129
3.24	$\int x^{3/2} \cos(a + bx^2) dx$	133
3.25	$\int \sqrt{x} \cos(a + bx^2) dx$	137

3.26	$\int \frac{\cos(a+bx^2)}{\sqrt{x}} dx$	140
3.27	$\int \frac{\cos(a+bx^2)}{x^{3/2}} dx$	143
3.28	$\int \frac{\cos(a+bx^2)}{x^{5/2}} dx$	147
3.29	$\int x^{5/2} \cos^2(a+bx^2) dx$	151
3.30	$\int x^{3/2} \cos^2(a+bx^2) dx$	155
3.31	$\int \sqrt{x} \cos^2(a+bx^2) dx$	159
3.32	$\int \frac{\cos^2(a+bx^2)}{\sqrt{x}} dx$	163
3.33	$\int \frac{\cos^2(a+bx^2)}{x^{3/2}} dx$	167
3.34	$\int \frac{\cos^2(a+bx^2)}{x^{5/2}} dx$	171
3.35	$\int \cos\left(a + \frac{b}{x}\right) dx$	175
3.36	$\int \frac{\cos\left(a + \frac{b}{x}\right)}{x} dx$	179
3.37	$\int \frac{\cos\left(a + \frac{b}{x}\right)}{x^2} dx$	182
3.38	$\int \frac{\cos\left(a + \frac{b}{x}\right)}{x^3} dx$	185
3.39	$\int \frac{\cos\left(a + \frac{b}{x}\right)}{x^4} dx$	188
3.40	$\int \cos\left(a + \frac{b}{x^2}\right) dx$	192
3.41	$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x} dx$	196
3.42	$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^2} dx$	199
3.43	$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^3} dx$	203
3.44	$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^4} dx$	206
3.45	$\int \frac{\cos^2(\sqrt{x})}{\sqrt{x}} dx$	210
3.46	$\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$	213
3.47	$\int \cos(\sqrt{x}) dx$	216
3.48	$\int \cos^2(\sqrt{x}) dx$	219
3.49	$\int x^{3/2} \cos(a+b\sqrt[3]{x}) dx$	222
3.50	$\int \sqrt{x} \cos(a+b\sqrt[3]{x}) dx$	229
3.51	$\int \frac{\cos(a+b\sqrt[3]{x})}{\sqrt{x}} dx$	234
3.52	$\int \frac{\cos(a+b\sqrt[3]{x})}{x^{3/2}} dx$	239
3.53	$\int \frac{\cos(a+b\sqrt[3]{x})}{x^{5/2}} dx$	243
3.54	$\int \frac{\cos(a+b\sqrt[3]{x})}{x^{7/2}} dx$	249
3.55	$\int x^{3/2} \cos^2(a+b\sqrt[3]{x}) dx$	257
3.56	$\int \sqrt{x} \cos^2(a+b\sqrt[3]{x}) dx$	264

3.57	$\int \frac{\cos^2(a+b\sqrt[3]{x})}{\sqrt{x}} dx$	269
3.58	$\int \frac{\cos^2(a+b\sqrt[3]{x})}{x^{3/2}} dx$	274
3.59	$\int \frac{\cos^2(a+b\sqrt[3]{x})}{x^{5/2}} dx$	279
3.60	$\int \frac{\cos^2(a+b\sqrt[3]{x})}{x^{7/2}} dx$	285
3.61	$\int \cos^3(\sqrt[3]{x}) dx$	293
3.62	$\int \frac{\cos(\sqrt[6]{x})}{x^{5/6}} dx$	297
3.63	$\int (ex)^m (b \cos(c + dx^n))^p dx$	300
3.64	$\int (ex)^m (a + b \cos(c + dx^n))^p dx$	302
3.65	$\int (ex)^{-1+n} (b \cos(c + dx^n))^p dx$	305
3.66	$\int (ex)^{-1+2n} (b \cos(c + dx^n))^p dx$	308
3.67	$\int (ex)^{-1+n} (a + b \cos(c + dx^n))^p dx$	311
3.68	$\int (ex)^{-1+2n} (a + b \cos(c + dx^n))^p dx$	315
3.69	$\int \frac{\cos(ax^n)}{x} dx$	318
3.70	$\int \frac{\cos^2(ax^n)}{x} dx$	321
3.71	$\int \frac{\cos^3(ax^n)}{x} dx$	324
3.72	$\int \frac{\cos^4(ax^n)}{x} dx$	328
3.73	$\int \cos(a + bx^n) dx$	332
3.74	$\int \cos^2(a + bx^n) dx$	335
3.75	$\int \cos^3(a + bx^n) dx$	338
3.76	$\int x^m \cos(a + bx^n) dx$	341
3.77	$\int x^m \cos^2(a + bx^n) dx$	344
3.78	$\int x^m \cos^3(a + bx^n) dx$	347
3.79	$\int x^{-1-n} \cos(a + bx^n) dx$	350
3.80	$\int x^{-1-n} \cos^2(a + bx^n) dx$	354
3.81	$\int x^{-1-n} \cos^3(a + bx^n) dx$	358
3.82	$\int x^{-1-2n} \cos(a + bx^n) dx$	362
3.83	$\int x^{-1-2n} \cos^2(a + bx^n) dx$	366
3.84	$\int x^{-1-2n} \cos^3(a + bx^n) dx$	370
3.85	$\int x^2 \cos((a + bx)^2) dx$	374
3.86	$\int x \cos((a + bx)^2) dx$	378
3.87	$\int \cos((a + bx)^2) dx$	382
3.88	$\int \frac{\cos((a+bx)^2)}{x} dx$	385
3.89	$\int \frac{\cos((a+bx)^2)}{x^2} dx$	388
3.90	$\int x^2 \cos(a + b\sqrt{c + dx}) dx$	391
3.91	$\int x \cos(a + b\sqrt{c + dx}) dx$	396
3.92	$\int \cos(a + b\sqrt{c + dx}) dx$	400
3.93	$\int \frac{\cos(a+b\sqrt{c+dx})}{x} dx$	404

3.94	$\int \frac{\cos(a+b\sqrt{c+dx})}{x^2} dx$	408
3.95	$\int x^2 \cos(a+b\sqrt[3]{c+dx}) dx$	413
3.96	$\int x \cos(a+b\sqrt[3]{c+dx}) dx$	420
3.97	$\int \cos(a+b\sqrt[3]{c+dx}) dx$	425
3.98	$\int \frac{\cos(a+b\sqrt[3]{c+dx})}{x} dx$	429
3.99	$\int \frac{\cos(a+b\sqrt[3]{c+dx})}{x^2} dx$	433

### 3.1 $\int x^3 \cos(a + bx^2) dx$

Optimal. Leaf size=34

$$\frac{\cos(a + bx^2)}{2b^2} + \frac{x^2 \sin(a + bx^2)}{2b}$$

[Out]  $1/2*\cos(b*x^2+a)/b^2+1/2*x^2*\sin(b*x^2+a)/b$

Rubi [A]

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3461, 3377, 2718}

$$\frac{\cos(a + bx^2)}{2b^2} + \frac{x^2 \sin(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] `Int[x^3*Cos[a + b*x^2],x]`

[Out] `Cos[a + b*x^2]/(2*b^2) + (x^2*Sin[a + b*x^2])/(2*b)`

Rule 2718

`Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 3377

`Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 3461

`Int[((a_.) + Cos[(c_.) + (d_.)*(x_.)^(n_.)]*(b_.))^(p_.)*(x_.)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))`

Rubi steps

$$\begin{aligned}
 \int x^3 \cos(a + bx^2) dx &= \frac{1}{2} \text{Subst} \left( \int x \cos(a + bx) dx, x, x^2 \right) \\
 &= \frac{x^2 \sin(a + bx^2)}{2b} - \frac{\text{Subst}(\int \sin(a + bx) dx, x, x^2)}{2b} \\
 &= \frac{\cos(a + bx^2)}{2b^2} + \frac{x^2 \sin(a + bx^2)}{2b}
 \end{aligned}$$

**Mathematica [A]**

time = 0.04, size = 29, normalized size = 0.85

$$\frac{\cos(a + bx^2) + bx^2 \sin(a + bx^2)}{2b^2}$$

Antiderivative was successfully verified.

`[In] Integrate[x^3*Cos[a + b*x^2],x]``[Out] (Cos[a + b*x^2] + b*x^2*Sin[a + b*x^2])/(2*b^2)`**Maple [A]**

time = 0.06, size = 31, normalized size = 0.91

method	result	size
default	$\frac{\cos(bx^2+a)}{2b^2} + \frac{x^2 \sin(bx^2+a)}{2b}$	31
risch	$\frac{\cos(bx^2+a)}{2b^2} + \frac{x^2 \sin(bx^2+a)}{2b}$	31
norman	$\frac{\frac{1}{b^2} + \frac{x^2 \tan\left(\frac{a}{2} + \frac{bx^2}{2}\right)}{b}}{1 + \tan^2\left(\frac{a}{2} + \frac{bx^2}{2}\right)}$	41
meijerg	$\frac{\cos(a) \sqrt{\pi} \left( -\frac{1}{2\sqrt{\pi}} + \frac{\cos(bx^2)}{2\sqrt{\pi}} + \frac{x^2 b \sin(bx^2)}{2\sqrt{\pi}} \right)}{b^2} - \frac{\sin(a) \sqrt{\pi} \left( -\frac{x^2 b \cos(bx^2)}{2\sqrt{\pi}} + \frac{\sin(bx^2)}{2\sqrt{\pi}} \right)}{b^2}$	80

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3*cos(b*x^2+a),x,method=_RETURNVERBOSE)``[Out] 1/2*cos(b*x^2+a)/b^2+1/2*x^2*sin(b*x^2+a)/b`**Maxima [A]**

time = 0.30, size = 27, normalized size = 0.79

$$\frac{bx^2 \sin(bx^2 + a) + \cos(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(b*x^2+a),x, algorithm="maxima")`

[Out]  $1/2*(b*x^2*\sin(b*x^2 + a) + \cos(b*x^2 + a))/b^2$

**Fricas** [A]

time = 0.36, size = 27, normalized size = 0.79

$$\frac{bx^2 \sin(bx^2 + a) + \cos(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(b*x^2+a),x, algorithm="fricas")`

[Out]  $1/2*(b*x^2*\sin(b*x^2 + a) + \cos(b*x^2 + a))/b^2$

**Sympy** [A]

time = 0.18, size = 36, normalized size = 1.06

$$\begin{cases} \frac{x^2 \sin(a+bx^2)}{2b} + \frac{\cos(a+bx^2)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^4 \cos(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*cos(b*x**2+a),x)`

[Out] `Piecewise((x**2*sin(a + b*x**2)/(2*b) + cos(a + b*x**2)/(2*b**2), Ne(b, 0)), (x**4*cos(a)/4, True))`

**Giac** [A]

time = 0.44, size = 45, normalized size = 1.32

$$-\frac{a \sin(bx^2 + a)}{2b^2} + \frac{(bx^2 + a) \sin(bx^2 + a) + \cos(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(b*x^2+a),x, algorithm="giac")`

[Out]  $-1/2*a*\sin(b*x^2 + a)/b^2 + 1/2*((b*x^2 + a)*\sin(b*x^2 + a) + \cos(b*x^2 + a))/b^2$

**Mupad** [B]

time = 0.10, size = 27, normalized size = 0.79

$$\frac{\cos(bx^2 + a) + bx^2 \sin(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cos(a + b*x^2),x)`

[Out]  $(\cos(a + b*x^2) + b*x^2*\sin(a + b*x^2))/(2*b^2)$

## 3.2 $\int x^2 \cos(a + bx^2) dx$

**Optimal.** Leaf size=91

$$-\frac{\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{2b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a)}{2b^{3/2}} + \frac{x \sin(a + bx^2)}{2b}$$

[Out]  $1/2*x*\sin(b*x^2+a)/b-1/4*\cos(a)*\operatorname{FresnelS}(x*b^{(1/2)}*2^{(1/2)}/\operatorname{Pi}^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(3/2)}-1/4*\operatorname{FresnelC}(x*b^{(1/2)}*2^{(1/2)}/\operatorname{Pi}^{(1/2)})*\sin(a)*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(3/2)}$

**Rubi [A]**

time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3467, 3434, 3433, 3432}

$$-\frac{\sqrt{\frac{\pi}{2}} \sin(a) \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{b} x\right)}{2b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{2b^{3/2}} + \frac{x \sin(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] `Int[x^2*Cos[a + b*x^2], x]`

[Out]  $-1/2*(\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Cos}[a]*\operatorname{FresnelS}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\operatorname{Pi}]*x])/b^{(3/2)} - (\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{FresnelC}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\operatorname{Pi}]*x]*\operatorname{Sin}[a])/(2*b^{(3/2)}) + (x*\operatorname{Sin}[a + b*x^2])/(2*b)$

Rule 3432

`Int[Sin[(d_.)*((e_.) + (f_.)*(x_)) ^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]`

Rule 3433

`Int[Cos[(d_.)*((e_.) + (f_.)*(x_)) ^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]`

Rule 3434

`Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_)) ^2], x_Symbol] := Dist[Sin[c], Int[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /; FreeQ[{c, d, e, f}, x]`

Rule 3467



```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.), x_Symbol] := Simp[e^(n
- 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned} \int x^2 \cos(a + bx^2) dx &= \frac{x \sin(a + bx^2)}{2b} - \frac{\int \sin(a + bx^2) dx}{2b} \\ &= \frac{x \sin(a + bx^2)}{2b} - \frac{\cos(a) \int \sin(bx^2) dx}{2b} - \frac{\sin(a) \int \cos(bx^2) dx}{2b} \\ &= -\frac{\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{2b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} C\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a)}{2b^{3/2}} + \frac{x \sin(a + bx^2)}{2b} \end{aligned}$$

**Mathematica [A]**

time = 0.10, size = 82, normalized size = 0.90

$$\frac{-\sqrt{2\pi} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) - \sqrt{2\pi} \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a) + 2\sqrt{b} x \sin(a + bx^2)}{4b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Cos[a + b\*x^2],x]

[Out]  $(-\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Cos}[a]*\operatorname{FresnelS}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\operatorname{Pi}]*x]) - \operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{FresnelC}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\operatorname{Pi}]*x]*\operatorname{Sin}[a] + 2*\operatorname{Sqrt}[b]*x*\operatorname{Sin}[a + b*x^2])/(4*b^{(3/2)})$

**Maple [A]**

time = 0.11, size = 58, normalized size = 0.64

method	result
default	$\frac{x \sin(bx^2+a)}{2b} - \frac{\sqrt{2} \sqrt{\pi} \left( \cos(a) S\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) + \sin(a) \operatorname{FresnelC}\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) \right)}{4b^{3/2}}$
risch	$-\frac{ie^{-ia} \sqrt{\pi} \operatorname{erf}\left(\sqrt{ib} x\right)}{8b\sqrt{ib}} + \frac{ie^{ia} \sqrt{\pi} \operatorname{erf}\left(\sqrt{-ib} x\right)}{8b\sqrt{-ib}} + \frac{x \sin(bx^2+a)}{2b}$

meijerg	$\frac{\cos(a)\sqrt{\pi}\sqrt{2}\left(\frac{x\sqrt{2}(b^2)^{\frac{3}{4}}\sin(bx^2)}{2\sqrt{\pi}b} - \frac{(b^2)^{\frac{3}{4}}S\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right)}{2b^{\frac{3}{2}}}\right) - \sin(a)\sqrt{\pi}\sqrt{2}\left(-\frac{x\sqrt{2}\sqrt{b}\cos(bx^2)}{2\sqrt{\pi}} + \frac{\text{FresnelC}\left(\frac{x\sqrt{2}\sqrt{b}}{\sqrt{\pi}}\right)}{2b^{\frac{3}{2}}}\right)}{2(b^2)^{\frac{3}{4}}}$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(b*x^2+a),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{2}x\sin(bx^2+a)/b - 1/4/b^{3/2} * 2^{1/2} * \pi^{1/2} * (\cos(a) * \text{FresnelS}(x * b^{1/2}) * 2^{1/2} / \pi^{1/2}) + \sin(a) * \text{FresnelC}(x * b^{1/2}) * 2^{1/2} / \pi^{1/2}$

**Maxima** [C] Result contains complex when optimal does not.

time = 0.30, size = 67, normalized size = 0.74

$$\frac{8b^2x\sin(bx^2+a) + \sqrt{2}\sqrt{\pi}\left((-i+1)\cos(a) + (i-1)\sin(a)\right)\text{erf}\left(\sqrt{ib}x\right) + \left((i-1)\cos(a) - (i+1)\sin(a)\right)\text{erf}\left(\sqrt{-ib}x\right)}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(b*x^2+a),x, algorithm="maxima")`

[Out]  $\frac{1}{16} * (8 * b^2 * x * \sin(b * x^2 + a) + \text{sqrt}(2) * \text{sqrt}(\pi) * ((-1 + 1) * \cos(a) + (1 - 1) * \sin(a)) * \text{erf}(\text{sqrt}(I * b) * x) + ((1 - 1) * \cos(a) - (1 + 1) * \sin(a)) * \text{erf}(\text{sqrt}(-I * b) * x)) * b^{3/2}) / b^3$

**Fricas** [A]

time = 0.37, size = 72, normalized size = 0.79

$$\frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)S\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right) + \sqrt{2}\pi\sqrt{\frac{b}{\pi}}C\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin(a) - 2bx\sin(bx^2+a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(b*x^2+a),x, algorithm="fricas")`

[Out]  $-1/4 * (\text{sqrt}(2) * \pi * \text{sqrt}(b/\pi) * \cos(a) * \text{fresnel\_sin}(\text{sqrt}(2) * x * \text{sqrt}(b/\pi)) + \text{sqrt}(2) * \pi * \text{sqrt}(b/\pi) * \text{fresnel\_cos}(\text{sqrt}(2) * x * \text{sqrt}(b/\pi)) * \sin(a) - 2 * b * x * \sin(b * x^2 + a)) / b^2$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs.  $2(90) = 180$ .

time = 1.22, size = 209, normalized size = 2.30

$$\frac{b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\sin(a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right) {}_2F_3\left(\frac{3}{4}, \frac{5}{4} \middle| -\frac{b^2x^4}{4}\right) - \sqrt{b}x^3\sqrt{\frac{1}{b}}\cos(a)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\frac{1}{4}, \frac{3}{4} \middle| -\frac{b^2x^4}{4}\right)}{8\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin(a)S\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{\pi}}\right) + \sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos(a)C\left(\frac{\sqrt{2}\sqrt{bx}}{\sqrt{\pi}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cos(b*x**2+a),x)`

[Out]  $b^{3/2}x^{5/2}\sqrt{1/b}\sin(a)\Gamma(3/4)\Gamma(5/4)\text{hyper}((3/4, 5/4), (3/2, 7/4, 9/4), -b^{3/2}x^{4/4})/(8\Gamma(7/4)\Gamma(9/4)) - \sqrt{b}x^{3/2}\sqrt{1/b}\cos(a)\Gamma(1/4)\Gamma(3/4)\text{hyper}((1/4, 3/4), (1/2, 5/4, 7/4), -b^{3/2}x^{4/4})/(8\Gamma(5/4)\Gamma(7/4)) - \sqrt{2}\sqrt{\pi}x^{2/2}\sqrt{1/b}\sin(a)\text{fresnelc}(\sqrt{2}\sqrt{b}x/\sqrt{\pi})/2 + \sqrt{2}\sqrt{\pi}x^{2/2}\sqrt{1/b}\cos(a)\text{fresnelc}(\sqrt{2}\sqrt{b}x/\sqrt{\pi})/2$

**Giac** [C] Result contains complex when optimal does not.

time = 0.43, size = 135, normalized size = 1.48

$$-\frac{ixe^{(ibx^2+ia)}}{4b} + \frac{ixe^{(-ibx^2-ia)}}{4b} - \frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(ia)}}{8b\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}} + \frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(-ia)}}{8b\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(b*x^2+a),x, algorithm="giac")`

[Out]  $-1/4Ix^2e^{(Ibx^2 + Ia)}/b + 1/4Ix^2e^{(-Ibx^2 - Ia)}/b - 1/8I\sqrt{2}\sqrt{\pi}\operatorname{erf}(-1/2\sqrt{2}x*(-Ib/abs(b) + 1)\sqrt{abs(b)})e^{(Ia)}/(b*(-Ib/abs(b) + 1)\sqrt{abs(b)}) + 1/8I\sqrt{2}\sqrt{\pi}\operatorname{erf}(-1/2\sqrt{2}x*(Ib/abs(b) + 1)\sqrt{abs(b)})e^{(-Ia)}/(b*(Ib/abs(b) + 1)\sqrt{abs(b)})$

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cos(bx^2 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(a + b*x^2),x)`

[Out] `int(x^2*cos(a + b*x^2), x)`

### 3.3 $\int x \cos(a + bx^2) dx$

Optimal. Leaf size=15

$$\frac{\sin(a + bx^2)}{2b}$$

[Out] 1/2\*sin(b\*x^2+a)/b

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3461, 2717}

$$\frac{\sin(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x\*Cos[a + b\*x^2],x]

[Out] Sin[a + b\*x^2]/(2\*b)

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p,
x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
&& (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \int x \cos(a + bx^2) dx &= \frac{1}{2} \text{Subst} \left( \int \cos(a + bx) dx, x, x^2 \right) \\ &= \frac{\sin(a + bx^2)}{2b} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sin(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Cos[a + b\*x^2],x]

[Out] Sin[a + b\*x^2]/(2\*b)

**Maple [A]**

time = 0.04, size = 14, normalized size = 0.93

method	result	size
derivativdivides	$\frac{\sin(bx^2+a)}{2b}$	14
default	$\frac{\sin(bx^2+a)}{2b}$	14
risch	$\frac{\sin(bx^2+a)}{2b}$	14
norman	$\frac{\tan\left(\frac{a}{2} + \frac{bx^2}{2}\right)}{b\left(1 + \tan^2\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)}$	33
meijerg	$\frac{\cos(a)\sin(bx^2)}{2b} - \frac{\sin(a)\sqrt{\pi}}{2b} \left( \frac{1}{\sqrt{\pi}} - \frac{\cos(bx^2)}{\sqrt{\pi}} \right)$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cos(b\*x^2+a),x,method=\_RETURNVERBOSE)

[Out] 1/2\*sin(b\*x^2+a)/b

**Maxima [A]**

time = 0.28, size = 13, normalized size = 0.87

$$\frac{\sin(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(b\*x^2+a),x, algorithm="maxima")

[Out] 1/2\*sin(b\*x^2 + a)/b

**Fricas [A]**

time = 0.37, size = 13, normalized size = 0.87

$$\frac{\sin(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(b\*x^2+a),x, algorithm="fricas")

[Out] 1/2\*sin(b\*x^2 + a)/b

**Sympy [A]**

time = 0.07, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\sin(a+bx^2)}{2b} & \text{for } b \neq 0 \\ \frac{x^2 \cos(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*cos(b*x**2+a),x)``[Out] Piecewise((sin(a + b*x**2)/(2*b), Ne(b, 0)), (x**2*cos(a)/2, True))`**Giac [A]**

time = 0.44, size = 13, normalized size = 0.87

$$\frac{\sin(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*cos(b*x^2+a),x, algorithm="giac")``[Out] 1/2*sin(b*x^2 + a)/b`**Mupad [B]**

time = 0.06, size = 13, normalized size = 0.87

$$\frac{\sin(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*cos(a + b*x^2),x)``[Out] sin(a + b*x^2)/(2*b)`

### 3.4 $\int \cos(a + bx^2) dx$

**Optimal.** Leaf size=70

$$\frac{\sqrt{\frac{\pi}{2}} \cos(a) \text{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a)}{\sqrt{b}}$$

[Out]  $1/2*\cos(a)*\text{FresnelC}(x*b^{(1/2)}*2^{(1/2)}/\text{Pi}^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(1/2)}-1/2*\text{FresnelS}(x*b^{(1/2)}*2^{(1/2)}/\text{Pi}^{(1/2)})*\sin(a)*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(1/2)}$

**Rubi [A]**

time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {3435, 3433, 3432}

$$\frac{\sqrt{\frac{\pi}{2}} \cos(a) \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{b} x\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}} \sin(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] `Int[Cos[a + b*x^2], x]`

[Out]  $(\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a]*\text{FresnelC}[\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*x])/\text{Sqrt}[b] - (\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*x]*\text{Sin}[a])/\text{Sqrt}[b]$

Rule 3432

`Int[Sin[(d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]`

Rule 3433

`Int[Cos[(d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]`

Rule 3435

`Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Dist[Cos[c], Int[Cos[d*(e + f*x)2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)2], x], x] /; FreeQ[{c, d, e, f}, x]`

Rubi steps

$$\begin{aligned} \int \cos(a + bx^2) dx &= \cos(a) \int \cos(bx^2) dx - \sin(a) \int \sin(bx^2) dx \\ &= \frac{\sqrt{\frac{\pi}{2}} \cos(a) C\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a)}{\sqrt{b}} \end{aligned}$$

**Mathematica [A]**

time = 0.06, size = 57, normalized size = 0.81

$$\frac{\sqrt{\frac{\pi}{2}} \left( \cos(a) \text{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) - S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a) \right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*x^2], x]``[Out] (Sqrt[Pi/2]*(Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] - FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]))/Sqrt[b]`**Maple [A]**

time = 0.05, size = 44, normalized size = 0.63

method	result	size
default	$\frac{\sqrt{2} \sqrt{\pi} \left( \cos(a) \text{FresnelC}\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) - \sin(a) S\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) \right)}{2\sqrt{b}}$	44
meijerg	$\frac{\cos(a) \text{FresnelC}\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) \sqrt{2} \sqrt{\pi}}{2\sqrt{b}} - \frac{S\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) \sin(a) \sqrt{2} \sqrt{\pi}}{2\sqrt{b}}$	52
risch	$\frac{e^{-ia} \sqrt{\pi} \operatorname{erf}\left(\sqrt{ib} x\right)}{4\sqrt{ib}} + \frac{e^{ia} \sqrt{\pi} \operatorname{erf}\left(\sqrt{-ib} x\right)}{4\sqrt{-ib}}$	52

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(b*x^2+a), x, method=_RETURNVERBOSE)``[Out] 1/2*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2)))`



**Maxima** [C] Result contains complex when optimal does not.

time = 0.29, size = 48, normalized size = 0.69

$$\frac{\sqrt{2} \sqrt{\pi} \left( ((i-1) \cos(a) + (i+1) \sin(a)) \operatorname{erf}(\sqrt{i b} x) + (-(i+1) \cos(a) - (i-1) \sin(a)) \operatorname{erf}(\sqrt{-i b} x) \right)}{8 \sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a),x, algorithm="maxima")

[Out] -1/8\*sqrt(2)\*sqrt(pi)\*(((I - 1)\*cos(a) + (I + 1)\*sin(a))\*erf(sqrt(I\*b)\*x) +  
(-(I + 1)\*cos(a) - (I - 1)\*sin(a))\*erf(sqrt(-I\*b)\*x))/sqrt(b)

**Fricas** [A]

time = 0.40, size = 61, normalized size = 0.87

$$\frac{\sqrt{2} \pi \sqrt{\frac{b}{\pi}} \cos(a) C\left(\sqrt{2} x \sqrt{\frac{b}{\pi}}\right) - \sqrt{2} \pi \sqrt{\frac{b}{\pi}} S\left(\sqrt{2} x \sqrt{\frac{b}{\pi}}\right) \sin(a)}{2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a),x, algorithm="fricas")

[Out] 1/2\*(sqrt(2)\*pi\*sqrt(b/pi)\*cos(a)\*fresnel\_cos(sqrt(2)\*x\*sqrt(b/pi)) - sqrt(2)\*pi\*sqrt(b/pi)\*fresnel\_sin(sqrt(2)\*x\*sqrt(b/pi))\*sin(a))/b

**Sympy** [A]

time = 0.24, size = 61, normalized size = 0.87

$$\frac{\sqrt{2} \sqrt{\pi} \left( -\sin(a) S\left(\frac{\sqrt{2} \sqrt{b} x}{\sqrt{\pi}}\right) + \cos(a) C\left(\frac{\sqrt{2} \sqrt{b} x}{\sqrt{\pi}}\right) \right) \sqrt{\frac{1}{b}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x\*\*2+a),x)

[Out] sqrt(2)\*sqrt(pi)\*(-sin(a)\*fresnels(sqrt(2)\*sqrt(b)\*x/sqrt(pi)) + cos(a)\*fresnelc(sqrt(2)\*sqrt(b)\*x/sqrt(pi)))\*sqrt(1/b)/2

**Giac** [C] Result contains complex when optimal does not.

time = 0.46, size = 95, normalized size = 1.36

$$\frac{\sqrt{2} \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{2} x \left(-\frac{i b}{|b|} + 1\right) \sqrt{|b|}\right) e^{(i a)}}{4 \left(-\frac{i b}{|b|} + 1\right) \sqrt{|b|}} - \frac{\sqrt{2} \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{2} x \left(\frac{i b}{|b|} + 1\right) \sqrt{|b|}\right) e^{(-i a)}}{4 \left(\frac{i b}{|b|} + 1\right) \sqrt{|b|}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a),x, algorithm="giac")

[Out]  $-1/4*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}(-1/2*\sqrt{2}*x*(-I*b/\operatorname{abs}(b) + 1)*\sqrt{\operatorname{abs}(b)})*e^{(I*a)/((-I*b/\operatorname{abs}(b) + 1)*\sqrt{\operatorname{abs}(b)})} - 1/4*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}(-1/2*\sqrt{2}*x*(I*b/\operatorname{abs}(b) + 1)*\sqrt{\operatorname{abs}(b)})*e^{(-I*a)/((I*b/\operatorname{abs}(b) + 1)*\sqrt{\operatorname{abs}(b)})}$

**Mupad [B]**

time = 0.36, size = 51, normalized size = 0.73

$$\frac{\sqrt{2} \sqrt{\pi} C\left(\frac{\sqrt{2} \sqrt{b} x}{\sqrt{\pi}}\right) \cos(a)}{2 \sqrt{b}} - \frac{\sqrt{2} \sqrt{\pi} S\left(\frac{\sqrt{2} \sqrt{b} x}{\sqrt{\pi}}\right) \sin(a)}{2 \sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^2),x)

[Out]  $(2^{(1/2)}*\pi^{(1/2)}*\operatorname{fresnelc}((2^{(1/2)}*b^{(1/2)}*x)/\pi^{(1/2)})*\cos(a))/(2*b^{(1/2)}) - (2^{(1/2)}*\pi^{(1/2)}*\operatorname{fresnels}((2^{(1/2)}*b^{(1/2)}*x)/\pi^{(1/2)})*\sin(a))/(2*b^{(1/2)})$

### 3.5 $\int \frac{\cos(a+bx^2)}{x} dx$

Optimal. Leaf size=25

$$\frac{1}{2} \cos(a) \text{CosIntegral}(bx^2) - \frac{1}{2} \sin(a) \text{Si}(bx^2)$$

[Out]  $1/2 * \text{Ci}(b * x^2) * \cos(a) - 1/2 * \text{Si}(b * x^2) * \sin(a)$

Rubi [A]

time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3459, 3457, 3456}

$$\frac{1}{2} \cos(a) \text{CosIntegral}(bx^2) - \frac{1}{2} \sin(a) \text{Si}(bx^2)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[a + b * x^2] / x, x]$

[Out]  $(\text{Cos}[a] * \text{CosIntegral}[b * x^2]) / 2 - (\text{Sin}[a] * \text{SinIntegral}[b * x^2]) / 2$

Rule 3456

$\text{Int}[\text{Sin}[(d_.) * (x_)^(n_)] / (x_), x\_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[d * x^n] / n, x] / ; \text{FreeQ}\{d, n\}, x]$

Rule 3457

$\text{Int}[\text{Cos}[(d_.) * (x_)^(n_)] / (x_), x\_Symbol] \rightarrow \text{Simp}[\text{CosIntegral}[d * x^n] / n, x] / ; \text{FreeQ}\{d, n\}, x]$

Rule 3459

$\text{Int}[\text{Cos}[(c_) + (d_.) * (x_)^(n_)] / (x_), x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[c], \text{Int}[\text{Cos}[d * x^n] / x, x], x] - \text{Dist}[\text{Sin}[c], \text{Int}[\text{Sin}[d * x^n] / x, x], x] / ; \text{FreeQ}\{c, d, n\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx^2)}{x} dx &= \cos(a) \int \frac{\cos(bx^2)}{x} dx - \sin(a) \int \frac{\sin(bx^2)}{x} dx \\ &= \frac{1}{2} \cos(a) \text{Ci}(bx^2) - \frac{1}{2} \sin(a) \text{Si}(bx^2) \end{aligned}$$

**Mathematica [A]**

time = 0.03, size = 24, normalized size = 0.96

$$\frac{1}{2} (\cos(a) \text{CosIntegral}(bx^2) - \sin(a) \text{Si}(bx^2))$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*x^2]/x,x]``[Out] (Cos[a]*CosIntegral[b*x^2] - Sin[a]*SinIntegral[b*x^2])/2`**Maple [A]**

time = 0.08, size = 22, normalized size = 0.88

method	result	size
default	$\frac{\text{cosineIntegral}(bx^2) \cos(a)}{2} - \frac{\text{sinIntegral}(bx^2) \sin(a)}{2}$	22
risch	$\frac{ie^{-ia}\pi \text{csgn}(bx^2)}{4} - \frac{ie^{-ia} \text{sinIntegral}(bx^2)}{2} - \frac{e^{-ia} \text{expIntegral}(1, -ibx^2)}{4} - \frac{e^{ia} \text{expIntegral}(1, -ibx^2)}{4}$	63
meijerg	$\frac{\sqrt{\pi} \cos(a) \left( \frac{2\gamma + 4\ln(x) + \ln(b^2)}{\sqrt{\pi}} - \frac{2\gamma}{\sqrt{\pi}} - \frac{2\ln(2)}{\sqrt{\pi}} - \frac{2\ln\left(\frac{bx^2}{2}\right)}{\sqrt{\pi}} + \frac{2 \text{cosineIntegral}(bx^2)}{\sqrt{\pi}} \right)}{4} - \frac{\text{sinIntegral}(bx^2) \sin(a)}{2}$	72

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(b*x^2+a)/x,x,method=_RETURNVERBOSE)``[Out] 1/2*Ci(b*x^2)*cos(a)-1/2*Si(b*x^2)*sin(a)`**Maxima [C]** Result contains complex when optimal does not.

time = 0.37, size = 43, normalized size = 1.72

$$\frac{1}{4} (\text{Ei}(ibx^2) + \text{Ei}(-ibx^2)) \cos(a) + \frac{1}{4} (i \text{Ei}(ibx^2) - i \text{Ei}(-ibx^2)) \sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(b*x^2+a)/x,x, algorithm="maxima")``[Out] 1/4*(Ei(I*b*x^2) + Ei(-I*b*x^2))*cos(a) + 1/4*(I*Ei(I*b*x^2) - I*Ei(-I*b*x^2))*sin(a)`**Fricas [A]**

time = 0.36, size = 29, normalized size = 1.16

$$\frac{1}{4} (\text{Ci}(bx^2) + \text{Ci}(-bx^2)) \cos(a) - \frac{1}{2} \sin(a) \text{Si}(bx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(b*x^2+a)/x,x, algorithm="fricas")`

[Out]  $\frac{1}{4}(\cos\_integral(b*x^2) + \cos\_integral(-b*x^2))*\cos(a) - \frac{1}{2}\sin(a)*\sin\_integral(b*x^2)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + bx^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x**2+a)/x,x)`

[Out] `Integral(cos(a + b*x**2)/x, x)`

**Giac** [A]

time = 0.42, size = 21, normalized size = 0.84

$$\frac{1}{2} \cos(a) \operatorname{Ci}(bx^2) - \frac{1}{2} \sin(a) \operatorname{Si}(bx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)/x,x, algorithm="giac")`

[Out] `1/2*cos(a)*cos_integral(b*x^2) - 1/2*sin(a)*sin_integral(b*x^2)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.04

$$\frac{\cos(a) \operatorname{cosint}(bx^2)}{2} - \frac{\sin(a) \operatorname{sinint}(bx^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x^2)/x,x)`

[Out] `(cos(a)*cosint(b*x^2))/2 - (sin(a)*sinint(b*x^2))/2`

### 3.6 $\int \frac{\cos(a+bx^2)}{x^2} dx$

**Optimal.** Leaf size=80

$$-\frac{\cos(a+bx^2)}{x} - \sqrt{b} \sqrt{2\pi} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) - \sqrt{b} \sqrt{2\pi} \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a)$$

[Out]  $-\cos(b*x^2+a)/x - \cos(a)*\operatorname{FresnelS}(x*b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)})*b^{(1/2)}*2^{(1/2)}$   
 $*\pi^{(1/2)} - \operatorname{FresnelC}(x*b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)})*\sin(a)*b^{(1/2)}*2^{(1/2)}*\pi^{(1/2)}$

**Rubi [A]**

time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3469, 3434, 3433, 3432}

$$-\sqrt{2\pi} \sqrt{b} \sin(a) \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{b} x\right) - \sqrt{2\pi} \sqrt{b} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) - \frac{\cos(a+bx^2)}{x}$$

Antiderivative was successfully verified.

[In] `Int[Cos[a + b*x^2]/x^2, x]`

[Out] `-(Cos[a + b*x^2]/x) - Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[b]*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]`

**Rule 3432**

`Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]`

**Rule 3433**

`Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]`

**Rule 3434**

`Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(2)], x_Symbol] := Dist[Sin[c], Int[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /; FreeQ[{c, d, e, f}, x]`

**Rule 3469**

`Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] := Simp[(e*x)^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(`

`e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]`

Rubi steps

$$\begin{aligned} \int \frac{\cos(a + bx^2)}{x^2} dx &= -\frac{\cos(a + bx^2)}{x} - (2b) \int \sin(a + bx^2) dx \\ &= -\frac{\cos(a + bx^2)}{x} - (2b \cos(a)) \int \sin(bx^2) dx - (2b \sin(a)) \int \cos(bx^2) dx \\ &= -\frac{\cos(a + bx^2)}{x} - \sqrt{b} \sqrt{2\pi} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) - \sqrt{b} \sqrt{2\pi} C\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a) \end{aligned}$$

**Mathematica [A]**

time = 0.12, size = 81, normalized size = 1.01

$$-\frac{\cos(a) \cos(bx^2)}{x} - \sqrt{b} \sqrt{2\pi} \left( \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) + \text{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a) \right) + \frac{\sin(a) \sin(bx^2)}{x}$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[a + b*x^2]/x^2, x]`

[Out] `-((Cos[a]*Cos[b*x^2])/x) - Sqrt[b]*Sqrt[2*Pi]*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]) + (Sin[a]*Sin[b*x^2])/x`

**Maple [A]**

time = 0.07, size = 57, normalized size = 0.71

method	result
default	$-\frac{\cos(bx^2+a)}{x} - \sqrt{b} \sqrt{2} \sqrt{\pi} \left( \cos(a) S\left(\frac{x\sqrt{b} \sqrt{2}}{\sqrt{\pi}}\right) + \sin(a) \text{FresnelC}\left(\frac{x\sqrt{b} \sqrt{2}}{\sqrt{\pi}}\right) \right)$
risch	$-\frac{ie^{-ia}b\sqrt{\pi} \operatorname{erf}(\sqrt{ib} x)}{2\sqrt{ib}} + \frac{ie^{ia}b\sqrt{\pi} \operatorname{erf}(\sqrt{-ib} x)}{2\sqrt{-ib}} - \frac{\cos(bx^2+a)}{x}$
meijerg	$\frac{\sqrt{\pi} \cos(a)(b^2)^{\frac{1}{4}} \sqrt{2} \left( -\frac{4\sqrt{2} \cos(bx^2)}{\sqrt{\pi} x (b^2)^{\frac{1}{4}}} - \frac{8\sqrt{b} s\left(\frac{x\sqrt{b} \sqrt{2}}{\sqrt{\pi}}\right)}{(b^2)^{\frac{1}{4}}} \right)}{8} - \frac{\sqrt{\pi} \sin(a) \sqrt{b} \sqrt{2} \left( -\frac{4\sqrt{2} \sin(bx^2)}{x\sqrt{b} \sqrt{\pi}} + 8 \text{FresnelC}\left(\frac{x\sqrt{b} \sqrt{2}}{\sqrt{\pi}}\right) \right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x^2+a)/x^2, x, method=_RETURNVERBOSE)`

[Out]  $-\cos(bx^2+a)/x - b^{1/2} 2^{1/2} \pi^{1/2} (\cos(a) \text{FresnelS}(x b^{1/2} 2^{1/2} / \pi^{1/2}) + \sin(a) \text{FresnelC}(x b^{1/2} 2^{1/2} / \pi^{1/2}))$

**Maxima [C]** Result contains complex when optimal does not.

time = 0.56, size = 73, normalized size = 0.91

$$\frac{\sqrt{bx^2} \left( \left( -(i+1) \sqrt{2} \Gamma(-\frac{1}{2}, i bx^2) + (i-1) \sqrt{2} \Gamma(-\frac{1}{2}, -i bx^2) \right) \cos(a) + \left( (i-1) \sqrt{2} \Gamma(-\frac{1}{2}, i bx^2) - (i+1) \sqrt{2} \Gamma(-\frac{1}{2}, -i bx^2) \right) \sin(a) \right)}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)/x^2,x, algorithm="maxima")`

[Out]  $1/8 \sqrt{bx^2} \left( (-1) \sqrt{2} \Gamma(-1/2, I b x^2) + (1) \sqrt{2} \Gamma(-1/2, -I b x^2) \right) \cos(a) + \left( (1) \sqrt{2} \Gamma(-1/2, I b x^2) - (1) \sqrt{2} \Gamma(-1/2, -I b x^2) \right) \sin(a) / x$

**Fricas [A]**

time = 0.36, size = 70, normalized size = 0.88

$$\frac{\sqrt{2} \pi x \sqrt{\frac{b}{\pi}} \cos(a) S \left( \sqrt{2} x \sqrt{\frac{b}{\pi}} \right) + \sqrt{2} \pi x \sqrt{\frac{b}{\pi}} C \left( \sqrt{2} x \sqrt{\frac{b}{\pi}} \right) \sin(a) + \cos(bx^2 + a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)/x^2,x, algorithm="fricas")`

[Out]  $-(\sqrt{2} \pi x \sqrt{b/\pi} \cos(a) \text{fresnel\_sin}(\sqrt{2} x \sqrt{b/\pi}) + \sqrt{2} \pi x \sqrt{b/\pi} \text{fresnel\_cos}(\sqrt{2} x \sqrt{b/\pi})) \sin(a) + \cos(bx^2 + a) / x$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + bx^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x**2+a)/x**2,x)`

[Out] `Integral(cos(a + b*x**2)/x**2, x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(cos(b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)/x^2, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(bx^2 + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)/x^2,x)
```

```
[Out] int(cos(a + b*x^2)/x^2, x)
```

### 3.7 $\int \frac{\cos(a+bx^2)}{x^3} dx$

Optimal. Leaf size=42

$$-\frac{\cos(a+bx^2)}{2x^2} - \frac{1}{2}b\text{CosIntegral}(bx^2)\sin(a) - \frac{1}{2}b\cos(a)\text{Si}(bx^2)$$

[Out]  $-1/2*\cos(b*x^2+a)/x^2-1/2*b*\cos(a)*\text{Si}(b*x^2)-1/2*b*\text{Ci}(b*x^2)*\sin(a)$

Rubi [A]

time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {3461, 3378, 3384, 3380, 3383}

$$-\frac{1}{2}b\sin(a)\text{CosIntegral}(bx^2) - \frac{1}{2}b\cos(a)\text{Si}(bx^2) - \frac{\cos(a+bx^2)}{2x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[a + b*x^2]/x^3, x]$

[Out]  $-1/2*\text{Cos}[a + b*x^2]/x^2 - (b*\text{CosIntegral}[b*x^2]*\text{Sin}[a])/2 - (b*\text{Cos}[a]*\text{SinIntegral}[b*x^2])/2$

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c
+ d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

NeQ[d\*e - c\*f, 0]

### Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{\cos(a + bx^2)}{x^3} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{\cos(a + bx)}{x^2} dx, x, x^2 \right) \\ &= -\frac{\cos(a + bx^2)}{2x^2} - \frac{1}{2} b \text{Subst} \left( \int \frac{\sin(a + bx)}{x} dx, x, x^2 \right) \\ &= -\frac{\cos(a + bx^2)}{2x^2} - \frac{1}{2} (b \cos(a)) \text{Subst} \left( \int \frac{\sin(bx)}{x} dx, x, x^2 \right) - \frac{1}{2} (b \sin(a)) \text{Subst} \left( \int \frac{\cos(bx)}{x} dx, x, x^2 \right) \\ &= -\frac{\cos(a + bx^2)}{2x^2} - \frac{1}{2} b \text{Ci}(bx^2) \sin(a) - \frac{1}{2} b \cos(a) \text{Si}(bx^2) \end{aligned}$$

### Mathematica [A]

time = 0.05, size = 42, normalized size = 1.00

$$-\frac{\cos(a + bx^2) + bx^2 \text{CosIntegral}(bx^2) \sin(a) + bx^2 \cos(a) \text{Si}(bx^2)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*x^2]/x^3,x]

[Out] -1/2\*(Cos[a + b\*x^2] + b\*x^2\*CosIntegral[b\*x^2]\*Sin[a] + b\*x^2\*Cos[a]\*SinIntegral[b\*x^2])/x^2

### Maple [A]

time = 0.09, size = 39, normalized size = 0.93

method	result
default	$-\frac{\cos(bx^2+a)}{2x^2} - b \left( \frac{\cos(a) \sin \text{Integral}(bx^2)}{2} + \frac{\sin(a) \cos \text{Integral}(bx^2)}{2} \right)$
risch	$\frac{e^{-ia} \pi \text{csgn}(bx^2)b}{4} - \frac{e^{-ia} \sin \text{Integral}(bx^2)b}{2} + \frac{ie^{-ia} \exp \text{Integral}(1, -ibx^2)b}{4} - \frac{ie^{ia} b \exp \text{Integral}(1, -ibx^2)}{4} - \frac{\cos(bx^2+a)}{2x^2}$

meijerg	$\frac{\cos(a)\sqrt{\pi}\sqrt{b^2}\left(-\frac{4b^2\cos(x^2\sqrt{b^2})}{x^2(b^2)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\sin\text{Integral}(x^2\sqrt{b^2})}{\sqrt{\pi}}\right)}{8}-\frac{\sin(a)\sqrt{\pi}b\left(\frac{4\gamma-4+8\ln(x)+4\ln(b)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}\right)}{8}$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x^2+a)/x^3,x,method=_RETURNVERBOSE)`

[Out]  $-1/2*\cos(b*x^2+a)/x^2-b*(1/2*\cos(a)*\text{Si}(b*x^2)+1/2*\sin(a)*\text{Ci}(b*x^2))$

**Maxima** [C] Result contains complex when optimal does not.

time = 0.37, size = 48, normalized size = 1.14

$$-\frac{1}{4}((i\Gamma(-1,ibx^2)-i\Gamma(-1,-ibx^2))\cos(a)+(\Gamma(-1,ibx^2)+\Gamma(-1,-ibx^2))\sin(a))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)/x^3,x,algorithm="maxima")`

[Out]  $-1/4*((I*\gamma(-1,I*b*x^2)-I*\gamma(-1,-I*b*x^2))*\cos(a)+(\gamma(-1,I*b*x^2)+\gamma(-1,-I*b*x^2))*\sin(a))*b$

**Fricas** [A]

time = 0.37, size = 57, normalized size = 1.36

$$\frac{2bx^2\cos(a)\text{Si}(bx^2)+(bx^2\text{Ci}(bx^2)+bx^2\text{Ci}(-bx^2))\sin(a)+2\cos(bx^2+a)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)/x^3,x,algorithm="fricas")`

[Out]  $-1/4*(2*b*x^2*\cos(a)*\sin\_integral(b*x^2)+(b*x^2*\cos\_integral(b*x^2)+b*x^2*\cos\_integral(-b*x^2))*\sin(a)+2*\cos(b*x^2+a))/x^2$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x**2+a)/x**3,x)`

[Out] `Integral(cos(a + b*x**2)/x**3, x)`

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs.  $2(36) = 72$ .

time = 0.41, size = 87, normalized size = 2.07

$$\frac{(bx^2+a)b^2\text{Ci}(bx^2)\sin(a)-ab^2\text{Ci}(bx^2)\sin(a)+(bx^2+a)b^2\cos(a)\text{Si}(bx^2)-ab^2\cos(a)\text{Si}(bx^2)+b^2\cos(bx^2+a)}{2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)/x^3,x, algorithm="giac")

[Out] 
$$-1/2*((b*x^2 + a)*b^2*\cos\_integral(b*x^2)*\sin(a) - a*b^2*\cos\_integral(b*x^2)*\sin(a) + (b*x^2 + a)*b^2*\cos(a)*\sin\_integral(b*x^2) - a*b^2*\cos(a)*\sin\_integral(b*x^2) + b^2*\cos(b*x^2 + a))/(b^2*x^2)$$

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(bx^2 + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^2)/x^3,x)

[Out] int(cos(a + b\*x^2)/x^3, x)

### 3.8 $\int x^3 \cos^2(a + bx^2) dx$

Optimal. Leaf size=51

$$\frac{x^4}{8} + \frac{\cos^2(a + bx^2)}{8b^2} + \frac{x^2 \cos(a + bx^2) \sin(a + bx^2)}{4b}$$

[Out]  $1/8*x^4+1/8*\cos(b*x^2+a)^2/b^2+1/4*x^2*\cos(b*x^2+a)*\sin(b*x^2+a)/b$

Rubi [A]

time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3461, 3391, 30}

$$\frac{\cos^2(a + bx^2)}{8b^2} + \frac{x^2 \sin(a + bx^2) \cos(a + bx^2)}{4b} + \frac{x^4}{8}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Cos[a + b\*x^2]^2,x]

[Out]  $x^4/8 + \text{Cos}[a + b*x^2]^2/(8*b^2) + (x^2*\text{Cos}[a + b*x^2]*\text{Sin}[a + b*x^2])/(4*b)$

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3391

Int[((c\_) + (d\_)\*(x\_))\*((b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[d\*((b\*Sine[e + f\*x])^n/(f^2\*n^2)), x] + (Dist[b^2\*((n - 1)/n), Int[(c + d\*x)\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Simp[b\*(c + d\*x)\*Cos[e + f\*x]\*((b\*Sine[e + f\*x])^(n - 1)/(f\*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 3461

Int[((a\_) + Cos[(c\_) + (d\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x^3 \cos^2(a + bx^2) dx &= \frac{1}{2} \text{Subst}\left(\int x \cos^2(a + bx) dx, x, x^2\right) \\ &= \frac{\cos^2(a + bx^2)}{8b^2} + \frac{x^2 \cos(a + bx^2) \sin(a + bx^2)}{4b} + \frac{1}{4} \text{Subst}\left(\int x dx, x, x^2\right) \\ &= \frac{x^4}{8} + \frac{\cos^2(a + bx^2)}{8b^2} + \frac{x^2 \cos(a + bx^2) \sin(a + bx^2)}{4b} \end{aligned}$$

**Mathematica [A]**

time = 0.08, size = 40, normalized size = 0.78

$$\frac{\cos(2(a + bx^2)) + 2bx^2(bx^2 + \sin(2(a + bx^2)))}{16b^2}$$

Antiderivative was successfully verified.

`[In] Integrate[x^3*Cos[a + b*x^2]^2,x]``[Out] (Cos[2*(a + b*x^2)] + 2*b*x^2*(b*x^2 + Sin[2*(a + b*x^2)]))/(16*b^2)`**Maple [A]**

time = 0.08, size = 42, normalized size = 0.82

method	result	size
default	$\frac{x^4}{8} + \frac{x^2 \sin(2bx^2 + 2a)}{8b} + \frac{\cos(2bx^2 + 2a)}{16b^2}$	42
risch	$\frac{x^4}{8} + \frac{x^2 \sin(2bx^2 + 2a)}{8b} + \frac{\cos(2bx^2 + 2a)}{16b^2}$	42
norman	$\frac{x^4 + \frac{x^4 \left(\tan^2\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)}{4} + \frac{x^4 \left(\tan^4\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)}{8} + \frac{x^2 \tan\left(\frac{a}{2} + \frac{bx^2}{2}\right)}{2b} - \frac{x^2 \left(\tan^3\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)}{2b} - \frac{\tan^2\left(\frac{a}{2} + \frac{bx^2}{2}\right)}{2b^2}}{\left(1 + \tan^2\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)^2}$	119

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3*cos(b*x^2+a)^2,x,method=_RETURNVERBOSE)``[Out] 1/8*x^4+1/8/b*x^2*sin(2*b*x^2+2*a)+1/16/b^2*cos(2*b*x^2+2*a)`**Maxima [A]**

time = 0.29, size = 42, normalized size = 0.82

$$\frac{2b^2x^4 + 2bx^2 \sin(2bx^2 + 2a) + \cos(2bx^2 + 2a)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^3*cos(b*x^2+a)^2,x, algorithm="maxima")`

[Out]  $1/16*(2*b^2*x^4 + 2*b*x^2*\sin(2*b*x^2 + 2*a) + \cos(2*b*x^2 + 2*a))/b^2$

**Fricas** [A]

time = 0.37, size = 45, normalized size = 0.88

$$\frac{b^2 x^4 + 2 b x^2 \cos(b x^2 + a) \sin(b x^2 + a) + \cos(b x^2 + a)^2}{8 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(b*x^2+a)^2,x, algorithm="fricas")`

[Out]  $1/8*(b^2*x^4 + 2*b*x^2*\cos(b*x^2 + a)*\sin(b*x^2 + a) + \cos(b*x^2 + a)^2)/b^2$

**Sympy** [A]

time = 0.26, size = 78, normalized size = 1.53

$$\begin{cases} \frac{x^4 \sin^2(a+bx^2)}{8} + \frac{x^4 \cos^2(a+bx^2)}{8} + \frac{x^2 \sin(a+bx^2) \cos(a+bx^2)}{4b} + \frac{\cos^2(a+bx^2)}{8b^2} & \text{for } b \neq 0 \\ \frac{x^4 \cos^2(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*cos(b*x**2+a)**2,x)`

[Out] `Piecewise((x**4*sin(a + b*x**2)**2/8 + x**4*cos(a + b*x**2)**2/8 + x**2*sin(a + b*x**2)*cos(a + b*x**2)/(4*b) + cos(a + b*x**2)**2/(8*b**2), Ne(b, 0)), (x**4*cos(a)**2/4, True))`

**Giac** [A]

time = 0.44, size = 76, normalized size = 1.49

$$-\frac{(2bx^2 + 2a + \sin(2bx^2 + 2a))a}{8b^2} + \frac{2(bx^2 + a)^2 + 2(bx^2 + a)\sin(2bx^2 + 2a) + \cos(2bx^2 + 2a)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(b*x^2+a)^2,x, algorithm="giac")`

[Out]  $-1/8*(2*b*x^2 + 2*a + \sin(2*b*x^2 + 2*a))*a/b^2 + 1/16*(2*(b*x^2 + a)^2 + 2*(b*x^2 + a)*\sin(2*b*x^2 + 2*a) + \cos(2*b*x^2 + 2*a))/b^2$

**Mupad** [B]

time = 0.15, size = 41, normalized size = 0.80

$$\frac{\cos(2bx^2 + 2a)}{16b^2} + \frac{x^4}{8} + \frac{x^2 \sin(2bx^2 + 2a)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cos(a + b*x^2)^2,x)`

[Out]  $\cos(2*a + 2*b*x^2)/(16*b^2) + x^4/8 + (x^2*\sin(2*a + 2*b*x^2))/(8*b)$



### 3.9 $\int x^2 \cos^2(a + bx^2) dx$

**Optimal.** Leaf size=91

$$\frac{x^3}{6} - \frac{\sqrt{\pi} \cos(2a) S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{16b^{3/2}} - \frac{\sqrt{\pi} \operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) \sin(2a)}{16b^{3/2}} + \frac{x \sin(2a + 2bx^2)}{8b}$$

[Out]  $1/6*x^3+1/8*x*\sin(2*b*x^2+2*a)/b-1/16*\cos(2*a)*\operatorname{FresnelS}(2*x*b^{(1/2)}/\pi^{(1/2)})*\pi^{(1/2)}/b^{(3/2)}-1/16*\operatorname{FresnelC}(2*x*b^{(1/2)}/\pi^{(1/2)})*\sin(2*a)*\pi^{(1/2)}/b^{(3/2)}$

**Rubi [A]**

time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {3485, 3467, 3434, 3433, 3432}

$$-\frac{\sqrt{\pi} \sin(2a) \operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{16b^{3/2}} - \frac{\sqrt{\pi} \cos(2a) S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{16b^{3/2}} + \frac{x \sin(2a + 2bx^2)}{8b} + \frac{x^3}{6}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*\operatorname{Cos}[a + b*x^2]^2, x]$

[Out]  $x^3/6 - (\operatorname{Sqrt}[\pi]*\operatorname{Cos}[2*a]*\operatorname{FresnelS}[(2*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[\pi]])/(16*b^{(3/2)}) - (\operatorname{Sqrt}[\pi]*\operatorname{FresnelC}[(2*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[\pi]]*\operatorname{Sin}[2*a])/(16*b^{(3/2)}) + (x*\operatorname{Sin}[2*a + 2*b*x^2])/(8*b)$

Rule 3432

$\operatorname{Int}[\operatorname{Sin}[(d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[\pi/2]/(f*\operatorname{Rt}[d, 2]))*\operatorname{FresnelS}[\operatorname{Sqrt}[2/\pi]*\operatorname{Rt}[d, 2]*(e + f*x)], x] /; \operatorname{FreeQ}\{d, e, f, x\}$

Rule 3433

$\operatorname{Int}[\operatorname{Cos}[(d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[\pi/2]/(f*\operatorname{Rt}[d, 2]))*\operatorname{FresnelC}[\operatorname{Sqrt}[2/\pi]*\operatorname{Rt}[d, 2]*(e + f*x)], x] /; \operatorname{FreeQ}\{d, e, f, x\}$

Rule 3434

$\operatorname{Int}[\operatorname{Sin}[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Sin}[c], \operatorname{Int}[\operatorname{Cos}[d*(e + f*x)^2], x], x] + \operatorname{Dist}[\operatorname{Cos}[c], \operatorname{Int}[\operatorname{Sin}[d*(e + f*x)^2], x], x] /; \operatorname{FreeQ}\{c, d, e, f, x\}$

Rule 3467

$\operatorname{Int}[\operatorname{Cos}[(c_.) + (d_.)*(x_)^{(n_)}]*((e_.)*(x_))^{(m_)}], x\_Symbol] \rightarrow \operatorname{Simp}[e^{(n-1)}*(e*x)^{(m-n+1)}*(\operatorname{Sin}[c + d*x^n]/(d*n)), x] - \operatorname{Dist}[e^n*((m-n+1)/$

(d\*n)), Int[(e\*x)^(m - n)\*Sin[c + d\*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

### Rule 3485

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandTrigReduce[(e\*x)^m, (a + b\*Cos[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]

### Rubi steps

$$\begin{aligned}
 \int x^2 \cos^2(a + bx^2) dx &= \int \left( \frac{x^2}{2} + \frac{1}{2}x^2 \cos(2a + 2bx^2) \right) dx \\
 &= \frac{x^3}{6} + \frac{1}{2} \int x^2 \cos(2a + 2bx^2) dx \\
 &= \frac{x^3}{6} + \frac{x \sin(2a + 2bx^2)}{8b} - \frac{\int \sin(2a + 2bx^2) dx}{8b} \\
 &= \frac{x^3}{6} + \frac{x \sin(2a + 2bx^2)}{8b} - \frac{\cos(2a) \int \sin(2bx^2) dx}{8b} - \frac{\sin(2a) \int \cos(2bx^2) dx}{8b} \\
 &= \frac{x^3}{6} - \frac{\sqrt{\pi} \cos(2a) S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{16b^{3/2}} - \frac{\sqrt{\pi} C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) \sin(2a)}{16b^{3/2}} + \frac{x \sin(2a + 2bx^2)}{8b}
 \end{aligned}$$

### Mathematica [A]

time = 0.11, size = 87, normalized size = 0.96

$$\frac{-3\sqrt{\pi} \cos(2a) S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) - 3\sqrt{\pi} \operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) \sin(2a) + 2\sqrt{b}x(4bx^2 + 3\sin(2(a + bx^2)))}{48b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Cos[a + b\*x^2]^2,x]

[Out] (-3\*Sqrt[Pi]\*Cos[2\*a]\*FresnelS[(2\*Sqrt[b]\*x)/Sqrt[Pi]] - 3\*Sqrt[Pi]\*FresnelC[(2\*Sqrt[b]\*x)/Sqrt[Pi]]\*Sin[2\*a] + 2\*Sqrt[b]\*x\*(4\*b\*x^2 + 3\*Sin[2\*(a + b\*x^2)]))/(48\*b^(3/2))

### Maple [A]

time = 0.12, size = 63, normalized size = 0.69

method	result	size
--------	--------	------

default	$\frac{x^3}{6} + \frac{x \sin(2bx^2+2a)}{8b} - \frac{\sqrt{\pi} \left( \cos(2a) S\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right) + \sin(2a) \operatorname{FresnelC}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right) \right)}{16b^{\frac{3}{2}}}$	63
risch	$\frac{x^3}{6} - \frac{ie^{-2ia}\sqrt{\pi}\sqrt{2}\operatorname{erf}\left(\sqrt{2}\sqrt{ib}x\right)}{64b\sqrt{ib}} + \frac{ie^{2ia}\sqrt{\pi}\operatorname{erf}\left(\sqrt{-2ib}x\right)}{32b\sqrt{-2ib}} + \frac{x \sin(2bx^2+2a)}{8b}$	88

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(b*x^2+a)^2,x,method=_RETURNVERBOSE)`

[Out] `1/6*x^3+1/8*x*sin(2*b*x^2+2*a)/b-1/16/b^(3/2)*Pi^(1/2)*(cos(2*a)*FresnelS(2*x*b^(1/2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x*b^(1/2)/Pi^(1/2)))`

**Maxima** [C] Result contains complex when optimal does not.

time = 0.52, size = 90, normalized size = 0.99

$$\frac{64b^3x^3 + 48b^2x \sin(2bx^2 + 2a) - 3 \cdot 4^{\frac{1}{2}} \sqrt{2} \sqrt{\pi} \left( (i+1) \cos(2a) - (i-1) \sin(2a) \right) \operatorname{erf}\left(\sqrt{2ib}x\right) + \left( -(i-1) \cos(2a) + (i+1) \sin(2a) \right) \operatorname{erf}\left(\sqrt{-2ib}x\right)}{384b^3} b^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(b*x^2+a)^2,x, algorithm="maxima")`

[Out] `1/384*(64*b^3*x^3 + 48*b^2*x*sin(2*b*x^2 + 2*a) - 3*4^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*cos(2*a) - (I - 1)*sin(2*a))*erf(sqrt(2*I*b)*x) + (-(I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf(sqrt(-2*I*b)*x))*b^(3/2))/b^3`

**Fricas** [A]

time = 0.39, size = 84, normalized size = 0.92

$$\frac{8b^2x^3 + 12bx \cos(bx^2 + a) \sin(bx^2 + a) - 3\pi \sqrt{\frac{b}{\pi}} \cos(2a) S\left(2x\sqrt{\frac{b}{\pi}}\right) - 3\pi \sqrt{\frac{b}{\pi}} C\left(2x\sqrt{\frac{b}{\pi}}\right) \sin(2a)}{48b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(b*x^2+a)^2,x, algorithm="fricas")`

[Out] `1/48*(8*b^2*x^3 + 12*b*x*cos(b*x^2 + a)*sin(b*x^2 + a) - 3*pi*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x*sqrt(b/pi)) - 3*pi*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b/pi))*sin(2*a))/b^2`

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(85) = 170.

time = 1.61, size = 201, normalized size = 2.21

$$\frac{b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\sin(2a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\frac{3}{4},\frac{5}{4}\middle|\frac{3}{2},\frac{7}{4},\frac{9}{4}\right)-b^2x^4}{8\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{9}{4}\right)} - \frac{\sqrt{b}x^3\sqrt{\frac{1}{b}}\cos(2a)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\frac{1}{4},\frac{3}{4}\middle|\frac{1}{2},\frac{5}{4},\frac{7}{4}\right)-b^2x^4}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)} + \frac{x^3}{6} - \frac{\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4} + \frac{\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos(2a)C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cos(b\*x\*\*2+a)\*\*2,x)

[Out]  $b^{3/2} x^5 \sqrt{1/b} \sin(2a) \gamma(3/4) \gamma(5/4) \text{hyper}((3/4, 5/4), (3/2, 7/4, 9/4), -b^{3/2} x^4) / (8 \gamma(7/4) \gamma(9/4)) - \sqrt{b} x^3 \sqrt{1/b} \cos(2a) \gamma(1/4) \gamma(3/4) \text{hyper}((1/4, 3/4), (1/2, 5/4, 7/4), -b^{3/2} x^4) / (16 \gamma(5/4) \gamma(7/4)) + x^3/6 - \sqrt{\pi} x^2 \sqrt{1/b} \sin(2a) \text{fresnels}(2\sqrt{b} x / \sqrt{\pi}) / 4 + \sqrt{\pi} x^2 \sqrt{1/b} \cos(2a) \text{fresnelc}(2\sqrt{b} x / \sqrt{\pi}) / 4$

**Giac** [C] Result contains complex when optimal does not.  
time = 0.49, size = 118, normalized size = 1.30

$$\frac{1}{6} x^3 - \frac{i x e^{(2i b x^2 + 2i a)}}{16 b} + \frac{i x e^{(-2i b x^2 - 2i a)}}{16 b} - \frac{i \sqrt{\pi} \operatorname{erf}\left(-\sqrt{b} x \left(-\frac{i b}{|b|} + 1\right)\right) e^{(2i a)}}{32 b^{3/2} \left(-\frac{i b}{|b|} + 1\right)} + \frac{i \sqrt{\pi} \operatorname{erf}\left(-\sqrt{b} x \left(\frac{i b}{|b|} + 1\right)\right) e^{(-2i a)}}{32 b^{3/2} \left(\frac{i b}{|b|} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(b\*x^2+a)^2,x, algorithm="giac")

[Out]  $1/6 x^3 - 1/16 I x e^{(2 I b x^2 + 2 I a) / b} + 1/16 I x e^{(-2 I b x^2 - 2 I a) / b} - 1/32 I \sqrt{\pi} \operatorname{erf}(-\sqrt{b} x (-I b / \operatorname{abs}(b) + 1)) e^{(2 I a) / (b^{3/2} (-I b / \operatorname{abs}(b) + 1))} + 1/32 I \sqrt{\pi} \operatorname{erf}(-\sqrt{b} x (I b / \operatorname{abs}(b) + 1)) e^{(-2 I a) / (b^{3/2} (I b / \operatorname{abs}(b) + 1))}$

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cos(b x^2 + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cos(a + b\*x^2)^2,x)

[Out] int(x^2\*cos(a + b\*x^2)^2, x)

### 3.10 $\int x \cos^2(a + bx^2) dx$

Optimal. Leaf size=31

$$\frac{x^2}{4} + \frac{\cos(a + bx^2) \sin(a + bx^2)}{4b}$$

[Out] 1/4\*x^2+1/4\*cos(b\*x^2+a)\*sin(b\*x^2+a)/b

Rubi [A]

time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3461, 2715, 8}

$$\frac{\sin(a + bx^2) \cos(a + bx^2)}{4b} + \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x\*Cos[a + b\*x^2]^2,x]

[Out] x^2/4 + (Cos[a + b\*x^2]\*Sin[a + b\*x^2])/(4\*b)

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(-b)\*Cos[c + d\*x]\*((b\*Sin[c + d\*x])^(n - 1)/(d\*n)), x] + Dist[b^2\*((n - 1)/n), Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

Rule 3461

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x \cos^2(a + bx^2) dx &= \frac{1}{2} \text{Subst} \left( \int \cos^2(a + bx) dx, x, x^2 \right) \\ &= \frac{\cos(a + bx^2) \sin(a + bx^2)}{4b} + \frac{1}{4} \text{Subst} \left( \int 1 dx, x, x^2 \right) \\ &= \frac{x^2}{4} + \frac{\cos(a + bx^2) \sin(a + bx^2)}{4b} \end{aligned}$$

**Mathematica [A]**

time = 0.03, size = 27, normalized size = 0.87

$$\frac{2(a + bx^2) + \sin(2(a + bx^2))}{8b}$$

Antiderivative was successfully verified.

`[In] Integrate[x*Cos[a + b*x^2]^2,x]``[Out] (2*(a + b*x^2) + Sin[2*(a + b*x^2)])/(8*b)`**Maple [A]**

time = 0.05, size = 34, normalized size = 1.10

method	result	size
risch	$\frac{x^2}{4} + \frac{\sin(2bx^2+2a)}{8b}$	23
derivativedivides	$\frac{\frac{\cos(bx^2+a)\sin(bx^2+a)}{2} + \frac{bx^2}{2} + \frac{a}{2}}{2b}$	34
default	$\frac{\frac{\cos(bx^2+a)\sin(bx^2+a)}{2} + \frac{bx^2}{2} + \frac{a}{2}}{2b}$	34
norman	$\frac{\frac{x^2}{4} + \frac{\tan\left(\frac{a}{2} + \frac{bx^2}{2}\right)}{2b} - \frac{\tan^3\left(\frac{a}{2} + \frac{bx^2}{2}\right)}{2b} + \frac{x^2\left(\tan^2\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)}{2} + \frac{x^2\left(\tan^4\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)}{4}}{\left(1 + \tan^2\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)^2}$	95

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*cos(b*x^2+a)^2,x,method=_RETURNVERBOSE)``[Out] 1/2/b*(1/2*cos(b*x^2+a)*sin(b*x^2+a)+1/2*b*x^2+1/2*a)`**Maxima [A]**

time = 0.29, size = 23, normalized size = 0.74

$$\frac{2bx^2 + \sin(2bx^2 + 2a)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(b*x^2+a)^2,x, algorithm="maxima")`

[Out]  $1/8*(2*b*x^2 + \sin(2*b*x^2 + 2*a))/b$

**Fricas** [A]

time = 0.39, size = 28, normalized size = 0.90

$$\frac{bx^2 + \cos(bx^2 + a) \sin(bx^2 + a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(b*x^2+a)^2,x, algorithm="fricas")`

[Out]  $1/4*(b*x^2 + \cos(b*x^2 + a)*\sin(b*x^2 + a))/b$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs.  $2(24) = 48$ .

time = 0.12, size = 60, normalized size = 1.94

$$\begin{cases} \frac{x^2 \sin^2(a+bx^2)}{4} + \frac{x^2 \cos^2(a+bx^2)}{4} + \frac{\sin(a+bx^2) \cos(a+bx^2)}{4b} & \text{for } b \neq 0 \\ \frac{x^2 \cos^2(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(b*x**2+a)**2,x)`

[Out] `Piecewise((x**2*sin(a + b*x**2)**2/4 + x**2*cos(a + b*x**2)**2/4 + sin(a + b*x**2)*cos(a + b*x**2)/(4*b), Ne(b, 0)), (x**2*cos(a)**2/2, True))`

**Giac** [A]

time = 0.40, size = 26, normalized size = 0.84

$$\frac{2bx^2 + 2a + \sin(2bx^2 + 2a)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(b*x^2+a)^2,x, algorithm="giac")`

[Out]  $1/8*(2*b*x^2 + 2*a + \sin(2*b*x^2 + 2*a))/b$

**Mupad** [B]

time = 0.27, size = 22, normalized size = 0.71

$$\frac{\sin(2bx^2 + 2a)}{8b} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(a + b*x^2)^2,x)`

[Out]  $\sin(2*a + 2*b*x^2)/(8*b) + x^2/4$

### 3.11 $\int \cos^2(a + bx^2) dx$

Optimal. Leaf size=70

$$\frac{x}{2} + \frac{\sqrt{\pi} \cos(2a) \operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}} - \frac{\sqrt{\pi} S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) \sin(2a)}{4\sqrt{b}}$$

[Out] 1/2\*x+1/4\*cos(2\*a)\*FresnelC(2\*x\*b^(1/2)/Pi^(1/2))\*Pi^(1/2)/b^(1/2)-1/4\*FresnelS(2\*x\*b^(1/2)/Pi^(1/2))\*sin(2\*a)\*Pi^(1/2)/b^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {3439, 3435, 3433, 3432}

$$\frac{\sqrt{\pi} \cos(2a) \operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}} - \frac{\sqrt{\pi} \sin(2a) S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}} + \frac{x}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^2]^2,x]

[Out] x/2 + (Sqrt[Pi]\*Cos[2\*a]\*FresnelC[(2\*Sqrt[b]\*x)/Sqrt[Pi]])/(4\*Sqrt[b]) - (Sqrt[Pi]\*FresnelS[(2\*Sqrt[b]\*x)/Sqrt[Pi]]\*Sin[2\*a])/(4\*Sqrt[b])

Rule 3432

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

Rule 3433

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

Rule 3435

Int[Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] := Dist[Cos[c], Int[Cos[d\*(e + f\*x)^2], x], x] - Dist[Sin[c], Int[Sin[d\*(e + f\*x)^2], x], x] /; FreeQ[{c, d, e, f}, x]

Rule 3439

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_))^(n\_)])\*(b\_.))^(p\_), x\_Symbol] := Int[ExpandTrigReduce[(a + b\*Cos[c + d\*(e + f\*x)^n])^p, x], x] /; F



```
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
 \int \cos^2(a + bx^2) dx &= \int \left( \frac{1}{2} + \frac{1}{2} \cos(2a + 2bx^2) \right) dx \\
 &= \frac{x}{2} + \frac{1}{2} \int \cos(2a + 2bx^2) dx \\
 &= \frac{x}{2} + \frac{1}{2} \cos(2a) \int \cos(2bx^2) dx - \frac{1}{2} \sin(2a) \int \sin(2bx^2) dx \\
 &= \frac{x}{2} + \frac{\sqrt{\pi} \cos(2a) C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) - \sqrt{\pi} S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) \sin(2a)}{4\sqrt{b}}
 \end{aligned}$$

**Mathematica [A]**

time = 0.04, size = 67, normalized size = 0.96

$$\frac{2\sqrt{b}x + \sqrt{\pi} \cos(2a) \text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) - \sqrt{\pi} S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) \sin(2a)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2, x]
```

```
[Out] (2*Sqrt[b]*x + Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]] - Sqrt[Pi]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])/(4*Sqrt[b])
```

**Maple [A]**

time = 0.07, size = 45, normalized size = 0.64

method	result	size
default	$\frac{x}{2} + \frac{\sqrt{\pi} \left( \cos(2a) \text{FresnelC}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right) - \sin(2a) S\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right) \right)}{4\sqrt{b}}$	45
risch	$\frac{x}{2} + \frac{e^{-2ia} \sqrt{\pi} \sqrt{2} \operatorname{erf}\left(\sqrt{2} \sqrt{ib} x\right)}{16\sqrt{ib}} + \frac{e^{2ia} \sqrt{\pi} \operatorname{erf}\left(\sqrt{-2ib} x\right)}{8\sqrt{-2ib}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^2, x, method=_RETURNVERBOSE)
```

[Out]  $\frac{1}{2}x + \frac{1}{4}\pi^{1/2}/b^{1/2} * (\cos(2a) * \text{FresnelC}(2*x*b^{1/2}/\pi^{1/2}) - \sin(2a) * \text{FresnelS}(2*x*b^{1/2}/\pi^{1/2}))$

**Maxima** [C] Result contains complex when optimal does not.

time = 0.53, size = 70, normalized size = 1.00

$$\frac{4^{1/4} \sqrt{2} \sqrt{\pi} \left( ((i-1) \cos(2a) + (i+1) \sin(2a)) \operatorname{erf}(\sqrt{2ib}x) + (-(i+1) \cos(2a) - (i-1) \sin(2a)) \operatorname{erf}(\sqrt{-2ib}x) \right) b^{3/2} - 16b^2x}{32b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^2,x, algorithm="maxima")`

[Out]  $-\frac{1}{32} * (4^{1/4} * \sqrt{2} * \sqrt{\pi} * (((I-1) * \cos(2a) + (I+1) * \sin(2a)) * \operatorname{erf}(\sqrt{2I*b}*x) + (-(I+1) * \cos(2a) - (I-1) * \sin(2a)) * \operatorname{erf}(\sqrt{-2I*b}*x)) * b^{3/2} - 16 * b^2 * x) / b^2$

**Fricas** [A]

time = 0.37, size = 59, normalized size = 0.84

$$\frac{\pi \sqrt{\frac{b}{\pi}} \cos(2a) C\left(2x \sqrt{\frac{b}{\pi}}\right) - \pi \sqrt{\frac{b}{\pi}} S\left(2x \sqrt{\frac{b}{\pi}}\right) \sin(2a) + 2bx}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^2,x, algorithm="fricas")`

[Out]  $\frac{1}{4} * (\pi * \sqrt{b/\pi} * \cos(2a) * \text{fresnel\_cos}(2*x*\sqrt{b/\pi}) - \pi * \sqrt{b/\pi} * \text{fresnel\_sin}(2*x*\sqrt{b/\pi}) * \sin(2a) + 2*b*x) / b$

**Sympy** [A]

time = 0.47, size = 56, normalized size = 0.80

$$\frac{x}{2} + \frac{\sqrt{\pi} \left( -\sin(2a) S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) + \cos(2a) C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) \right) \sqrt{\frac{1}{b}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x**2+a)**2,x)`

[Out]  $x/2 + \sqrt{\pi} * (-\sin(2a) * \text{fresnels}(2*\sqrt{b}*x/\sqrt{\pi}) + \cos(2a) * \text{fresnelc}(2*\sqrt{b}*x/\sqrt{\pi})) * \sqrt{1/b} / 4$

**Giac** [C] Result contains complex when optimal does not.

time = 0.41, size = 82, normalized size = 1.17

$$\frac{1}{2}x - \frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{b}x\left(-\frac{ib}{|b|} + 1\right)\right) e^{(2ia)}}{8\sqrt{b}\left(-\frac{ib}{|b|} + 1\right)} - \frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{b}x\left(\frac{ib}{|b|} + 1\right)\right) e^{(-2ia)}}{8\sqrt{b}\left(\frac{ib}{|b|} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/8*sqrt(pi)*erf(-sqrt(b)*x*(-I*b/abs(b) + 1))*e^(2*I*a)/(sqrt(b)*(-I*b/abs(b) + 1)) - 1/8*sqrt(pi)*erf(-sqrt(b)*x*(I*b/abs(b) + 1))*e^(-2*I*a)/(sqrt(b)*(I*b/abs(b) + 1))
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(bx^2 + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^2,x)
```

```
[Out] int(cos(a + b*x^2)^2, x)
```

### 3.12 $\int \frac{\cos^2(a+bx^2)}{x} dx$

Optimal. Leaf size=37

$$\frac{1}{4} \cos(2a) \text{CosIntegral}(2bx^2) + \frac{\log(x)}{2} - \frac{1}{4} \sin(2a) \text{Si}(2bx^2)$$

[Out] 1/4\*Ci(2\*b\*x^2)\*cos(2\*a)+1/2\*ln(x)-1/4\*Si(2\*b\*x^2)\*sin(2\*a)

Rubi [A]

time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3485, 3459, 3457, 3456}

$$\frac{1}{4} \cos(2a) \text{CosIntegral}(2bx^2) - \frac{1}{4} \sin(2a) \text{Si}(2bx^2) + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^2]^2/x,x]

[Out] (Cos[2\*a]\*CosIntegral[2\*b\*x^2])/4 + Log[x]/2 - (Sin[2\*a]\*SinIntegral[2\*b\*x^2])/4

Rule 3456

Int[Sin[(d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] := Simp[SinIntegral[d\*x^n]/n, x] / ; FreeQ[{d, n}, x]

Rule 3457

Int[Cos[(d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] := Simp[CosIntegral[d\*x^n]/n, x] / ; FreeQ[{d, n}, x]

Rule 3459

Int[Cos[(c\_) + (d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] := Dist[Cos[c], Int[Cos[d\*x^n]/x, x], x] - Dist[Sin[c], Int[Sin[d\*x^n]/x, x], x] /; FreeQ[{c, d, n}, x]

Rule 3485

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)]\*(b\_.))^p]\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandTrigReduce[(e\*x)^m, (a + b\*Cos[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a + bx^2)}{x} dx &= \int \left( \frac{1}{2x} + \frac{\cos(2a + 2bx^2)}{2x} \right) dx \\
&= \frac{\log(x)}{2} + \frac{1}{2} \int \frac{\cos(2a + 2bx^2)}{x} dx \\
&= \frac{\log(x)}{2} + \frac{1}{2} \cos(2a) \int \frac{\cos(2bx^2)}{x} dx - \frac{1}{2} \sin(2a) \int \frac{\sin(2bx^2)}{x} dx \\
&= \frac{1}{4} \cos(2a) \text{Ci}(2bx^2) + \frac{\log(x)}{2} - \frac{1}{4} \sin(2a) \text{Si}(2bx^2)
\end{aligned}$$

**Mathematica [A]**

time = 0.04, size = 34, normalized size = 0.92

$$\frac{1}{4}(\cos(2a)\text{CosIntegral}(2bx^2) + 2\log(x) - \sin(2a)\text{Si}(2bx^2))$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*x^2]^2/x, x]``[Out] (Cos[2*a]*CosIntegral[2*b*x^2] + 2*Log[x] - Sin[2*a]*SinIntegral[2*b*x^2])/4`**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.13, size = 68, normalized size = 1.84

method	result	size
risch	$\frac{\ln(x)}{2} + \frac{ie^{-2ia}\pi \text{csgn}(bx^2)}{8} - \frac{ie^{-2ia} \text{sinIntegral}(2bx^2)}{4} - \frac{e^{-2ia} \text{expIntegral}(1, -2ibx^2)}{8} - \frac{e^{2ia} \text{expIntegral}(1, -2ibx^2)}{8}$	68

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(b*x^2+a)^2/x, x, method=_RETURNVERBOSE)``[Out] 1/2*ln(x)+1/8*I*exp(-2*I*a)*Pi*csgn(b*x^2)-1/4*I*exp(-2*I*a)*Si(2*b*x^2)-1/8*exp(-2*I*a)*Ei(1, -2*I*b*x^2)-1/8*exp(2*I*a)*Ei(1, -2*I*b*x^2)`**Maxima [C]** Result contains complex when optimal does not.

time = 0.37, size = 51, normalized size = 1.38

$$\frac{1}{8}(\text{Ei}(2i bx^2) + \text{Ei}(-2i bx^2)) \cos(2a) + \frac{1}{8}(i \text{Ei}(2i bx^2) - i \text{Ei}(-2i bx^2)) \sin(2a) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(b*x^2+a)^2/x, x, algorithm="maxima")`

[Out]  $\frac{1}{8}*(\text{Ei}(2*I*b*x^2) + \text{Ei}(-2*I*b*x^2))*\cos(2*a) + \frac{1}{8}*(I*\text{Ei}(2*I*b*x^2) - I*\text{Ei}(-2*I*b*x^2))*\sin(2*a) + \frac{1}{2}*\log(x)$

**Fricas** [A]

time = 0.37, size = 39, normalized size = 1.05

$$\frac{1}{8} (\text{Ci}(2bx^2) + \text{Ci}(-2bx^2)) \cos(2a) - \frac{1}{4} \sin(2a) \text{Si}(2bx^2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^2/x,x, algorithm="fricas")`

[Out]  $\frac{1}{8}*(\cos\_integral(2*b*x^2) + \cos\_integral(-2*b*x^2))*\cos(2*a) - \frac{1}{4}*\sin(2*a)*\sin\_integral(2*b*x^2) + \frac{1}{2}*\log(x)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a + bx^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x**2+a)**2/x,x)`

[Out] `Integral(cos(a + b*x**2)**2/x, x)`

**Giac** [A]

time = 0.44, size = 35, normalized size = 0.95

$$\frac{1}{4} \cos(2a) \text{Ci}(2bx^2) + \frac{1}{4} \sin(2a) \text{Si}(-2bx^2) + \frac{1}{4} \log(bx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^2/x,x, algorithm="giac")`

[Out]  $\frac{1}{4}*\cos(2*a)*\cos\_integral(2*b*x^2) + \frac{1}{4}*\sin(2*a)*\sin\_integral(-2*b*x^2) + \frac{1}{4}*\log(b*x^2)$

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(bx^2 + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x^2)^2/x,x)`

[Out] `int(cos(a + b*x^2)^2/x, x)`

### 3.13 $\int \frac{\cos^2(a+bx^2)}{x^2} dx$

**Optimal.** Leaf size=76

$$-\frac{\cos^2(a+bx^2)}{x} - \sqrt{b} \sqrt{\pi} \cos(2a) S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) - \sqrt{b} \sqrt{\pi} \operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) \sin(2a)$$

[Out]  $-\cos(b*x^2+a)^2/x - \cos(2*a)*\operatorname{FresnelS}(2*x*b^{(1/2)}/\pi^{(1/2)})*b^{(1/2)}*\pi^{(1/2)} - \operatorname{FresnelC}(2*x*b^{(1/2)}/\pi^{(1/2)})*\sin(2*a)*b^{(1/2)}*\pi^{(1/2)}$

**Rubi [A]**

time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {3475, 4669, 3454, 3434, 3433, 3432}

$$-\sqrt{\pi} \sqrt{b} \sin(2a) \operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) - \sqrt{\pi} \sqrt{b} \cos(2a) S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) - \frac{\cos^2(a+bx^2)}{x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[a + b*x^2]^2/x^2, x]$

[Out]  $-(\operatorname{Cos}[a + b*x^2]^2/x) - \operatorname{Sqrt}[b]*\operatorname{Sqrt}[\pi]*\operatorname{Cos}[2*a]*\operatorname{FresnelS}[(2*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[\pi]] - \operatorname{Sqrt}[b]*\operatorname{Sqrt}[\pi]*\operatorname{FresnelC}[(2*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[\pi]]*\operatorname{Sin}[2*a]$

Rule 3432

$\operatorname{Int}[\operatorname{Sin}[(d_.)*((e_.) + (f_.)*(x_))^2], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[\pi/2]/(f*\operatorname{Rt}[d, 2]))*\operatorname{FresnelS}[\operatorname{Sqrt}[2/\pi]*\operatorname{Rt}[d, 2]*(e + f*x)], x] /; \operatorname{FreeQ}[\{d, e, f\}, x]$

Rule 3433

$\operatorname{Int}[\operatorname{Cos}[(d_.)*((e_.) + (f_.)*(x_))^2], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[\pi/2]/(f*\operatorname{Rt}[d, 2]))*\operatorname{FresnelC}[\operatorname{Sqrt}[2/\pi]*\operatorname{Rt}[d, 2]*(e + f*x)], x] /; \operatorname{FreeQ}[\{d, e, f\}, x]$

Rule 3434

$\operatorname{Int}[\operatorname{Sin}[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^2], x\_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Sin}[c], \operatorname{Int}[\operatorname{Cos}[d*(e + f*x)^2], x], x] + \operatorname{Dist}[\operatorname{Cos}[c], \operatorname{Int}[\operatorname{Sin}[d*(e + f*x)^2], x], x] /; \operatorname{FreeQ}[\{c, d, e, f\}, x]$

Rule 3454

$\operatorname{Int}[(a_.) + (b_.)*\operatorname{Sin}[u_]^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Int}[(a + b*\operatorname{Sin}[\operatorname{ExpandToSum}[u, x]])^p, x] /; \operatorname{FreeQ}[\{a, b, p\}, x] \&\& \operatorname{BinomialQ}[u, x] \&\& !\operatorname{BinomialMatchQ}[u, x]$

Rule 3475

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)*(Cos[a + b*x^n]^(p/(m + 1))), x] + Dist[b*n*(p/(m + 1)), Int[Cos[a + b*x^n]^(p - 1)*Sin[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4669

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] := Dist[1/2^p, Int[u*Sin[2*v]^(p), x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a + bx^2)}{x^2} dx &= -\frac{\cos^2(a + bx^2)}{x} - (4b) \int \cos(a + bx^2) \sin(a + bx^2) dx \\ &= -\frac{\cos^2(a + bx^2)}{x} - (2b) \int \sin(2(a + bx^2)) dx \\ &= -\frac{\cos^2(a + bx^2)}{x} - (2b) \int \sin(2a + 2bx^2) dx \\ &= -\frac{\cos^2(a + bx^2)}{x} - (2b \cos(2a)) \int \sin(2bx^2) dx - (2b \sin(2a)) \int \cos(2bx^2) dx \\ &= -\frac{\cos^2(a + bx^2)}{x} - \sqrt{b} \sqrt{\pi} \cos(2a) S\left(\frac{2\sqrt{b} x}{\sqrt{\pi}}\right) - \sqrt{b} \sqrt{\pi} C\left(\frac{2\sqrt{b} x}{\sqrt{\pi}}\right) \sin(2a) \end{aligned}$$

Mathematica [A]

time = 0.11, size = 76, normalized size = 1.00

$$-\frac{\cos^2(a + bx^2) + \sqrt{b} \sqrt{\pi} x \cos(2a) S\left(\frac{2\sqrt{b} x}{\sqrt{\pi}}\right) + \sqrt{b} \sqrt{\pi} x \operatorname{FresnelC}\left(\frac{2\sqrt{b} x}{\sqrt{\pi}}\right) \sin(2a)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2/x^2,x]
```

```
[Out] -((Cos[a + b*x^2]^2 + Sqrt[b]*Sqrt[Pi]*x*Cos[2*a]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]] + Sqrt[b]*Sqrt[Pi]*x*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])/x)
```

Maple [A]

time = 0.09, size = 62, normalized size = 0.82

---



method	result	size
default	$-\frac{1}{2x} - \frac{\cos(2bx^2+2a)}{2x} - \sqrt{b} \sqrt{\pi} \left( \cos(2a) S\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right) + \sin(2a) \text{FresnelC}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right) \right)$	62
risch	$-\frac{1}{2x} - \frac{ie^{-2ia}b\sqrt{\pi} \sqrt{2} \operatorname{erf}\left(\sqrt{2} \sqrt{ib} x\right)}{4\sqrt{ib}} + \frac{ie^{2ia}b\sqrt{\pi} \operatorname{erf}\left(\sqrt{-2ib} x\right)}{2\sqrt{-2ib}} - \frac{\cos(2bx^2+2a)}{2x}$	83

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)`

[Out] `-1/2/x-1/2/x*cos(2*b*x^2+2*a)-b^(1/2)*Pi^(1/2)*(cos(2*a)*FresnelS(2*x*b^(1/2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x*b^(1/2)/Pi^(1/2)))`

**Maxima** [C] Result contains complex when optimal does not.

time = 0.56, size = 83, normalized size = 1.09

$$\frac{\sqrt{2} \sqrt{bx^2} \left( (-i+1) \sqrt{2} \Gamma(-\frac{1}{2}, 2ibx^2) + (i-1) \sqrt{2} \Gamma(-\frac{1}{2}, -2ibx^2) \right) \cos(2a) + \left( (i-1) \sqrt{2} \Gamma(-\frac{1}{2}, 2ibx^2) - (i+1) \sqrt{2} \Gamma(-\frac{1}{2}, -2ibx^2) \right) \sin(2a) - 8}{16x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^2/x^2,x, algorithm="maxima")`

[Out] `1/16*(sqrt(2)*sqrt(b*x^2)*((-I + 1)*sqrt(2)*gamma(-1/2, 2*I*b*x^2) + (I - 1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*cos(2*a) + ((I - 1)*sqrt(2)*gamma(-1/2, 2*I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*sin(2*a) - 8)/x`

**Fricas** [A]

time = 0.38, size = 66, normalized size = 0.87

$$\frac{\pi x \sqrt{\frac{b}{\pi}} \cos(2a) S\left(2x \sqrt{\frac{b}{\pi}}\right) + \pi x \sqrt{\frac{b}{\pi}} C\left(2x \sqrt{\frac{b}{\pi}}\right) \sin(2a) + \cos(bx^2 + a)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^2/x^2,x, algorithm="fricas")`

[Out] `-(pi*x*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x*sqrt(b/pi)) + pi*x*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b/pi))*sin(2*a) + cos(b*x^2 + a)^2)/x`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a + bx^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x\*\*2+a)\*\*2/x\*\*2,x)

[Out] Integral(cos(a + b\*x\*\*2)\*\*2/x\*\*2, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)^2/x^2,x, algorithm="giac")

[Out] integrate(cos(b\*x^2 + a)^2/x^2, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(bx^2 + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^2)^2/x^2,x)

[Out] int(cos(a + b\*x^2)^2/x^2, x)

### 3.14 $\int \frac{\cos^2(a+bx^2)}{x^3} dx$

**Optimal.** Leaf size=57

$$-\frac{1}{4x^2} - \frac{\cos(2(a+bx^2))}{4x^2} - \frac{1}{2}b \text{CosIntegral}(2bx^2) \sin(2a) - \frac{1}{2}b \cos(2a) \text{Si}(2bx^2)$$

[Out]  $-1/4/x^2 - 1/4*\cos(2*b*x^2+2*a)/x^2 - 1/2*b*\cos(2*a)*\text{Si}(2*b*x^2) - 1/2*b*\text{Ci}(2*b*x^2)*\sin(2*a)$

**Rubi [A]**

time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {3485, 3461, 3378, 3384, 3380, 3383}

$$-\frac{1}{2}b \sin(2a) \text{CosIntegral}(2bx^2) - \frac{1}{2}b \cos(2a) \text{Si}(2bx^2) - \frac{\cos(2(a+bx^2))}{4x^2} - \frac{1}{4x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[a + b*x^2]^2/x^3, x]$

[Out]  $-1/4*1/x^2 - \text{Cos}[2*(a + b*x^2)]/(4*x^2) - (b*\text{CosIntegral}[2*b*x^2]*\text{Sin}[2*a])/2 - (b*\text{Cos}[2*a]*\text{SinIntegral}[2*b*x^2])/2$

Rule 3378

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)*\sin[(e_.) + (f_.)*(x_.)]}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m + 1)}*(\text{Sin}[e + f*x]/(d*(m + 1))), x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{LtQ}[m, -1]$

Rule 3380

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{EqQ}[d*e - c*f, 0]$

Rule 3383

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{CosIntegral}[e - \text{Pi}/2 + f*x]/d, x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{EqQ}[d*(e - \text{Pi}/2) - c*f, 0]$

Rule 3384

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[c*(f/d) + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[c*(f/d) + f*x]/(c + d*x), x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\&$

NeQ[d\*e - c\*f, 0]

### Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x]
/; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

### Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*((e_.)*(x_)^(m_.), x_Symbol]
:= Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\cos^2(a + bx^2)}{x^3} dx &= \int \left( \frac{1}{2x^3} + \frac{\cos(2a + 2bx^2)}{2x^3} \right) dx \\
 &= -\frac{1}{4x^2} + \frac{1}{2} \int \frac{\cos(2a + 2bx^2)}{x^3} dx \\
 &= -\frac{1}{4x^2} + \frac{1}{4} \text{Subst} \left( \int \frac{\cos(2a + 2bx)}{x^2} dx, x, x^2 \right) \\
 &= -\frac{1}{4x^2} - \frac{\cos(2(a + bx^2))}{4x^2} - \frac{1}{2} b \text{Subst} \left( \int \frac{\sin(2a + 2bx)}{x} dx, x, x^2 \right) \\
 &= -\frac{1}{4x^2} - \frac{\cos(2(a + bx^2))}{4x^2} - \frac{1}{2} (b \cos(2a)) \text{Subst} \left( \int \frac{\sin(2bx)}{x} dx, x, x^2 \right) - \frac{1}{2} (b \sin(2a)) \text{Si}(2bx^2) \\
 &= -\frac{1}{4x^2} - \frac{\cos(2(a + bx^2))}{4x^2} - \frac{1}{2} b \text{Ci}(2bx^2) \sin(2a) - \frac{1}{2} b \cos(2a) \text{Si}(2bx^2)
 \end{aligned}$$

### Mathematica [A]

time = 0.08, size = 50, normalized size = 0.88

$$\frac{-\cos^2(a + bx^2) + bx^2 \text{CosIntegral}(2bx^2) \sin(2a) + bx^2 \cos(2a) \text{Si}(2bx^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2/x^3,x]
```

```
[Out] -1/2*(Cos[a + b*x^2]^2 + b*x^2*CosIntegral[2*b*x^2]*Sin[2*a] + b*x^2*Cos[2*a]*SinIntegral[2*b*x^2])/x^2
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.  
time = 0.12, size = 89, normalized size = 1.56

method	result
risch	$-\frac{1}{4x^2} + \frac{e^{-2ia}\pi \operatorname{csgn}(bx^2)b}{4} - \frac{e^{-2ia} \operatorname{sinIntegral}(2bx^2)b}{2} + \frac{ie^{-2ia} \operatorname{expIntegral}(1,-2ibx^2)b}{4} - \frac{ie^{2ia}b \operatorname{expIntegral}(1,-2ibx^2)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)`

[Out]  $-1/4/x^2 + 1/4*\exp(-2*I*a)*\pi*\operatorname{csgn}(b*x^2)*b - 1/2*\exp(-2*I*a)*\operatorname{Si}(2*b*x^2)*b + 1/4*I*\exp(-2*I*a)*\operatorname{Ei}(1,-2*I*b*x^2)*b - 1/4*I*\exp(2*I*a)*b*\operatorname{Ei}(1,-2*I*b*x^2) - 1/4*\cos(2*b*x^2+2*a)/x^2$

**Maxima [C]** Result contains complex when optimal does not.  
time = 0.34, size = 61, normalized size = 1.07

$$\frac{((i\Gamma(-1, 2i bx^2) - i\Gamma(-1, -2i bx^2)) \cos(2a) + (\Gamma(-1, 2i bx^2) + \Gamma(-1, -2i bx^2)) \sin(2a))bx^2 + 1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^2/x^3,x, algorithm="maxima")`

[Out]  $-1/4*((I*\gamma(-1, 2*I*b*x^2) - I*\gamma(-1, -2*I*b*x^2))*\cos(2*a) + (\gamma(-1, 2*I*b*x^2) + \gamma(-1, -2*I*b*x^2))*\sin(2*a))*b*x^2 + 1)/x^2$

**Fricas [A]**

time = 0.37, size = 65, normalized size = 1.14

$$\frac{2bx^2 \cos(2a) \operatorname{Si}(2bx^2) + 2 \cos(bx^2 + a)^2 + (bx^2 \operatorname{Ci}(2bx^2) + bx^2 \operatorname{Ci}(-2bx^2)) \sin(2a)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^2/x^3,x, algorithm="fricas")`

[Out]  $-1/4*(2*b*x^2*\cos(2*a)*\operatorname{sin\_integral}(2*b*x^2) + 2*\cos(b*x^2 + a)^2 + (b*x^2*\cos\_integral(2*b*x^2) + b*x^2*\cos\_integral(-2*b*x^2))*\sin(2*a))/x^2$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a + bx^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x**2+a)**2/x**3,x)`

[Out] Integral(cos(a + b\*x\*\*2)\*\*2/x\*\*3, x)

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(50) = 100.

time = 0.43, size = 107, normalized size = 1.88

$$\frac{2(bx^2 + a)b^2 \operatorname{Ci}(2bx^2) \sin(2a) - 2ab^2 \operatorname{Ci}(2bx^2) \sin(2a) - 2(bx^2 + a)b^2 \cos(2a) \operatorname{Si}(-2bx^2) + 2ab^2 \cos(2a) \operatorname{Si}(-2bx^2) + b^2 \cos(2bx^2 + 2a) + b^2}{4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)^2/x^3,x, algorithm="giac")

[Out] -1/4\*(2\*(b\*x^2 + a)\*b^2\*cos\_integral(2\*b\*x^2)\*sin(2\*a) - 2\*a\*b^2\*cos\_integral(2\*b\*x^2)\*sin(2\*a) - 2\*(b\*x^2 + a)\*b^2\*cos(2\*a)\*sin\_integral(-2\*b\*x^2) + 2\*a\*b^2\*cos(2\*a)\*sin\_integral(-2\*b\*x^2) + b^2\*cos(2\*b\*x^2 + 2\*a) + b^2)/(b^2\*x^2)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(bx^2 + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^2)^2/x^3,x)

[Out] int(cos(a + b\*x^2)^2/x^3, x)

### 3.15 $\int x^3 \cos^3(a + bx^2) dx$

**Optimal.** Leaf size=79

$$\frac{\cos(a + bx^2)}{3b^2} + \frac{\cos^3(a + bx^2)}{18b^2} + \frac{x^2 \sin(a + bx^2)}{3b} + \frac{x^2 \cos^2(a + bx^2) \sin(a + bx^2)}{6b}$$

[Out] 1/3\*cos(b\*x^2+a)/b^2+1/18\*cos(b\*x^2+a)^3/b^2+1/3\*x^2\*sin(b\*x^2+a)/b+1/6\*x^2\*cos(b\*x^2+a)^2\*sin(b\*x^2+a)/b

**Rubi [A]**

time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3461, 3391, 3377, 2718}

$$\frac{\cos^3(a + bx^2)}{18b^2} + \frac{\cos(a + bx^2)}{3b^2} + \frac{x^2 \sin(a + bx^2)}{3b} + \frac{x^2 \sin(a + bx^2) \cos^2(a + bx^2)}{6b}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Cos[a + b\*x^2]^3,x]

[Out] Cos[a + b\*x^2]/(3\*b^2) + Cos[a + b\*x^2]^3/(18\*b^2) + (x^2\*Sin[a + b\*x^2])/(3\*b) + (x^2\*Cos[a + b\*x^2]^2\*Sin[a + b\*x^2])/(6\*b)

Rule 2718

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(-(c + d\*x)^m)\*(Cos[e + f\*x]/f), x] + Dist[d\*(m/f), Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3391

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[d\*((b\*Sin[e + f\*x])^n/(f^2\*n^2)), x] + (Dist[b^2\*((n - 1)/n), Int[(c + d\*x)\*(b\*Sin[e + f\*x])^(n - 2), x], x] - Simp[b\*(c + d\*x)\*Cos[e + f\*x]\*((b\*Sin[e + f\*x])^(n - 1)/(f\*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 3461

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \int x^3 \cos^3(a + bx^2) dx &= \frac{1}{2} \text{Subst}\left(\int x \cos^3(a + bx) dx, x, x^2\right) \\ &= \frac{\cos^3(a + bx^2)}{18b^2} + \frac{x^2 \cos^2(a + bx^2) \sin(a + bx^2)}{6b} + \frac{1}{3} \text{Subst}\left(\int x \cos(a + bx) dx, x, x^2\right) \\ &= \frac{\cos^3(a + bx^2)}{18b^2} + \frac{x^2 \sin(a + bx^2)}{3b} + \frac{x^2 \cos^2(a + bx^2) \sin(a + bx^2)}{6b} - \frac{\text{Subst}\left(\int \sin(a + bx) dx, x, x^2\right)}{3} \\ &= \frac{\cos(a + bx^2)}{3b^2} + \frac{\cos^3(a + bx^2)}{18b^2} + \frac{x^2 \sin(a + bx^2)}{3b} + \frac{x^2 \cos^2(a + bx^2) \sin(a + bx^2)}{6b} \end{aligned}$$

**Mathematica [A]**

time = 0.10, size = 55, normalized size = 0.70

$$\frac{27 \cos(a + bx^2) + \cos(3(a + bx^2)) + 3bx^2(9 \sin(a + bx^2) + \sin(3(a + bx^2)))}{72b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Cos[a + b\*x^2]^3,x]

[Out] (27\*Cos[a + b\*x^2] + Cos[3\*(a + b\*x^2)] + 3\*b\*x^2\*(9\*Sin[a + b\*x^2] + Sin[3\*(a + b\*x^2)]))/(72\*b^2)

**Maple [A]**

time = 0.12, size = 66, normalized size = 0.84

method	result	size
default	$\frac{3x^2 \sin(bx^2+a)}{8b} + \frac{3 \cos(bx^2+a)}{8b^2} + \frac{x^2 \sin(3bx^2+3a)}{24b} + \frac{\cos(3bx^2+3a)}{72b^2}$	66
risch	$\frac{3x^2 \sin(bx^2+a)}{8b} + \frac{3 \cos(bx^2+a)}{8b^2} + \frac{x^2 \sin(3bx^2+3a)}{24b} + \frac{\cos(3bx^2+3a)}{72b^2}$	66
norman	$\frac{x^2 \tan\left(\frac{a}{2} + \frac{bx^2}{2}\right)}{b} + \frac{x^2 \left(\tan^5\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)}{b} + \frac{\tan^4\left(\frac{a}{2} + \frac{bx^2}{2}\right)}{b^2} + \frac{7}{9b^2} + \frac{2x^2 \left(\tan^3\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)}{3b} + \frac{4 \left(\tan^2\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)}{3b^2}$ $\frac{\quad}{\left(1 + \tan^2\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)^3}$	119

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*cos(b\*x^2+a)^3,x,method=\_RETURNVERBOSE)



[Out]  $\frac{3}{8}x^2\sin(bx^2+a)/b + \frac{3}{8}\cos(bx^2+a)/b^2 + \frac{1}{24}bx^2\sin(3bx^2+3a) + \frac{1}{72}bx^2\cos(3bx^2+3a)$

**Maxima** [A]

time = 0.29, size = 58, normalized size = 0.73

$$\frac{3bx^2\sin(3bx^2+3a) + 27bx^2\sin(bx^2+a) + \cos(3bx^2+3a) + 27\cos(bx^2+a)}{72b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(b*x^2+a)^3,x, algorithm="maxima")`

[Out]  $\frac{1}{72}(3bx^2\sin(3bx^2+3a) + 27bx^2\sin(bx^2+a) + \cos(3bx^2+3a) + 27\cos(bx^2+a))/b^2$

**Fricas** [A]

time = 0.36, size = 58, normalized size = 0.73

$$\frac{\cos(bx^2+a)^3 + 3\left(bx^2\cos(bx^2+a)^2 + 2bx^2\right)\sin(bx^2+a) + 6\cos(bx^2+a)}{18b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(b*x^2+a)^3,x, algorithm="fricas")`

[Out]  $\frac{1}{18}(\cos(bx^2+a)^3 + 3(bx^2\cos(bx^2+a)^2 + 2bx^2)\sin(bx^2+a) + 6\cos(bx^2+a))/b^2$

**Sympy** [A]

time = 0.41, size = 92, normalized size = 1.16

$$\begin{cases} \frac{x^2\sin^3(a+bx^2)}{3b} + \frac{x^2\sin(a+bx^2)\cos^2(a+bx^2)}{2b} + \frac{\sin^2(a+bx^2)\cos(a+bx^2)}{3b^2} + \frac{7\cos^3(a+bx^2)}{18b^2} & \text{for } b \neq 0 \\ \frac{x^4\cos^3(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*cos(b*x**2+a)**3,x)`

[Out] `Piecewise((x**2*sin(a + b*x**2)**3/(3*b) + x**2*sin(a + b*x**2)*cos(a + b*x**2)**2/(2*b) + sin(a + b*x**2)**2*cos(a + b*x**2)/(3*b**2) + 7*cos(a + b*x**2)**3/(18*b**2), Ne(b, 0)), (x**4*cos(a)**3/4, True))`

**Giac** [A]

time = 0.41, size = 92, normalized size = 1.16

$$\frac{(\sin(bx^2+a)^3 - 3\sin(bx^2+a))a}{6b^2} + \frac{3(bx^2+a)\sin(3bx^2+3a) + 27(bx^2+a)\sin(bx^2+a) + \cos(3bx^2+3a) + 27\cos(bx^2+a)}{72b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cos(b\*x^2+a)^3,x, algorithm="giac")

[Out] 1/6\*(sin(b\*x^2 + a)^3 - 3\*sin(b\*x^2 + a))\*a/b^2 + 1/72\*(3\*(b\*x^2 + a)\*sin(3\*b\*x^2 + 3\*a) + 27\*(b\*x^2 + a)\*sin(b\*x^2 + a) + cos(3\*b\*x^2 + 3\*a) + 27\*cos(b\*x^2 + a))/b^2

Mupad [B]

time = 0.41, size = 66, normalized size = 0.84

$$\frac{\frac{\cos(bx^2+a)}{3} + \frac{\cos(bx^2+a)^3}{18} + b \left( \frac{x^2 \sin(bx^2+a)}{3} + \frac{x^2 \cos(bx^2+a)^2 \sin(bx^2+a)}{6} \right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*cos(a + b\*x^2)^3,x)

[Out] (cos(a + b\*x^2)/3 + cos(a + b\*x^2)^3/18 + b\*((x^2\*sin(a + b\*x^2))/3 + (x^2\*cos(a + b\*x^2)^2\*sin(a + b\*x^2))/6))/b^2

### 3.16 $\int x^2 \cos^3(a + bx^2) dx$

**Optimal.** Leaf size=188

$$\frac{3\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{8b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}} \cos(3a) S\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right)}{24b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a)}{8b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}} \text{FresnelC}\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right) \sin(3a)}{24b^{3/2}}$$

[Out]  $3/8*x*\sin(b*x^2+a)/b+1/24*x*\sin(3*b*x^2+3*a)/b-1/144*\cos(3*a)*\text{FresnelS}(x*b^{1/2})*6^{1/2}/\text{Pi}^{1/2})*6^{1/2}*\text{Pi}^{1/2}/b^{3/2}-1/144*\text{FresnelC}(x*b^{1/2})*6^{1/2}/\text{Pi}^{1/2})*\sin(3*a)*6^{1/2}*\text{Pi}^{1/2}/b^{3/2}-3/16*\cos(a)*\text{FresnelS}(x*b^{1/2})*2^{1/2}/\text{Pi}^{1/2})*2^{1/2}*\text{Pi}^{1/2}/b^{3/2}-3/16*\text{FresnelC}(x*b^{1/2})*2^{1/2}/\text{Pi}^{1/2})*\sin(a)*2^{1/2}*\text{Pi}^{1/2}/b^{3/2}$

**Rubi [A]**

time = 0.14, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {3485, 3467, 3434, 3433, 3432}

$$\frac{3\sqrt{\frac{\pi}{2}} \sin(a) \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{b} x\right)}{8b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}} \sin(3a) \text{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{b} x\right)}{24b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{8b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}} \cos(3a) S\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right)}{24b^{3/2}} + \frac{3x \sin(a + bx^2)}{8b} + \frac{x \sin(3a + 3bx^2)}{24b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{Cos}[a + b*x^2]^3, x]$

[Out]  $(-3*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a]*\text{FresnelS}[\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*x])/(8*b^{3/2}) - (\text{Sqrt}[\text{Pi}/6]*\text{Cos}[3*a]*\text{FresnelS}[\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*x])/(24*b^{3/2}) - (3*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*x]*\text{Sin}[a])/(8*b^{3/2}) - (\text{Sqrt}[\text{Pi}/6]*\text{FresnelC}[\text{Sqrt}[b]*\text{Sqrt}[6/\text{Pi}]*x]*\text{Sin}[3*a])/(24*b^{3/2}) + (3*x*\text{Sin}[a + b*x^2])/(8*b) + (x*\text{Sin}[3*a + 3*b*x^2])/(24*b)$

**Rule 3432**

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_))^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]/(f*\text{Rt}[d, 2]))*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)], x] /; \text{FreeQ}[\{d, e, f\}, x]$

**Rule 3433**

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_))^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]/(f*\text{Rt}[d, 2]))*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)], x] /; \text{FreeQ}[\{d, e, f\}, x]$

**Rule 3434**

$\text{Int}[\text{Sin}[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^2], x\_Symbol] \rightarrow \text{Dist}[\text{Sin}[c], \text{Int}[\text{Cos}[d*(e + f*x)^2], x], x] + \text{Dist}[\text{Cos}[c], \text{Int}[\text{Sin}[d*(e + f*x)^2], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x]$

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] := Simp[e^(n
- 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cos^3(a + bx^2) dx &= \int \left( \frac{3}{4} x^2 \cos(a + bx^2) + \frac{1}{4} x^2 \cos(3a + 3bx^2) \right) dx \\
&= \frac{1}{4} \int x^2 \cos(3a + 3bx^2) dx + \frac{3}{4} \int x^2 \cos(a + bx^2) dx \\
&= \frac{3x \sin(a + bx^2)}{8b} + \frac{x \sin(3a + 3bx^2)}{24b} - \frac{\int \sin(3a + 3bx^2) dx}{24b} - \frac{3 \int \sin(a + bx^2) dx}{8b} \\
&= \frac{3x \sin(a + bx^2)}{8b} + \frac{x \sin(3a + 3bx^2)}{24b} - \frac{(3 \cos(a)) \int \sin(bx^2) dx}{8b} - \frac{\cos(3a) \int \sin(3bx^2) dx}{24b} \\
&= -\frac{3 \sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{8b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}} \cos(3a) S\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right)}{24b^{3/2}} - \frac{3 \sqrt{\frac{\pi}{2}} C\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.29, size = 160, normalized size = 0.85

$$\frac{-27\sqrt{2\pi} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) - \sqrt{6\pi} \cos(3a) S\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right) - 27\sqrt{2\pi} \text{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a) - \sqrt{6\pi} \text{FresnelC}\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right) \sin(3a) + 54\sqrt{b} x \sin(a + bx^2) + 6\sqrt{b} x \sin(3(a + bx^2))}{144b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x^2]^3,x]
```

```
[Out] (-27*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[6*Pi]*Cos[3*a]
*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x] - 27*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]
*x]*Sin[a] - Sqrt[6*Pi]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a] + 54*Sqrt[b]
]*x*Ssin[a + b*x^2] + 6*Sqrt[b]*x*Ssin[3*(a + b*x^2)])/(144*b^(3/2))
```

Maple [A]

time = 0.15, size = 130, normalized size = 0.69

method	result
default	$\frac{3x \sin(bx^2+a)}{8b} - \frac{3\sqrt{2} \sqrt{\pi} \left( \cos(a) S\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) + \sin(a) \operatorname{FresnelC}\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) \right)}{16b^{\frac{3}{2}}} + \frac{x \sin(3bx^2+3a)}{24b} - \frac{\sqrt{2} \sqrt{\pi} \operatorname{erf}\left(\sqrt{3}\sqrt{ib}x\right)}{288b\sqrt{ib}} - \frac{3ie^{-ia}\sqrt{\pi} \operatorname{erf}\left(\sqrt{ib}x\right)}{32b\sqrt{ib}} + \frac{ie^{3ia}\sqrt{\pi} \operatorname{erf}\left(\sqrt{-3ib}x\right)}{96b\sqrt{-3ib}} + \frac{3ie^{ia}\sqrt{\pi} \operatorname{erf}\left(\sqrt{-ib}x\right)}{32b\sqrt{-ib}}$
risch	

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(b*x^2+a)^3,x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{3}{8}x\sin(bx^2+a)/b - 3/16/b^{(3/2)}*2^{(1/2)}*\pi^{(1/2)}*(\cos(a)*\operatorname{FresnelS}(x*b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)}) + \sin(a)*\operatorname{FresnelC}(x*b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)})) + 1/24*x\sin(3*b*x^2+3*a)/b - 1/144/b^{(3/2)}*2^{(1/2)}*\pi^{(1/2)}*3^{(1/2)}*(\cos(3*a)*\operatorname{FresnelS}(2^{(1/2)}/\pi^{(1/2)}*3^{(1/2)}*b^{(1/2)}*x) + \sin(3*a)*\operatorname{FresnelC}(2^{(1/2)}/\pi^{(1/2)}*3^{(1/2)}*b^{(1/2)}*x))$$

**Maxima [C]** Result contains complex when optimal does not.

time = 0.50, size = 143, normalized size = 0.76

$$\frac{24b^2x\sin(3bx^2+3a) + 216b^2x\sin(bx^2+a) + 9^{1/4}\sqrt{\pi}((-i+1)\cos(3a) + (i-1)\sin(3a))\operatorname{erf}(\sqrt{3ib}x) + ((i-1)\cos(3a) - (i+1)\sin(3a))\operatorname{erf}(\sqrt{-3ib}x)}{576b^3} - 27\sqrt{2}\sqrt{\pi}((i+1)\cos(a) - (i-1)\sin(a))\operatorname{erf}(\sqrt{ib}x) + (-i-1)\cos(a) + (i+1)\sin(a)\operatorname{erf}(\sqrt{-ib}x)}{144b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(b*x^2+a)^3,x, algorithm="maxima")`

[Out] 
$$\frac{1}{576}*(24*b^2*x*\sin(3*b*x^2 + 3*a) + 216*b^2*x*\sin(b*x^2 + a) + 9^{(1/4)}*\sqrt{\pi}*((-i + 1)*\cos(3*a) + (i - 1)*\sin(3*a))*\operatorname{erf}(\sqrt{3*I*b}*x) + ((i - 1)*\cos(3*a) - (i + 1)*\sin(3*a))*\operatorname{erf}(\sqrt{-3*I*b}*x))*b^{(3/2)} - 27*\sqrt{\pi}*((i + 1)*\cos(a) - (i - 1)*\sin(a))*\operatorname{erf}(\sqrt{I*b}*x) + (-i - 1)*\cos(a) + (i + 1)*\sin(a))*\operatorname{erf}(\sqrt{-I*b}*x))*b^{(3/2)}/b^3$$

**Fricas [A]**

time = 0.40, size = 148, normalized size = 0.79

$$\frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\cos(3a)S\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right) + 27\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)S\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right) + \sqrt{6}\pi\sqrt{\frac{b}{\pi}}C\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin(3a) + 27\sqrt{2}\pi\sqrt{\frac{b}{\pi}}C\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin(a) - 24(bx\cos(bx^2+a)^2 + 2bx)\sin(bx^2+a)}{144b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(b*x^2+a)^3,x, algorithm="fricas")`

[Out] 
$$-1/144*(\sqrt{6}*\pi*\sqrt{b/\pi}*\cos(3*a)*\operatorname{fresnel\_sin}(\sqrt{6}*x*\sqrt{b/\pi})) + 27*\sqrt{2}*\pi*\sqrt{b/\pi}*\cos(a)*\operatorname{fresnel\_sin}(\sqrt{2}*x*\sqrt{b/\pi}) + \sqrt{6}*\pi*\sqrt{b/\pi}*\operatorname{fresnel\_cos}(\sqrt{6}*x*\sqrt{b/\pi})*\sin(3*a) + 27*\sqrt{2}*\pi*\sqrt{b/\pi}*\operatorname{fresnel\_cos}(\sqrt{2}*x*\sqrt{b/\pi})*\sin(a) - 24*(b*x*\cos(b*x^2 + a)^2 + 2*b*x)*\sin(b*x^2 + a))/b^2$$

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 439 vs.  $2(194) = 388$ .

time = 2.70, size = 439, normalized size = 2.34

$$\frac{3b^2 x^2 \sqrt{\frac{\pi}{2}} \sin(a) \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{5}{4}; \frac{3}{2}; -\frac{bx^2}{b}\right)}{32 \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{5}{4}\right)} + \frac{3b^2 x^2 \sqrt{\frac{\pi}{2}} \sin(3a) \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{5}{4}; \frac{3}{2}; -\frac{3bx^2}{b}\right)}{32 \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{5}{4}\right)} - \frac{3\sqrt{6} x^2 \sqrt{\frac{\pi}{8}} \cos(a) \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{b}\right)}{32 \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{6} x^2 \sqrt{\frac{\pi}{8}} \cos(3a) \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{3}{2}; -\frac{3bx^2}{b}\right)}{32 \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{2} \sqrt{\pi} x^2 \sqrt{\frac{\pi}{8}} \sin(a) S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{8} - \frac{\sqrt{6} \sqrt{\pi} x^2 \sqrt{\frac{\pi}{8}} \sin(3a) S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{24} + \frac{3\sqrt{2} \sqrt{\pi} x^2 \sqrt{\frac{\pi}{8}} \cos(a) C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{8} + \frac{\sqrt{6} \sqrt{\pi} x^2 \sqrt{\frac{\pi}{8}} \cos(3a) C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cos(b\*x\*\*2+a)\*\*3,x)

[Out]  $3*b**(3/2)*x**5*\text{sqrt}(1/b)*\sin(a)*\text{gamma}(3/4)*\text{gamma}(5/4)*\text{hyper}((3/4, 5/4), (3/2, 7/4, 9/4), -b**2*x**4/4)/(32*\text{gamma}(7/4)*\text{gamma}(9/4)) + 3*b**(3/2)*x**5*\text{sqrt}(1/b)*\sin(3*a)*\text{gamma}(3/4)*\text{gamma}(5/4)*\text{hyper}((3/4, 5/4), (3/2, 7/4, 9/4), -9*b**2*x**4/4)/(32*\text{gamma}(7/4)*\text{gamma}(9/4)) - 3*\text{sqrt}(b)*x**3*\text{sqrt}(1/b)*\cos(a)*\text{gamma}(1/4)*\text{gamma}(3/4)*\text{hyper}((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*x**4/4)/(32*\text{gamma}(5/4)*\text{gamma}(7/4)) - \text{sqrt}(b)*x**3*\text{sqrt}(1/b)*\cos(3*a)*\text{gamma}(1/4)*\text{gamma}(3/4)*\text{hyper}((1/4, 3/4), (1/2, 5/4, 7/4), -9*b**2*x**4/4)/(32*\text{gamma}(5/4)*\text{gamma}(7/4)) - 3*\text{sqrt}(2)*\text{sqrt}(\pi)*x**2*\text{sqrt}(1/b)*\sin(a)*\text{fresnels}(\text{sqrt}(2)*\text{sqrt}(b)*x/\text{sqrt}(\pi))/8 - \text{sqrt}(6)*\text{sqrt}(\pi)*x**2*\text{sqrt}(1/b)*\sin(3*a)*\text{fresnels}(\text{sqrt}(6)*\text{sqrt}(b)*x/\text{sqrt}(\pi))/24 + 3*\text{sqrt}(2)*\text{sqrt}(\pi)*x**2*\text{sqrt}(1/b)*\cos(a)*\text{fresnelc}(\text{sqrt}(2)*\text{sqrt}(b)*x/\text{sqrt}(\pi))/8 + \text{sqrt}(6)*\text{sqrt}(\pi)*x**2*\text{sqrt}(1/b)*\cos(3*a)*\text{fresnelc}(\text{sqrt}(6)*\text{sqrt}(b)*x/\text{sqrt}(\pi))/24$

**Giac [C]** Result contains complex when optimal does not.

time = 0.42, size = 259, normalized size = 1.38

$$\frac{i x e^{(3i b x^2 + 3i a)}}{48 b} - \frac{3i x e^{(i b x^2 + i a)}}{16 b} + \frac{3i x e^{(-i b x^2 - i a)}}{16 b} + \frac{i x e^{(-3i b x^2 - 3i a)}}{48 b} - \frac{i \sqrt{6} \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{6} \sqrt{b} x \left(-\frac{i}{b} + 1\right)\right) e^{(3i a)}}{288 b^2 \left(-\frac{i}{b} + 1\right)} - \frac{3i \sqrt{2} \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{2} x \left(-\frac{i}{b} + 1\right) \sqrt{|b|}\right) e^{(i a)}}{32 b \left(-\frac{i}{b} + 1\right) \sqrt{|b|}} + \frac{3i \sqrt{2} \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{2} x \left(\frac{i}{b} + 1\right) \sqrt{|b|}\right) e^{(-i a)}}{32 b \left(\frac{i}{b} + 1\right) \sqrt{|b|}} + \frac{i \sqrt{6} \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{6} \sqrt{b} x \left(\frac{i}{b} + 1\right)\right) e^{(-3i a)}}{288 b^2 \left(\frac{i}{b} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(b\*x^2+a)^3,x, algorithm="giac")

[Out]  $-1/48*I*x*e^{(3*I*b*x^2 + 3*I*a)/b} - 3/16*I*x*e^{(I*b*x^2 + I*a)/b} + 3/16*I*x*e^{(-I*b*x^2 - I*a)/b} + 1/48*I*x*e^{(-3*I*b*x^2 - 3*I*a)/b} - 1/288*I*\text{sqrt}(6)*\text{sqrt}(\pi)*\text{erf}(-1/2*\text{sqrt}(6)*\text{sqrt}(b)*x*(-I*b/\text{abs}(b) + 1))*e^{(3*I*a)/(b^{(3/2)}*(-I*b/\text{abs}(b) + 1))} - 3/32*I*\text{sqrt}(2)*\text{sqrt}(\pi)*\text{erf}(-1/2*\text{sqrt}(2)*x*(-I*b/\text{abs}(b) + 1)*\text{sqrt}(\text{abs}(b)))*e^{(I*a)/(b*(-I*b/\text{abs}(b) + 1)*\text{sqrt}(\text{abs}(b)))} + 3/32*I*\text{sqrt}(2)*\text{sqrt}(\pi)*\text{erf}(-1/2*\text{sqrt}(2)*x*(I*b/\text{abs}(b) + 1)*\text{sqrt}(\text{abs}(b)))*e^{(-I*a)/(b*(I*b/\text{abs}(b) + 1)*\text{sqrt}(\text{abs}(b)))} + 1/288*I*\text{sqrt}(6)*\text{sqrt}(\pi)*\text{erf}(-1/2*\text{sqrt}(6)*\text{sqrt}(b)*x*(I*b/\text{abs}(b) + 1))*e^{(-3*I*a)/(b^{(3/2)}*(I*b/\text{abs}(b) + 1))}$

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cos(bx^2 + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cos(a + b\*x^2)^3,x)

[Out] int(x^2\*cos(a + b\*x^2)^3, x)

### 3.17 $\int x \cos^3(a + bx^2) dx$

Optimal. Leaf size=33

$$\frac{\sin(a + bx^2)}{2b} - \frac{\sin^3(a + bx^2)}{6b}$$

[Out]  $1/2*\sin(b*x^2+a)/b-1/6*\sin(b*x^2+a)^3/b$

Rubi [A]

time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3461, 2713}

$$\frac{\sin(a + bx^2)}{2b} - \frac{\sin^3(a + bx^2)}{6b}$$

Antiderivative was successfully verified.

[In] `Int[x*Cos[a + b*x^2]^3,x]`

[Out] `Sin[a + b*x^2]/(2*b) - Sin[a + b*x^2]^3/(6*b)`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 3461

`Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))`

Rubi steps

$$\begin{aligned} \int x \cos^3(a + bx^2) dx &= \frac{1}{2} \text{Subst} \left( \int \cos^3(a + bx) dx, x, x^2 \right) \\ &= -\frac{\text{Subst}(\int (1 - x^2) dx, x, -\sin(a + bx^2))}{2b} \\ &= \frac{\sin(a + bx^2)}{2b} - \frac{\sin^3(a + bx^2)}{6b} \end{aligned}$$

**Mathematica [A]**

time = 0.01, size = 33, normalized size = 1.00

$$\frac{\sin(a + bx^2)}{2b} - \frac{\sin^3(a + bx^2)}{6b}$$

Antiderivative was successfully verified.

`[In] Integrate[x*Cos[a + b*x^2]^3,x]``[Out] Sin[a + b*x^2]/(2*b) - Sin[a + b*x^2]^3/(6*b)`**Maple [A]**

time = 0.06, size = 26, normalized size = 0.79

method	result	size
derivativedivides	$\frac{(2+\cos^2(bx^2+a))\sin(bx^2+a)}{6b}$	26
default	$\frac{(2+\cos^2(bx^2+a))\sin(bx^2+a)}{6b}$	26
risch	$\frac{3\sin(bx^2+a)}{8b} + \frac{\sin(3bx^2+3a)}{24b}$	31
norman	$\frac{\tan\left(\frac{a}{2} + \frac{bx^2}{2}\right)}{b} + \frac{\tan^5\left(\frac{a}{2} + \frac{bx^2}{2}\right)}{b} + \frac{2\left(\tan^3\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{a}{2} + \frac{bx^2}{2}\right)\right)^3}$	70

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*cos(b*x^2+a)^3,x,method=_RETURNVERBOSE)``[Out] 1/6/b*(2+cos(b*x^2+a)^2)*sin(b*x^2+a)`**Maxima [A]**

time = 0.29, size = 27, normalized size = 0.82

$$\frac{\sin(3bx^2 + 3a) + 9\sin(bx^2 + a)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*cos(b*x^2+a)^3,x, algorithm="maxima")``[Out] 1/24*(sin(3*b*x^2 + 3*a) + 9*sin(b*x^2 + a))/b`**Fricas [A]**

time = 0.37, size = 25, normalized size = 0.76

$$\frac{\left(\cos(bx^2 + a)^2 + 2\right)\sin(bx^2 + a)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x\*cos(b\*x^2+a)^3,x, algorithm="fricas")

[Out] 1/6\*(cos(b\*x^2 + a)^2 + 2)\*sin(b\*x^2 + a)/b

**Sympy [A]**

time = 0.18, size = 44, normalized size = 1.33

$$\begin{cases} \frac{\sin^3(a+bx^2)}{3b} + \frac{\sin(a+bx^2)\cos^2(a+bx^2)}{2b} & \text{for } b \neq 0 \\ \frac{x^2 \cos^3(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(b\*x\*\*2+a)\*\*3,x)

[Out] Piecewise((sin(a + b\*x\*\*2)\*\*3/(3\*b) + sin(a + b\*x\*\*2)\*cos(a + b\*x\*\*2)\*\*2/(2\*b), Ne(b, 0)), (x\*\*2\*cos(a)\*\*3/2, True))

**Giac [A]**

time = 0.45, size = 26, normalized size = 0.79

$$-\frac{\sin(bx^2 + a)^3 - 3 \sin(bx^2 + a)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(b\*x^2+a)^3,x, algorithm="giac")

[Out] -1/6\*(sin(b\*x^2 + a)^3 - 3\*sin(b\*x^2 + a))/b

**Mupad [B]**

time = 0.29, size = 28, normalized size = 0.85

$$\frac{3 \sin(bx^2 + a) - \sin(bx^2 + a)^3}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cos(a + b\*x^2)^3,x)

[Out] (3\*sin(a + b\*x^2) - sin(a + b\*x^2)^3)/(6\*b)

### 3.18 $\int \cos^3(a + bx^2) dx$

**Optimal.** Leaf size=153

$$\frac{3\sqrt{\frac{\pi}{2}} \cos(a) \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{4\sqrt{b}} + \frac{\sqrt{\frac{\pi}{6}} \cos(3a) \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right)}{4\sqrt{b}} - \frac{3\sqrt{\frac{\pi}{2}} S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a)}{4\sqrt{b}}$$

[Out]  $1/24*\cos(3*a)*\operatorname{FresnelC}(x*b^{(1/2)}*6^{(1/2)}/\operatorname{Pi}^{(1/2)})*6^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(1/2)}$   
 $-1/24*\operatorname{FresnelS}(x*b^{(1/2)}*6^{(1/2)}/\operatorname{Pi}^{(1/2)})*\sin(3*a)*6^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(1/2)}$   
 $+3/8*\cos(a)*\operatorname{FresnelC}(x*b^{(1/2)}*2^{(1/2)}/\operatorname{Pi}^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(1/2)}$   
 $-3/8*\operatorname{FresnelS}(x*b^{(1/2)}*2^{(1/2)}/\operatorname{Pi}^{(1/2)})*\sin(a)*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(1/2)}$

**Rubi [A]**

time = 0.07, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {3439, 3435, 3433, 3432}

$$\frac{3\sqrt{\frac{\pi}{2}} \cos(a) \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{b} x\right)}{4\sqrt{b}} + \frac{\sqrt{\frac{\pi}{6}} \cos(3a) \operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}} \sqrt{b} x\right)}{4\sqrt{b}} - \frac{3\sqrt{\frac{\pi}{2}} \sin(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{4\sqrt{b}} - \frac{\sqrt{\frac{\pi}{6}} \sin(3a) S\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[a + b*x^2]^3, x]$

[Out]  $(3*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Cos}[a]*\operatorname{FresnelC}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\operatorname{Pi}]*x])/(4*\operatorname{Sqrt}[b]) + (\operatorname{Sqrt}[\operatorname{Pi}/6]*\operatorname{Cos}[3*a]*\operatorname{FresnelC}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[6/\operatorname{Pi}]*x])/(4*\operatorname{Sqrt}[b]) - (3*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{FresnelS}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\operatorname{Pi}]*x]*\operatorname{Sin}[a])/(4*\operatorname{Sqrt}[b]) - (\operatorname{Sqrt}[\operatorname{Pi}/6]*\operatorname{FresnelS}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[6/\operatorname{Pi}]*x]*\operatorname{Sin}[3*a])/(4*\operatorname{Sqrt}[b])$

Rule 3432

$\operatorname{Int}[\operatorname{Sin}[(d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[\operatorname{Pi}/2]/(f*\operatorname{Rt}[d, 2]))*\operatorname{FresnelS}[\operatorname{Sqrt}[2/\operatorname{Pi}]*\operatorname{Rt}[d, 2]*(e + f*x)], x] /; \operatorname{FreeQ}\{d, e, f\}, x]$

Rule 3433

$\operatorname{Int}[\operatorname{Cos}[(d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[\operatorname{Pi}/2]/(f*\operatorname{Rt}[d, 2]))*\operatorname{FresnelC}[\operatorname{Sqrt}[2/\operatorname{Pi}]*\operatorname{Rt}[d, 2]*(e + f*x)], x] /; \operatorname{FreeQ}\{d, e, f\}, x]$

Rule 3435

$\operatorname{Int}[\operatorname{Cos}[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[c], \operatorname{Int}[\operatorname{Cos}[d*(e + f*x)^2], x], x] - \operatorname{Dist}[\operatorname{Sin}[c], \operatorname{Int}[\operatorname{Sin}[d*(e + f*x)^2], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x]$

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_)^(n_)])*(b_.))^(p_), x_Sy
mbol] :=> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n]]^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned} \int \cos^3(a + bx^2) dx &= \int \left( \frac{3}{4} \cos(a + bx^2) + \frac{1}{4} \cos(3a + 3bx^2) \right) dx \\ &= \frac{1}{4} \int \cos(3a + 3bx^2) dx + \frac{3}{4} \int \cos(a + bx^2) dx \\ &= \frac{1}{4} (3 \cos(a)) \int \cos(bx^2) dx + \frac{1}{4} \cos(3a) \int \cos(3bx^2) dx - \frac{1}{4} (3 \sin(a)) \int \sin(bx^2) dx \\ &= \frac{3 \sqrt{\frac{\pi}{2}} \cos(a) C\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{4 \sqrt{b}} + \frac{\sqrt{\frac{\pi}{6}} \cos(3a) C\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right)}{4 \sqrt{b}} - \frac{3 \sqrt{\frac{\pi}{2}} S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)}{4 \sqrt{b}} \end{aligned}$$

**Mathematica [A]**

time = 0.15, size = 116, normalized size = 0.76

$$\frac{\sqrt{\frac{\pi}{6}} \left( 3\sqrt{3} \cos(a) \text{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) + \cos(3a) \text{FresnelC}\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right) - 3\sqrt{3} S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a) - S\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right) \sin(3a) \right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*x^2]^3, x]

[Out] (Sqrt[Pi/6]\*(3\*Sqrt[3]\*Cos[a]\*FresnelC[Sqrt[b]\*Sqrt[2/Pi]\*x] + Cos[3\*a]\*FresnelC[Sqrt[b]\*Sqrt[6/Pi]\*x] - 3\*Sqrt[3]\*FresnelS[Sqrt[b]\*Sqrt[2/Pi]\*x]\*Sin[a] - FresnelS[Sqrt[b]\*Sqrt[6/Pi]\*x]\*Sin[3\*a]))/(4\*Sqrt[b])

**Maple [A]**

time = 0.09, size = 101, normalized size = 0.66

method	result
default	$\frac{3\sqrt{2} \sqrt{\pi} \left( \cos(a) \text{FresnelC}\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) - \sin(a) S\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) \right)}{8\sqrt{b}} + \frac{\sqrt{2} \sqrt{\pi} \sqrt{3} \left( \cos(3a) \text{FresnelC}\left(\frac{\sqrt{2}\sqrt{3}}{\sqrt{\pi}}\right) - \sin(3a) S\left(\frac{\sqrt{2}\sqrt{3}}{\sqrt{\pi}}\right) \right)}{24\sqrt{b}}$
risch	$\frac{e^{-3ia} \sqrt{\pi} \sqrt{3} \operatorname{erf}\left(\sqrt{3} \sqrt{ib} x\right)}{48\sqrt{ib}} + \frac{3e^{-ia} \sqrt{\pi} \operatorname{erf}\left(\sqrt{ib} x\right)}{16\sqrt{ib}} + \frac{e^{3ia} \sqrt{\pi} \operatorname{erf}\left(\sqrt{-3ib} x\right)}{16\sqrt{-3ib}} + \frac{3e^{ia} \sqrt{\pi} \operatorname{erf}\left(\sqrt{-ib} x\right)}{16\sqrt{-ib}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x^2+a)^3,x,method=_RETURNVERBOSE)`

[Out]  $3/8*2^{(1/2)}*Pi^{(1/2)}/b^{(1/2)}*(cos(a)*FresnelC(x*b^{(1/2)}*2^{(1/2)}/Pi^{(1/2)})-sin(a)*FresnelS(x*b^{(1/2)}*2^{(1/2)}/Pi^{(1/2)}))+1/24*2^{(1/2)}*Pi^{(1/2)}*3^{(1/2)}/b^{(1/2)}*(cos(3*a)*FresnelC(2^{(1/2)}/Pi^{(1/2)}*3^{(1/2)}*b^{(1/2)}*x)-sin(3*a)*FresnelS(2^{(1/2)}/Pi^{(1/2)}*3^{(1/2)}*b^{(1/2)}*x))$

**Maxima [C]** Result contains complex when optimal does not.  
time = 0.51, size = 112, normalized size = 0.73

$$\frac{9^{1/2}\sqrt{\pi}\left(\left((i-1)\cos(3a)+(i+1)\sin(3a)\right)\operatorname{erf}\left(\sqrt{3i}x\right)+\left(-i+1\right)\cos(3a)-\left(i-1\right)\sin(3a)\right)\operatorname{erf}\left(\frac{\sqrt{-3i}x}{b}\right)-9\sqrt{2}\sqrt{\pi}\left(-\left(i-1\right)\cos(a)-\left(i+1\right)\sin(a)\right)\operatorname{erf}\left(\sqrt{i}x\right)+\left(\left(i+1\right)\cos(a)+\left(i-1\right)\sin(a)\right)\operatorname{erf}\left(\frac{\sqrt{-i}x}{b}\right)}{96b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^3,x, algorithm="maxima")`

[Out]  $-1/96*(9^{(1/4)}*\sqrt{2}*\sqrt{\pi}*((I-1)*\cos(3*a)+(I+1)*\sin(3*a))*\operatorname{erf}(\sqrt{3*I*b}*x)+(-I+1)*\cos(3*a)-(I-1)*\sin(3*a))*\operatorname{erf}(\sqrt{-3*I*b}*x))*b^{(3/2)}-9*\sqrt{2}*\sqrt{\pi}*((-I-1)*\cos(a)-(I+1)*\sin(a))*\operatorname{erf}(\sqrt{I*b}*x)+((I+1)*\cos(a)+(I-1)*\sin(a))*\operatorname{erf}(\sqrt{-I*b}*x))*b^{(3/2)})/b^2$

**Fricas [A]**

time = 0.35, size = 121, normalized size = 0.79

$$\frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\cos(3a)C\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)+9\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)C\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)-\sqrt{6}\pi\sqrt{\frac{b}{\pi}}S\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin(3a)-9\sqrt{2}\pi\sqrt{\frac{b}{\pi}}S\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin(a)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^3,x, algorithm="fricas")`

[Out]  $1/24*(\sqrt{6}*\pi*\sqrt{b/\pi}*\cos(3*a)*\operatorname{fresnel\_cos}(\sqrt{6}*x*\sqrt{b/\pi}))+9*\sqrt{2}*\pi*\sqrt{b/\pi}*\cos(a)*\operatorname{fresnel\_cos}(\sqrt{2}*x*\sqrt{b/\pi})-\sqrt{6}*\pi*\sqrt{b/\pi}*\operatorname{fresnel\_sin}(\sqrt{6}*x*\sqrt{b/\pi})*\sin(3*a)-9*\sqrt{2}*\pi*\sqrt{b/\pi}*\operatorname{fresnel\_sin}(\sqrt{2}*x*\sqrt{b/\pi})*\sin(a))/b$

**Sympy [A]**

time = 0.71, size = 129, normalized size = 0.84

$$\frac{3\sqrt{2}\sqrt{\pi}\left(-\sin(a)S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)+\cos(a)C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{8}+\frac{\sqrt{6}\sqrt{\pi}\left(-\sin(3a)S\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)+\cos(3a)C\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x**2+a)**3,x)`

[Out]  $3*\sqrt{2}*\sqrt{\pi}*(-\sin(a)*\operatorname{fresnel}(\sqrt{2}*\sqrt{b}*x/\sqrt{\pi}))+\cos(a)*\operatorname{resnelc}(\sqrt{2}*\sqrt{b}*x/\sqrt{\pi}))*\sqrt{1/b}/8+\sqrt{6}*\sqrt{\pi}*(-\sin(3$

`*a)*fresnels(sqrt(6)*sqrt(b)*x/sqrt(pi)) + cos(3*a)*fresnelc(sqrt(6)*sqrt(b)*x/sqrt(pi))*sqrt(1/b)/24`

**Giac [C]** Result contains complex when optimal does not.

time = 0.45, size = 185, normalized size = 1.21

$$-\frac{\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b}x\left(-\frac{ib}{|b|}+1\right)\right)e^{3ia}}{48\sqrt{b}\left(-\frac{ib}{|b|}+1\right)} - \frac{3\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{ia}}{16\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}} - \frac{3\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{-ia}}{16\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}} - \frac{\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b}x\left(\frac{ib}{|b|}+1\right)\right)e^{-3ia}}{48\sqrt{b}\left(\frac{ib}{|b|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** `integrate(cos(b*x^2+a)^3,x, algorithm="giac")`

**[Out]** `-1/48*sqrt(6)*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(3*I*a)/(sqrt(b)*(-I*b/abs(b) + 1)) - 3/16*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/((-I*b/abs(b) + 1)*sqrt(abs(b))) - 3/16*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/((I*b/abs(b) + 1)*sqrt(abs(b))) - 1/48*sqrt(6)*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b)*x*(I*b/abs(b) + 1))*e^(-3*I*a)/(sqrt(b)*(I*b/abs(b) + 1))`

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(bx^2 + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** `int(cos(a + b*x^2)^3,x)`

**[Out]** `int(cos(a + b*x^2)^3, x)`

### 3.19 $\int \frac{\cos^3(a+bx^2)}{x} dx$

**Optimal.** Leaf size=55

$$\frac{3}{8} \cos(a) \operatorname{CosIntegral}(bx^2) + \frac{1}{8} \cos(3a) \operatorname{CosIntegral}(3bx^2) - \frac{3}{8} \sin(a) \operatorname{Si}(bx^2) - \frac{1}{8} \sin(3a) \operatorname{Si}(3bx^2)$$

[Out] 3/8\*Ci(b\*x^2)\*cos(a)+1/8\*Ci(3\*b\*x^2)\*cos(3\*a)-3/8\*Si(b\*x^2)\*sin(a)-1/8\*Si(3\*b\*x^2)\*sin(3\*a)

**Rubi [A]**

time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3485, 3459, 3457, 3456}

$$\frac{3}{8} \cos(a) \operatorname{CosIntegral}(bx^2) + \frac{1}{8} \cos(3a) \operatorname{CosIntegral}(3bx^2) - \frac{3}{8} \sin(a) \operatorname{Si}(bx^2) - \frac{1}{8} \sin(3a) \operatorname{Si}(3bx^2)$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^2]^3/x,x]

[Out] (3\*Cos[a]\*CosIntegral[b\*x^2])/8 + (Cos[3\*a]\*CosIntegral[3\*b\*x^2])/8 - (3\*Sin[a]\*SinIntegral[b\*x^2])/8 - (Sin[3\*a]\*SinIntegral[3\*b\*x^2])/8

Rule 3456

Int[Sin[(d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] := Simp[SinIntegral[d\*x^n]/n, x] / ; FreeQ[{d, n}, x]

Rule 3457

Int[Cos[(d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] := Simp[CosIntegral[d\*x^n]/n, x] / ; FreeQ[{d, n}, x]

Rule 3459

Int[Cos[(c\_) + (d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] := Dist[Cos[c], Int[Cos[d\*x^n]/x, x], x] - Dist[Sin[c], Int[Sin[d\*x^n]/x, x], x] / ; FreeQ[{c, d, n}, x]

Rule 3485

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)]\*(b\_.))^p]\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[ExpandTrigReduce[(e\*x)^m, (a + b\*Cos[c + d\*x^n])^p, x], x] / ; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a + bx^2)}{x} dx &= \int \left( \frac{3 \cos(a + bx^2)}{4x} + \frac{\cos(3a + 3bx^2)}{4x} \right) dx \\
&= \frac{1}{4} \int \frac{\cos(3a + 3bx^2)}{x} dx + \frac{3}{4} \int \frac{\cos(a + bx^2)}{x} dx \\
&= \frac{1}{4} (3 \cos(a)) \int \frac{\cos(bx^2)}{x} dx + \frac{1}{4} \cos(3a) \int \frac{\cos(3bx^2)}{x} dx - \frac{1}{4} (3 \sin(a)) \int \frac{\sin(bx^2)}{x} dx \\
&= \frac{3}{8} \cos(a) \text{Ci}(bx^2) + \frac{1}{8} \cos(3a) \text{Ci}(3bx^2) - \frac{3}{8} \sin(a) \text{Si}(bx^2) - \frac{1}{8} \sin(3a) \text{Si}(3bx^2)
\end{aligned}$$

**Mathematica [A]**

time = 0.06, size = 50, normalized size = 0.91

$$\frac{1}{8} (3 \cos(a) \text{CosIntegral}(bx^2) + \cos(3a) \text{CosIntegral}(3bx^2) - 3 \sin(a) \text{Si}(bx^2) - \sin(3a) \text{Si}(3bx^2))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*x^2]^3/x, x]

[Out] (3\*Cos[a]\*CosIntegral[b\*x^2] + Cos[3\*a]\*CosIntegral[3\*b\*x^2] - 3\*Sin[a]\*SinIntegral[b\*x^2] - Sin[3\*a]\*SinIntegral[3\*b\*x^2])/8

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.20, size = 125, normalized size = 2.27

method	result
risch	$\frac{ie^{-3ia}\pi \text{csgn}(bx^2)}{16} - \frac{ie^{-3ia} \text{sinIntegral}(3bx^2)}{8} - \frac{e^{-3ia} \text{expIntegral}(1, -3ibx^2)}{16} + \frac{3ie^{-ia}\pi \text{csgn}(bx^2)}{16} - \frac{3ie^{-ia} \text{sinIntegral}(bx^2)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b\*x^2+a)^3/x, x, method=\_RETURNVERBOSE)

[Out] 1/16\*I\*exp(-3\*I\*a)\*Pi\*csgn(b\*x^2)-1/8\*I\*exp(-3\*I\*a)\*Si(3\*b\*x^2)-1/16\*exp(-3\*I\*a)\*Ei(1, -3\*I\*x^2\*b)+3/16\*I\*Pi\*csgn(b\*x^2)\*exp(-I\*a)-3/8\*I\*exp(-I\*a)\*Si(b\*x^2)-3/16\*exp(-I\*a)\*Ei(1, -I\*b\*x^2)-1/16\*exp(3\*I\*a)\*Ei(1, -3\*I\*x^2\*b)-3/16\*exp(I\*a)\*Ei(1, -I\*b\*x^2)

**Maxima [C]** Result contains complex when optimal does not.

time = 0.38, size = 89, normalized size = 1.62

$$\frac{1}{16} (\text{Ei}(3i bx^2) + \text{Ei}(-3i bx^2)) \cos(3a) + \frac{3}{16} (\text{Ei}(i bx^2) + \text{Ei}(-i bx^2)) \cos(a) + \frac{1}{16} (i \text{Ei}(3i bx^2) - i \text{Ei}(-3i bx^2)) \sin(3a) - \frac{3}{16} (-i \text{Ei}(i bx^2) + i \text{Ei}(-i bx^2)) \sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)^3/x, x, algorithm="maxima")

[Out]  $\frac{1}{16}(\text{Ei}(3Ibx^2) + \text{Ei}(-3Ibx^2))\cos(3a) + \frac{3}{16}(\text{Ei}(Ibx^2) + \text{Ei}(-Ibx^2))\cos(a) + \frac{1}{16}(I\text{Ei}(3Ibx^2) - I\text{Ei}(-3Ibx^2))\sin(3a) - \frac{3}{16}(-I\text{Ei}(Ibx^2) + I\text{Ei}(-Ibx^2))\sin(a)$

**Fricas** [A]

time = 0.35, size = 63, normalized size = 1.15

$$\frac{1}{16}(\text{Ci}(3bx^2) + \text{Ci}(-3bx^2))\cos(3a) + \frac{3}{16}(\text{Ci}(bx^2) + \text{Ci}(-bx^2))\cos(a) - \frac{1}{8}\sin(3a)\text{Si}(3bx^2) - \frac{3}{8}\sin(a)\text{Si}(bx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^3/x,x, algorithm="fricas")`

[Out]  $\frac{1}{16}(\cos\_integral(3bx^2) + \cos\_integral(-3bx^2))\cos(3a) + \frac{3}{16}(\cos\_integral(bx^2) + \cos\_integral(-bx^2))\cos(a) - \frac{1}{8}\sin(3a)\sin\_integral(3bx^2) - \frac{3}{8}\sin(a)\sin\_integral(bx^2)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(a + bx^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x**2+a)**3/x,x)`

[Out] `Integral(cos(a + b*x**2)**3/x, x)`

**Giac** [A]

time = 0.42, size = 47, normalized size = 0.85

$$\frac{1}{8}\cos(3a)\text{Ci}(3bx^2) + \frac{3}{8}\cos(a)\text{Ci}(bx^2) - \frac{3}{8}\sin(a)\text{Si}(bx^2) + \frac{1}{8}\sin(3a)\text{Si}(-3bx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^3/x,x, algorithm="giac")`

[Out]  $\frac{1}{8}\cos(3a)\cos\_integral(3bx^2) + \frac{3}{8}\cos(a)\cos\_integral(bx^2) - \frac{3}{8}\sin(a)\sin\_integral(bx^2) + \frac{1}{8}\sin(3a)\sin\_integral(-3bx^2)$

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(bx^2 + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x^2)^3/x,x)`

[Out] `int(cos(a + b*x^2)^3/x, x)`



### 3.20 $\int \frac{\cos^3(a+bx^2)}{x^2} dx$

**Optimal.** Leaf size=168

$$-\frac{\cos^3(a+bx^2)}{x} - \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right) - \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right) - \frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right) - \frac{\cos^3(a+bx^2)}{x}$$

```
[Out] -cos(b*x^2+a)^3/x-3/4*cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)-3/4*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*b^(1/2)*2^(1/2)*Pi^(1/2)-1/4*cos(3*a)*FresnelS(x*b^(1/2)*6^(1/2)/Pi^(1/2))*b^(1/2)*6^(1/2)*Pi^(1/2)-1/4*FresnelC(x*b^(1/2)*6^(1/2)/Pi^(1/2))*sin(3*a)*b^(1/2)*6^(1/2)*Pi^(1/2)
```

**Rubi [A]**

time = 0.11, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {3475, 4670, 3434, 3433, 3432}

$$-\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right) - \frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right) - \frac{\cos^3(a+bx^2)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^3/x^2,x]
```

```
[Out] -(Cos[a + b*x^2]^3/x) - (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x])/2 - (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[3*a]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x])/2 - (3*Sqrt[b]*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/2 - (Sqrt[b]*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/2
```

**Rule 3432**

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

**Rule 3433**

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

**Rule 3434**

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Dist[Sin[c], Int[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /; FreeQ[{c, d, e, f}, x]
```

**Rule 3475**

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] := Simp[x^(m +
1)*(Cos[a + b*x^n]^(p/(m + 1))), x] + Dist[b*n*(p/(m + 1)), Int[Cos[a + b*x^n
]^(p - 1)*Sin[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
+ n, 0] && NeQ[n, 1] && IntegerQ[n]
```

### Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] := Int[ExpandTrigReduce[Sin[v]^(p
)*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a + bx^2)}{x^2} dx &= -\frac{\cos^3(a + bx^2)}{x} - (6b) \int \cos^2(a + bx^2) \sin(a + bx^2) dx \\
&= -\frac{\cos^3(a + bx^2)}{x} - (6b) \int \left( \frac{1}{4} \sin(a + bx^2) + \frac{1}{4} \sin(3a + 3bx^2) \right) dx \\
&= -\frac{\cos^3(a + bx^2)}{x} - \frac{1}{2}(3b) \int \sin(a + bx^2) dx - \frac{1}{2}(3b) \int \sin(3a + 3bx^2) dx \\
&= -\frac{\cos^3(a + bx^2)}{x} - \frac{1}{2}(3b \cos(a)) \int \sin(bx^2) dx - \frac{1}{2}(3b \cos(3a)) \int \sin(3bx^2) dx - \frac{1}{2}(3b \sin(a)) \int \cos(bx^2) dx - \frac{1}{2}(3b \sin(3a)) \int \cos(3bx^2) dx \\
&= -\frac{\cos^3(a + bx^2)}{x} - \frac{3}{2} \sqrt{b} \sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) - \frac{1}{2} \sqrt{b} \sqrt{\frac{3\pi}{2}} \cos(3a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) - \frac{3}{2} \sqrt{b} \sqrt{\frac{\pi}{2}} \sin(a) \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) - \frac{1}{2} \sqrt{b} \sqrt{\frac{3\pi}{2}} \sin(3a) \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right)
\end{aligned}$$

### Mathematica [A]

time = 0.43, size = 166, normalized size = 0.99

$$\frac{3 \cos(a + bx^2) + \cos(3(a + bx^2)) + 3\sqrt{b} \sqrt{2\pi} x \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) + \sqrt{b} \sqrt{6\pi} x \cos(3a) S\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right) + 3\sqrt{b} \sqrt{2\pi} x \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} x\right) \sin(a) + \sqrt{b} \sqrt{6\pi} x \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{6}{\pi}} x\right) \sin(3a)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^3/x^2, x]
```

```
[Out] -1/4*(3*Cos[a + b*x^2] + Cos[3*(a + b*x^2)]) + 3*Sqrt[b]*Sqrt[2*Pi]*x*Cos[a]
*SresnelS[Sqrt[b]*Sqrt[2/Pi]*x] + Sqrt[b]*Sqrt[6*Pi]*x*Cos[3*a]*FresnelS[Sq
rt[b]*Sqrt[6/Pi]*x] + 3*Sqrt[b]*Sqrt[2*Pi]*x*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]
*Sin[a] + Sqrt[b]*Sqrt[6*Pi]*x*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/x
```

### Maple [A]

time = 0.13, size = 128, normalized size = 0.76

method	result
default	$-\frac{3 \cos(bx^2+a)}{4x} - \frac{3\sqrt{b} \sqrt{2} \sqrt{\pi} \left( \cos(a) S\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) + \sin(a) \operatorname{FresnelC}\left(\frac{x\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right) \right)}{4} - \frac{\cos(3bx^2+3a)}{4x} - \frac{\sqrt{b}}{8\sqrt{ib}}$
risch	$-\frac{ie^{-3ia}b\sqrt{\pi} \sqrt{3} \operatorname{erf}\left(\sqrt{3} \sqrt{ib} x\right)}{8\sqrt{ib}} - \frac{3ie^{-ia}b\sqrt{\pi} \operatorname{erf}\left(\sqrt{ib} x\right)}{8\sqrt{ib}} + \frac{3ie^{3ia}b\sqrt{\pi} \operatorname{erf}\left(\sqrt{-3ib} x\right)}{8\sqrt{-3ib}} + \frac{3ie^{ia}b\sqrt{\pi}}{8\sqrt{ib}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)`

[Out] `-3/4*cos(b*x^2+a)/x-3/4*b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))-1/4*cos(3*b*x^2+3*a)/x-1/4*b^(1/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)+sin(3*a)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)`

**Maxima** [C] Result contains complex when optimal does not.

time = 0.63, size = 152, normalized size = 0.90

$$\frac{\sqrt{3} \sqrt{b^2} \left( (-i+1) \sqrt{2} \Gamma\left(-\frac{1}{2}, 3ibx^2\right) + (i-1) \sqrt{2} \Gamma\left(-\frac{1}{2}, -3ibx^2\right) \right) \cos(3a) + \left( (i-1) \sqrt{2} \Gamma\left(-\frac{1}{2}, 3ibx^2\right) - (i+1) \sqrt{2} \Gamma\left(-\frac{1}{2}, -3ibx^2\right) \right) \sin(3a) - 3 \sqrt{b^2} \left( (i+1) \sqrt{2} \Gamma\left(-\frac{1}{2}, ibx^2\right) - (i-1) \sqrt{2} \Gamma\left(-\frac{1}{2}, -ibx^2\right) \right) \cos(a) + \left( -(i-1) \sqrt{2} \Gamma\left(-\frac{1}{2}, ibx^2\right) + (i+1) \sqrt{2} \Gamma\left(-\frac{1}{2}, -ibx^2\right) \right) \sin(a)}{32x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^3/x^2,x, algorithm="maxima")`

[Out] `1/32*(sqrt(3)*sqrt(b*x^2)*((-I + 1)*sqrt(2)*gamma(-1/2, 3*I*b*x^2) + (I - 1)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*cos(3*a) + ((I - 1)*sqrt(2)*gamma(-1/2, 3*I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*sin(3*a) - 3*sqrt(b*x^2)*(((I + 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I - 1)*sqrt(2)*gamma(-1/2, -I*b*x^2))*cos(a) + (-(I - 1)*sqrt(2)*gamma(-1/2, I*b*x^2) + (I + 1)*sqrt(2)*gamma(-1/2, -I*b*x^2))*sin(a))/x`

**Fricas** [A]

time = 0.39, size = 136, normalized size = 0.81

$$\frac{\sqrt{6} \pi x \sqrt{\frac{b}{\pi}} \cos(3a) S\left(\sqrt{6} x \sqrt{\frac{b}{\pi}}\right) + 3 \sqrt{2} \pi x \sqrt{\frac{b}{\pi}} \cos(a) S\left(\sqrt{2} x \sqrt{\frac{b}{\pi}}\right) + \sqrt{6} \pi x \sqrt{\frac{b}{\pi}} C\left(\sqrt{6} x \sqrt{\frac{b}{\pi}}\right) \sin(3a) + 3 \sqrt{2} \pi x \sqrt{\frac{b}{\pi}} C\left(\sqrt{2} x \sqrt{\frac{b}{\pi}}\right) \sin(a) + 4 \cos(bx^2+a)^3}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^3/x^2,x, algorithm="fricas")`

[Out] `-1/4*(sqrt(6)*pi*x*sqrt(b/pi)*cos(3*a)*fresnel_sin(sqrt(6)*x*sqrt(b/pi)) + 3*sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(6)*pi*x*sqrt(b/pi)*fresnel_cos(sqrt(6)*x*sqrt(b/pi))*sin(3*a) + 3*sqrt(2)*pi`

`*x*sqrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a) + 4*cos(b*x^2 + a)^3)/x`

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(a + bx^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x**2+a)**3/x**2,x)`

[Out] `Integral(cos(a + b*x**2)**3/x**2, x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^3/x^2,x, algorithm="giac")`

[Out] `integrate(cos(b*x^2 + a)^3/x^2, x)`

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(bx^2 + a)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x^2)^3/x^2,x)`

[Out] `int(cos(a + b*x^2)^3/x^2, x)`

## 3.21 $\int \frac{\cos^3(a+bx^2)}{x^3} dx$

**Optimal.** Leaf size=91

$$-\frac{3 \cos(a+bx^2)}{8x^2} - \frac{\cos(3(a+bx^2))}{8x^2} - \frac{3}{8}b \operatorname{CosIntegral}(bx^2) \sin(a) - \frac{3}{8}b \operatorname{CosIntegral}(3bx^2) \sin(3a) - \frac{3}{8}b \cos(a) \operatorname{Si}(bx^2) - \frac{3}{8}b \cos(3a) \operatorname{Si}(3bx^2) - \frac{3}{8}b \operatorname{Ci}(bx^2) \sin(a) - \frac{3}{8}b \operatorname{Ci}(3bx^2) \sin(3a)$$

[Out]  $-3/8*\cos(b*x^2+a)/x^2-1/8*\cos(3*b*x^2+3*a)/x^2-3/8*b*\cos(a)*\operatorname{Si}(b*x^2)-3/8*b*\cos(3*a)*\operatorname{Si}(3*b*x^2)-3/8*b*\operatorname{Ci}(b*x^2)*\sin(a)-3/8*b*\operatorname{Ci}(3*b*x^2)*\sin(3*a)$

**Rubi [A]**

time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {3485, 3461, 3378, 3384, 3380, 3383}

$$-\frac{3}{8}b \sin(a) \operatorname{CosIntegral}(bx^2) - \frac{3}{8}b \sin(3a) \operatorname{CosIntegral}(3bx^2) - \frac{3}{8}b \cos(a) \operatorname{Si}(bx^2) - \frac{3}{8}b \cos(3a) \operatorname{Si}(3bx^2) - \frac{3 \cos(a+bx^2)}{8x^2} - \frac{\cos(3(a+bx^2))}{8x^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[a + b*x^2]^3/x^3, x]$

[Out]  $(-3*\operatorname{Cos}[a + b*x^2])/(8*x^2) - \operatorname{Cos}[3*(a + b*x^2)]/(8*x^2) - (3*b*\operatorname{CosIntegral}[b*x^2]*\operatorname{Sin}[a])/8 - (3*b*\operatorname{CosIntegral}[3*b*x^2]*\operatorname{Sin}[3*a])/8 - (3*b*\operatorname{Cos}[a]*\operatorname{SinIntegral}[b*x^2])/8 - (3*b*\operatorname{Cos}[3*a]*\operatorname{SinIntegral}[3*b*x^2])/8$

Rule 3378

$\operatorname{Int}[(c + d*x)^m \sin(e + f*x), x] \rightarrow \operatorname{Simp}[(c + d*x)^{m+1} (\sin[e + f*x]/(d*(m+1))), x] - \operatorname{Dist}[f/(d*(m+1)), \operatorname{Int}[(c + d*x)^{m+1} \cos[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3380

$\operatorname{Int}[\sin(e + f*x)/(c + d*x), x] \rightarrow \operatorname{Simp}[\operatorname{SinIntegral}[e + f*x]/d, x] /;$  FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

Rule 3383

$\operatorname{Int}[\sin(e + f*x)/(c + d*x), x] \rightarrow \operatorname{Simp}[\operatorname{CosIntegral}[e - \pi/2 + f*x]/d, x] /;$  FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - \pi/2) - c\*f, 0]

Rule 3384

$\operatorname{Int}[\sin(e + f*x)/(c + d*x), x] \rightarrow \operatorname{Dist}[\cos((d*e - c*f)/d), \operatorname{Int}[\sin[c*(f/d) + f*x]/(c + d*x), x], x] + \operatorname{Dist}[\sin((d*e - c*f)/d), \operatorname{Int}[\cos[c*(f/d) + f*x]/(c + d*x), x], x]$

/d], Int[Cos[c\*(f/d) + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

### Rule 3461

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol  
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p  
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(  
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(  
m + 1)/n], 0]))

### Rule 3485

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x  
\_Symbol] := Int[ExpandTrigReduce[(e\*x)^m, (a + b\*Cos[c + d\*x^n])^p, x], x]  
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cos^3(a + bx^2)}{x^3} dx &= \int \left( \frac{3 \cos(a + bx^2)}{4x^3} + \frac{\cos(3a + 3bx^2)}{4x^3} \right) dx \\
 &= \frac{1}{4} \int \frac{\cos(3a + 3bx^2)}{x^3} dx + \frac{3}{4} \int \frac{\cos(a + bx^2)}{x^3} dx \\
 &= \frac{1}{8} \text{Subst} \left( \int \frac{\cos(3a + 3bx)}{x^2} dx, x, x^2 \right) + \frac{3}{8} \text{Subst} \left( \int \frac{\cos(a + bx)}{x^2} dx, x, x^2 \right) \\
 &= -\frac{3 \cos(a + bx^2)}{8x^2} - \frac{\cos(3(a + bx^2))}{8x^2} - \frac{1}{8}(3b) \text{Subst} \left( \int \frac{\sin(a + bx)}{x} dx, x, x^2 \right) - \frac{1}{8}(3b) \\
 &= -\frac{3 \cos(a + bx^2)}{8x^2} - \frac{\cos(3(a + bx^2))}{8x^2} - \frac{1}{8}(3b \cos(a)) \text{Subst} \left( \int \frac{\sin(bx)}{x} dx, x, x^2 \right) - \frac{1}{8}(3b) \\
 &= -\frac{3 \cos(a + bx^2)}{8x^2} - \frac{\cos(3(a + bx^2))}{8x^2} - \frac{3}{8} b \text{Ci}(bx^2) \sin(a) - \frac{3}{8} b \text{Ci}(3bx^2) \sin(3a) - \frac{3}{8} b \cos(a)
 \end{aligned}$$

### Mathematica [A]

time = 0.12, size = 90, normalized size = 0.99

$$\frac{3 \cos(a + bx^2) + \cos(3(a + bx^2)) + 3bx^2 \text{CosIntegral}(bx^2) \sin(a) + 3bx^2 \text{CosIntegral}(3bx^2) \sin(3a) + 3bx^2 \cos(a) \text{Si}(bx^2) + 3bx^2 \cos(3a) \text{Si}(3bx^2)}{8x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*x^2]^3/x^3, x]

[Out] -1/8\*(3\*Cos[a + b\*x^2] + Cos[3\*(a + b\*x^2)]) + 3\*b\*x^2\*CosIntegral[b\*x^2]\*Sin[a] + 3\*b\*x^2\*CosIntegral[3\*b\*x^2]\*Sin[3\*a] + 3\*b\*x^2\*Cos[a]\*SinIntegral[b\*x^2] + 3\*b\*x^2\*Cos[3\*a]\*SinIntegral[3\*b\*x^2])/x^2

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.  
time = 0.19, size = 162, normalized size = 1.78

method	result
risch	$\frac{3e^{-3ia}\pi\operatorname{csgn}(bx^2)b}{16} - \frac{3e^{-3ia}\operatorname{sinIntegral}(3bx^2)b}{8} + \frac{3ie^{-3ia}\operatorname{expIntegral}(1,-3ibx^2)b}{16} + \frac{3e^{-ia}\pi\operatorname{csgn}(bx^2)b}{16} - \frac{3e^{-ia}\operatorname{sinIntegral}(bx^2)b}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{3}{16}\exp(-3Ia)\pi\operatorname{csgn}(bx^2)b - \frac{3}{8}\exp(-3Ia)\operatorname{Si}(3bx^2)b + \frac{3}{16}\exp(-3Ia)\operatorname{Ei}(1,-3Ibx^2)b + \frac{3}{16}\exp(-Ia)\pi\operatorname{csgn}(bx^2)b - \frac{3}{8}\exp(-Ia)\operatorname{Si}(bx^2)b + \frac{3}{16}\exp(-Ia)\operatorname{Ei}(1,-Ibx^2)b - \frac{3}{16}\exp(3Ia)\operatorname{Ei}(1,-3Ibx^2)b - \frac{3}{16}\exp(Ia)\operatorname{Ei}(1,-Ibx^2)b - \frac{3}{8}\cos(bx^2+a)/x^2 - \frac{1}{8}\cos(3bx^2+3a)/x^2$$

**Maxima [C]** Result contains complex when optimal does not.  
time = 0.39, size = 98, normalized size = 1.08

$$\frac{3}{16}((-i\Gamma(-1,3ibx^2) + i\Gamma(-1,-3ibx^2))\cos(3a) + (-i\Gamma(-1,ibx^2) + i\Gamma(-1,-ibx^2))\cos(a) - (\Gamma(-1,3ibx^2) + \Gamma(-1,-3ibx^2))\sin(3a) - (\Gamma(-1,ibx^2) + \Gamma(-1,-ibx^2))\sin(a))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^3/x^3,x, algorithm="maxima")`

[Out] 
$$\frac{3}{16}((-I\operatorname{gamma}(-1,3Ibx^2) + I\operatorname{gamma}(-1,-3Ibx^2))\cos(3a) + (-I\operatorname{gamma}(-1,Ibx^2) + I\operatorname{gamma}(-1,-Ibx^2))\cos(a) - (\operatorname{gamma}(-1,3Ibx^2) + \operatorname{gamma}(-1,-3Ibx^2))\sin(3a) - (\operatorname{gamma}(-1,Ibx^2) + \operatorname{gamma}(-1,-Ibx^2))\sin(a))b$$

**Fricas [A]**

time = 0.39, size = 108, normalized size = 1.19

$$\frac{6bx^2\cos(3a)\operatorname{Si}(3bx^2) + 6bx^2\cos(a)\operatorname{Si}(bx^2) + 8\cos(bx^2+a)^3 + 3(bx^2\operatorname{Ci}(3bx^2) + bx^2\operatorname{Ci}(-3bx^2))\sin(3a) + 3(bx^2\operatorname{Ci}(bx^2) + bx^2\operatorname{Ci}(-bx^2))\sin(a)}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^3/x^3,x, algorithm="fricas")`

[Out] 
$$-1/16(6bx^2\cos(3a)\operatorname{sin\_integral}(3bx^2) + 6bx^2\cos(a)\operatorname{sin\_integral}(bx^2) + 8\cos(bx^2+a)^3 + 3(bx^2\cos\_integral(3bx^2) + bx^2\cos\_integral(-3bx^2))\sin(3a) + 3(bx^2\cos\_integral(bx^2) + bx^2\cos\_integral(-bx^2))\sin(a))/x^2$$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(a + bx^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x\*\*2+a)\*\*3/x\*\*3,x)

[Out] Integral(cos(a + b\*x\*\*2)\*\*3/x\*\*3, x)

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(80) = 160.

time = 0.42, size = 185, normalized size = 2.03

$$\frac{3(bx^2+a)^2 \operatorname{Ci}(3bx^2) \sin(3a) - 3ab^2 \operatorname{Ci}(3bx^2) \sin(3a) + 3(bx^2+a)^2 \operatorname{Ci}(bx^2) \sin(a) - 3ab^2 \operatorname{Ci}(bx^2) \sin(a) + 3(bx^2+a)^2 \cos(a) \operatorname{Si}(bx^2) - 3ab^2 \cos(a) \operatorname{Si}(bx^2) - 3(bx^2+a)b^2 \cos(3a) \operatorname{Si}(-3bx^2) + 3ab^2 \cos(3a) \operatorname{Si}(-3bx^2) + b^2 \cos(3bx^2+3a) + 3b^2 \cos(bx^2+a)}{8b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)^3/x^3,x, algorithm="giac")

[Out]  $-1/8*(3*(bx^2 + a)*b^2*\cos\_integral(3*bx^2)*\sin(3*a) - 3*a*b^2*\cos\_integral(3*bx^2)*\sin(3*a) + 3*(bx^2 + a)*b^2*\cos\_integral(bx^2)*\sin(a) - 3*a*b^2*\cos\_integral(bx^2)*\sin(a) + 3*(bx^2 + a)*b^2*\cos(a)*\sin\_integral(bx^2) - 3*a*b^2*\cos(a)*\sin\_integral(bx^2) - 3*(bx^2 + a)*b^2*\cos(3*a)*\sin\_integral(-3*bx^2) + 3*a*b^2*\cos(3*a)*\sin\_integral(-3*bx^2) + b^2*\cos(3*bx^2 + 3*a) + 3*b^2*\cos(bx^2 + a))/(b^2*x^2)$

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(bx^2 + a)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^2)^3/x^3,x)

[Out] int(cos(a + b\*x^2)^3/x^3, x)



### 3.22 $\int x \cos^7(a + bx^2) dx$

**Optimal.** Leaf size=67

$$\frac{\sin(a + bx^2)}{2b} - \frac{\sin^3(a + bx^2)}{2b} + \frac{3 \sin^5(a + bx^2)}{10b} - \frac{\sin^7(a + bx^2)}{14b}$$

[Out]  $1/2*\sin(b*x^2+a)/b-1/2*\sin(b*x^2+a)^3/b+3/10*\sin(b*x^2+a)^5/b-1/14*\sin(b*x^2+a)^7/b$

**Rubi [A]**

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3461, 2713}

$$-\frac{\sin^7(a + bx^2)}{14b} + \frac{3 \sin^5(a + bx^2)}{10b} - \frac{\sin^3(a + bx^2)}{2b} + \frac{\sin(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x\*Cos[a + b\*x^2]^7,x]

[Out] Sin[a + b\*x^2]/(2\*b) - Sin[a + b\*x^2]^3/(2\*b) + (3\*Sin[a + b\*x^2]^5)/(10\*b) - Sin[a + b\*x^2]^7/(14\*b)

Rule 2713

Int[sin[(c\_.) + (d\_.)\*(x\_)^(n\_.)], x\_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 3461

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x \cos^7(a + bx^2) dx &= \frac{1}{2} \text{Subst} \left( \int \cos^7(a + bx) dx, x, x^2 \right) \\ &= -\frac{\text{Subst} \left( \int (1 - 3x^2 + 3x^4 - x^6) dx, x, -\sin(a + bx^2) \right)}{2b} \\ &= \frac{\sin(a + bx^2)}{2b} - \frac{\sin^3(a + bx^2)}{2b} + \frac{3 \sin^5(a + bx^2)}{10b} - \frac{\sin^7(a + bx^2)}{14b} \end{aligned}$$

**Mathematica [A]**

time = 0.04, size = 67, normalized size = 1.00

$$\frac{35 \sin(a + bx^2)}{128b} + \frac{7 \sin(3(a + bx^2))}{128b} + \frac{7 \sin(5(a + bx^2))}{640b} + \frac{\sin(7(a + bx^2))}{896b}$$

Antiderivative was successfully verified.

`[In] Integrate[x*Cos[a + b*x^2]^7,x]`

```
[Out] (35*Sin[a + b*x^2])/(128*b) + (7*Sin[3*(a + b*x^2)])/(128*b) + (7*Sin[5*(a + b*x^2)])/(640*b) + Sin[7*(a + b*x^2)]/(896*b)
```

**Maple [A]**

time = 0.08, size = 50, normalized size = 0.75

method	result	size
derivativedivides	$\frac{\left(\frac{16}{5} + \cos^6(bx^2+a) + \frac{6(\cos^4(bx^2+a))}{5} + \frac{8(\cos^2(bx^2+a))}{5}\right) \sin(bx^2+a)}{14b}$	50
default	$\frac{\left(\frac{16}{5} + \cos^6(bx^2+a) + \frac{6(\cos^4(bx^2+a))}{5} + \frac{8(\cos^2(bx^2+a))}{5}\right) \sin(bx^2+a)}{14b}$	50
risch	$\frac{35 \sin(bx^2+a)}{128b} + \frac{\sin(7bx^2+7a)}{896b} + \frac{7 \sin(5bx^2+5a)}{640b} + \frac{7 \sin(3bx^2+3a)}{128b}$	63

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x*cos(b*x^2+a)^7,x,method=_RETURNVERBOSE)`

```
[Out] 1/14/b*(16/5+cos(b*x^2+a)^6+6/5*cos(b*x^2+a)^4+8/5*cos(b*x^2+a)^2)*sin(b*x^2+a)
```

**Maxima [A]**

time = 0.30, size = 55, normalized size = 0.82

$$\frac{5 \sin(7bx^2 + 7a) + 49 \sin(5bx^2 + 5a) + 245 \sin(3bx^2 + 3a) + 1225 \sin(bx^2 + a)}{4480b}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x*cos(b*x^2+a)^7,x, algorithm="maxima")`

```
[Out] 1/4480*(5*sin(7*b*x^2 + 7*a) + 49*sin(5*b*x^2 + 5*a) + 245*sin(3*b*x^2 + 3*a) + 1225*sin(b*x^2 + a))/b
```

**Fricas [A]**

time = 0.38, size = 51, normalized size = 0.76

$$\frac{\left(5 \cos(bx^2 + a)^6 + 6 \cos(bx^2 + a)^4 + 8 \cos(bx^2 + a)^2 + 16\right) \sin(bx^2 + a)}{70b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(b\*x^2+a)^7,x, algorithm="fricas")

[Out] 1/70\*(5\*cos(b\*x^2 + a)^6 + 6\*cos(b\*x^2 + a)^4 + 8\*cos(b\*x^2 + a)^2 + 16)\*sin(b\*x^2 + a)/b

**Sympy** [A]

time = 0.90, size = 94, normalized size = 1.40

$$\begin{cases} \frac{8 \sin^7(a+bx^2)}{35b} + \frac{4 \sin^5(a+bx^2) \cos^2(a+bx^2)}{5b} + \frac{\sin^3(a+bx^2) \cos^4(a+bx^2)}{b} + \frac{\sin(a+bx^2) \cos^6(a+bx^2)}{2b} & \text{for } b \neq 0 \\ \frac{x^2 \cos^7(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(b\*x\*\*2+a)\*\*7,x)

[Out] Piecewise((8\*sin(a + b\*x\*\*2)\*\*7/(35\*b) + 4\*sin(a + b\*x\*\*2)\*\*5\*cos(a + b\*x\*\*2)\*\*2/(5\*b) + sin(a + b\*x\*\*2)\*\*3\*cos(a + b\*x\*\*2)\*\*4/b + sin(a + b\*x\*\*2)\*cos(a + b\*x\*\*2)\*\*6/(2\*b), Ne(b, 0)), (x\*\*2\*cos(a)\*\*7/2, True))

**Giac** [A]

time = 0.43, size = 52, normalized size = 0.78

$$\frac{5 \sin(bx^2 + a)^7 - 21 \sin(bx^2 + a)^5 + 35 \sin(bx^2 + a)^3 - 35 \sin(bx^2 + a)}{70b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(b\*x^2+a)^7,x, algorithm="giac")

[Out] -1/70\*(5\*sin(b\*x^2 + a)^7 - 21\*sin(b\*x^2 + a)^5 + 35\*sin(b\*x^2 + a)^3 - 35\*sin(b\*x^2 + a))/b

**Mupad** [B]

time = 0.75, size = 55, normalized size = 0.82

$$\frac{245 \sin(3bx^2 + 3a) + 49 \sin(5bx^2 + 5a) + 5 \sin(7bx^2 + 7a) + 1225 \sin(bx^2 + a)}{4480b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cos(a + b\*x^2)^7,x)

[Out] (245\*sin(3\*a + 3\*b\*x^2) + 49\*sin(5\*a + 5\*b\*x^2) + 5\*sin(7\*a + 7\*b\*x^2) + 1225\*sin(a + b\*x^2))/(4480\*b)

### 3.23 $\int x^{5/2} \cos(a + bx^2) dx$

Optimal. Leaf size=111

$$-\frac{3ie^{ia}x^{3/2}\Gamma\left(\frac{3}{4}, -ibx^2\right)}{16b(-ibx^2)^{3/4}} + \frac{3ie^{-ia}x^{3/2}\Gamma\left(\frac{3}{4}, ibx^2\right)}{16b(ibx^2)^{3/4}} + \frac{x^{3/2} \sin(a + bx^2)}{2b}$$

[Out]  $-3/16*I*\exp(I*a)*x^{(3/2)*\text{GAMMA}(3/4, -I*b*x^2)}/b/(-I*b*x^2)^{(3/4)}+3/16*I*x^{(3/2)*\text{GAMMA}(3/4, I*b*x^2)}/b/\exp(I*a)/(I*b*x^2)^{(3/4)}+1/2*x^{(3/2)*\sin(b*x^2+a)}/b$

**Rubi [A]**

time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3467, 3470, 2250}

$$-\frac{3ie^{ia}x^{3/2}\Gamma\left(\frac{3}{4}, -ibx^2\right)}{16b(-ibx^2)^{3/4}} + \frac{3ie^{-ia}x^{3/2}\Gamma\left(\frac{3}{4}, ibx^2\right)}{16b(ibx^2)^{3/4}} + \frac{x^{3/2} \sin(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(5/2)*\text{Cos}[a + b*x^2]}, x]$

[Out]  $(((-3*I)/16)*E^{(I*a)*x^{(3/2)*\text{Gamma}[3/4, (-I)*b*x^2]})/(b*((-I)*b*x^2)^{(3/4)}) + (((3*I)/16)*x^{(3/2)*\text{Gamma}[3/4, I*b*x^2]})/(b*E^{(I*a)*(I*b*x^2)^{(3/4)})} + (x^{(3/2)*\text{Sin}[a + b*x^2]})/(2*b)$

Rule 2250

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((e_.) + (f_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \text{Simp}[(-F^a)*((e + f*x)^{(m + 1)})/(f*n*((-b)*(c + d*x)^n*\text{Log}[F])^{((m + 1)/n)})*\Gamma[(m + 1)/n, (-b)*(c + d*x)^n*\text{Log}[F]], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, m, n\}, x] \&\& \text{EqQ}[d*e - c*f, 0]$

Rule 3467

$\text{Int}[\text{Cos}[(c_.) + (d_.)*(x_)]^{(n_.)}*((e_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \text{Simp}[e^{(n - 1)*(e*x)^{(m - n + 1)}*(\text{Sin}[c + d*x^n]/(d*n))}, x] - \text{Dist}[e^n*((m - n + 1)/(d*n)), \text{Int}[(e*x)^{(m - n)*\text{Sin}[c + d*x^n]}, x], x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[n, m + 1]$

Rule 3470

$\text{Int}[(e_.)*(x_)]^{(m_.)*\text{Sin}[(c_.) + (d_.)*(x_)]^{(n_.)}], x\_Symbol] \rightarrow \text{Dist}[I/2, \text{Int}[(e*x)^m*E^{((-c)*I - d*I*x^n)}, x], x] - \text{Dist}[I/2, \text{Int}[(e*x)^m*E^{(c*I + d*I*x^n)}, x], x] /; \text{FreeQ}\{c, d, e, m\}, x] \&\& \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int x^{5/2} \cos(a + bx^2) dx &= \frac{x^{3/2} \sin(a + bx^2)}{2b} - \frac{3 \int \sqrt{x} \sin(a + bx^2) dx}{4b} \\
&= \frac{x^{3/2} \sin(a + bx^2)}{2b} - \frac{(3i) \int e^{-ia-ibx^2} \sqrt{x} dx}{8b} + \frac{(3i) \int e^{ia+ibx^2} \sqrt{x} dx}{8b} \\
&= -\frac{3ie^{ia} x^{3/2} \Gamma(\frac{3}{4}, -ibx^2)}{16b(-ibx^2)^{3/4}} + \frac{3ie^{-ia} x^{3/2} \Gamma(\frac{3}{4}, ibx^2)}{16b(ibx^2)^{3/4}} + \frac{x^{3/2} \sin(a + bx^2)}{2b}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 113, normalized size = 1.02

$$\frac{bx^{11/2} \left( 3(ibx^2)^{3/4} \Gamma(\frac{3}{4}, -ibx^2) (-i \cos(a) + \sin(a)) + 3(-ibx^2)^{3/4} \Gamma(\frac{3}{4}, ibx^2) (i \cos(a) + \sin(a)) + 8(b^2 x^4)^{3/4} \sin(a + bx^2) \right)}{16(b^2 x^4)^{7/4}}$$

Antiderivative was successfully verified.

`[In] Integrate[x^(5/2)*Cos[a + b*x^2], x]`

```
[Out] (b*x^(11/2)*(3*(I*b*x^2)^(3/4)*Gamma[3/4, (-I)*b*x^2]*((-I)*Cos[a] + Sin[a])
+ 3*((-I)*b*x^2)^(3/4)*Gamma[3/4, I*b*x^2]*(I*Cos[a] + Sin[a]) + 8*(b^2*x
^4)^(3/4)*Sin[a + b*x^2]))/(16*(b^2*x^4)^(7/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.07, size = 229, normalized size = 2.06

method	result
meijerg	$ \frac{2^{\frac{3}{4}} \cos(a) \sqrt{\pi} \left( \frac{2x^{\frac{3}{2}} 2^{\frac{1}{4}} (b^2)^{\frac{7}{8}} \sin(bx^2)}{7\sqrt{\pi} b} + \frac{3x^{\frac{7}{2}} (b^2)^{\frac{7}{8}} 2^{\frac{1}{4}} \sin(bx^2)}{14\sqrt{\pi} (bx^2)^{\frac{5}{4}}} + \frac{3x^{\frac{7}{2}} (b^2)^{\frac{7}{8}} 2^{\frac{1}{4}} (\cos(bx^2)x^2b - \sin(bx^2))}{8\sqrt{\pi} (bx^2)^{\frac{9}{4}}} \right)}{2(b^2)^{\frac{7}{8}}} $

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^(5/2)*cos(b*x^2+a), x, method=_RETURNVERBOSE)`

```
[Out] 1/2*2^(3/4)/(b^2)^(7/8)*cos(a)*Pi^(1/2)*(2/7/Pi^(1/2)*x^(3/2)*2^(1/4)*(b^2)
^(7/8)/b*sin(b*x^2)+3/14/Pi^(1/2)*x^(7/2)*(b^2)^(7/8)*2^(1/4)/(b*x^2)^(5/4)
*sin(b*x^2)*LommelS1(5/4, 3/2, b*x^2)+3/8/Pi^(1/2)*x^(7/2)*(b^2)^(7/8)*2^(1/4)
)/(b*x^2)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(1/4, 1/2, b*x^2))-1/2*
2^(3/4)/b^(7/4)*sin(a)*Pi^(1/2)*(-1/8/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x
^2)^(5/4)*sin(b*x^2)*LommelS1(1/4, 3/2, b*x^2)-1/2/Pi^(1/2)*x^(7/2)*b^(7/4)*2
^(1/4)/(b*x^2)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(5/4, 1/2, b*x^2))
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(73) = 146.  
time = 0.12, size = 153, normalized size = 1.38

$$\frac{16bx^2 \sin(bx^2 + a) + 3(bx^2)^{\frac{1}{4}} \left( \left( \sqrt{\sqrt{2} + 2} \Gamma\left(\frac{3}{4}, ibx^2\right) + \Gamma\left(\frac{3}{4}, -ibx^2\right) \right) + \sqrt{-\sqrt{2} + 2} \left( i\Gamma\left(\frac{3}{4}, ibx^2\right) - i\Gamma\left(\frac{3}{4}, -ibx^2\right) \right) \right) \cos(a) + \left( \sqrt{-\sqrt{2} + 2} \left( \Gamma\left(\frac{3}{4}, ibx^2\right) + \Gamma\left(\frac{3}{4}, -ibx^2\right) \right) + \sqrt{\sqrt{2} + 2} \left( -i\Gamma\left(\frac{3}{4}, ibx^2\right) + i\Gamma\left(\frac{3}{4}, -ibx^2\right) \right) \right) \sin(a)}{32b^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*cos(b\*x^2+a),x, algorithm="maxima")

[Out] 1/32\*(16\*b\*x^2\*sin(b\*x^2 + a) + 3\*(b\*x^2)^(1/4)\*((sqrt(sqrt(2) + 2))\*(gamma(3/4, I\*b\*x^2) + gamma(3/4, -I\*b\*x^2)) + sqrt(-sqrt(2) + 2)\*(I\*gamma(3/4, I\*b\*x^2) - I\*gamma(3/4, -I\*b\*x^2)))\*cos(a) + (sqrt(-sqrt(2) + 2)\*(gamma(3/4, I\*b\*x^2) + gamma(3/4, -I\*b\*x^2)) + sqrt(sqrt(2) + 2)\*(-I\*gamma(3/4, I\*b\*x^2) + I\*gamma(3/4, -I\*b\*x^2)))\*sin(a))/(b^2\*sqrt(x))

**Fricas [A]**

time = 0.12, size = 58, normalized size = 0.52

$$\frac{8bx^{\frac{3}{2}} \sin(bx^2 + a) + 3(ib)^{\frac{1}{4}} e^{(-ia)} \Gamma\left(\frac{3}{4}, ibx^2\right) + 3(-ib)^{\frac{1}{4}} e^{(ia)} \Gamma\left(\frac{3}{4}, -ibx^2\right)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*cos(b\*x^2+a),x, algorithm="fricas")

[Out] 1/16\*(8\*b\*x^(3/2)\*sin(b\*x^2 + a) + 3\*(I\*b)^(1/4)\*e^(-I\*a)\*gamma(3/4, I\*b\*x^2) + 3\*(-I\*b)^(1/4)\*e^(I\*a)\*gamma(3/4, -I\*b\*x^2))/b^2

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} \cos(a + bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)\*cos(b\*x\*\*2+a),x)

[Out] Integral(x\*\*(5/2)\*cos(a + b\*x\*\*2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*cos(b\*x^2+a),x, algorithm="giac")

[Out] integrate(x^(5/2)\*cos(b\*x^2 + a), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2} \cos(bx^2 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*cos(a + b\*x^2),x)

[Out] int(x^(5/2)\*cos(a + b\*x^2), x)

### 3.24 $\int x^{3/2} \cos(a + bx^2) dx$

**Optimal.** Leaf size=111

$$-\frac{ie^{ia}\sqrt{x}\Gamma\left(\frac{1}{4},-ibx^2\right)}{16b\sqrt[4]{-ibx^2}} + \frac{ie^{-ia}\sqrt{x}\Gamma\left(\frac{1}{4},ibx^2\right)}{16b\sqrt[4]{ibx^2}} + \frac{\sqrt{x}\sin(a+bx^2)}{2b}$$

[Out]  $-1/16*I*\exp(I*a)*\text{GAMMA}(1/4,-I*b*x^2)*x^{(1/2)}/b/(-I*b*x^2)^{(1/4)}+1/16*I*\text{GAMMA}(1/4,I*b*x^2)*x^{(1/2)}/b/\exp(I*a)/(I*b*x^2)^{(1/4)}+1/2*\sin(b*x^2+a)*x^{(1/2)}/b$

**Rubi [A]**

time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3467, 3470, 2250}

$$-\frac{ie^{ia}\sqrt{x}\Gamma\left(\frac{1}{4},-ibx^2\right)}{16b\sqrt[4]{-ibx^2}} + \frac{ie^{-ia}\sqrt{x}\Gamma\left(\frac{1}{4},ibx^2\right)}{16b\sqrt[4]{ibx^2}} + \frac{\sqrt{x}\sin(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(3/2)}*\text{Cos}[a + b*x^2], x]$

[Out]  $((-1/16*I)*E^{(I*a)}*\text{Sqrt}[x]*\text{Gamma}[1/4, (-I)*b*x^2])/(b*((-I)*b*x^2)^{(1/4)}) + ((I/16)*\text{Sqrt}[x]*\text{Gamma}[1/4, I*b*x^2])/(b*E^{(I*a)}*(I*b*x^2)^{(1/4)}) + (\text{Sqrt}[x]*\text{Sin}[a + b*x^2])/(2*b)$

Rule 2250

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((e_.) + (f_.)*(x_))^{(m_.)}], x\_Symbol] := \text{Simp}[(-F^a)*((e + f*x)^{(m + 1)})/(f*n*((-b)*(c + d*x)^n*\text{Log}[F])^{((m + 1)/n)})*\text{Gamma}[(m + 1)/n, (-b)*(c + d*x)^n*\text{Log}[F]], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, m, n\}, x] \&\& \text{EqQ}[d*e - c*f, 0]$

Rule 3467

$\text{Int}[\text{Cos}[(c_.) + (d_.)*(x_)^{(n_.)}]*((e_.)*(x_))^{(m_.)}], x\_Symbol] := \text{Simp}[e^{(n - 1)}*(e*x)^{(m - n + 1)}*(\text{Sin}[c + d*x^n]/(d*n)), x] - \text{Dist}[e^n*((m - n + 1)/(d*n)), \text{Int}[(e*x)^{(m - n)}*\text{Sin}[c + d*x^n], x], x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[n, m + 1]$

Rule 3470

$\text{Int}[(e_.)*(x_)^{(m_.)}*\text{Sin}[(c_.) + (d_.)*(x_)^{(n_.)}], x\_Symbol] := \text{Dist}[I/2, \text{Int}[(e*x)^m * E^{((-c)*I - d*I*x^n)}, x], x] - \text{Dist}[I/2, \text{Int}[(e*x)^m * E^{(c*I + d*I*x^n)}, x], x] /; \text{FreeQ}\{c, d, e, m\}, x] \&\& \text{IGtQ}[n, 0]$



Rubi steps

$$\begin{aligned}
\int x^{3/2} \cos(a + bx^2) dx &= \frac{\sqrt{x} \sin(a + bx^2)}{2b} - \frac{\int \frac{\sin(a + bx^2)}{\sqrt{x}} dx}{4b} \\
&= \frac{\sqrt{x} \sin(a + bx^2)}{2b} - \frac{i \int \frac{e^{-ia - ibx^2}}{\sqrt{x}} dx}{8b} + \frac{i \int \frac{e^{ia + ibx^2}}{\sqrt{x}} dx}{8b} \\
&= -\frac{ie^{ia} \sqrt{x} \Gamma\left(\frac{1}{4}, -ibx^2\right)}{16b^4 \sqrt{-ibx^2}} + \frac{ie^{-ia} \sqrt{x} \Gamma\left(\frac{1}{4}, ibx^2\right)}{16b^4 \sqrt{ibx^2}} + \frac{\sqrt{x} \sin(a + bx^2)}{2b}
\end{aligned}$$

**Mathematica [A]**

time = 0.11, size = 111, normalized size = 1.00

$$\frac{bx^{9/2} \left( \sqrt[4]{ibx^2} \Gamma\left(\frac{1}{4}, -ibx^2\right) (-i \cos(a) + \sin(a)) + \sqrt[4]{-ibx^2} \Gamma\left(\frac{1}{4}, ibx^2\right) (i \cos(a) + \sin(a)) + 8 \sqrt[4]{b^2 x^4} \sin(a + bx^2) \right)}{16 (b^2 x^4)^{5/4}}$$

Antiderivative was successfully verified.

**[In]** Integrate[x^(3/2)\*Cos[a + b\*x^2], x]

**[Out]** (b\*x^(9/2)\*((I\*b\*x^2)^(1/4)\*Gamma[1/4, (-I)\*b\*x^2]\*((-I)\*Cos[a] + Sin[a]) + ((-I)\*b\*x^2)^(1/4)\*Gamma[1/4, I\*b\*x^2]\*(I\*Cos[a] + Sin[a]) + 8\*(b^2\*x^4)^(1/4)\*Sin[a + b\*x^2])/(16\*(b^2\*x^4)^(5/4))

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.06, size = 290, normalized size = 2.61

method	result
meijerg	$ \frac{2^{1/4} \cos(a) \sqrt{\pi} \left( \frac{2 \sqrt{x} 2^{3/4} (b^2)^{5/8} \sin(bx^2)}{5 \sqrt{\pi} b} + \frac{2 \sqrt{x} 2^{3/4} (b^2)^{5/8} (\cos(bx^2) x^2 b - \sin(bx^2))}{5 \sqrt{\pi} b} + \frac{x^{9/2} (b^2)^{5/8} 2^{3/4} b \sin(bx^2) \text{LommelS1}\left(\frac{3}{4}, \frac{3}{2}, bx^2\right)}{10 \sqrt{\pi} (bx^2)^{7/4}} \right)}{2(b^2)^{5/8}} $

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(x^(3/2)\*cos(b\*x^2+a), x, method=\_RETURNVERBOSE)

**[Out]** 1/2\*2^(1/4)/(b^2)^(5/8)\*cos(a)\*Pi^(1/2)\*(2/5/Pi^(1/2)\*x^(1/2)\*2^(3/4)\*(b^2)^(5/8)/b\*sin(b\*x^2)+2/5/Pi^(1/2)\*x^(1/2)\*2^(3/4)\*(b^2)^(5/8)/b\*(cos(b\*x^2)\*x^2\*b-sin(b\*x^2))+1/10/Pi^(1/2)\*x^(9/2)\*(b^2)^(5/8)\*2^(3/4)\*b/(b\*x^2)^(7/4)\*sin(b\*x^2)\*LommelS1(3/4, 3/2, b\*x^2)-2/5/Pi^(1/2)\*x^(9/2)\*(b^2)^(5/8)\*2^(3/4)\*b/(b\*x^2)^(11/4)\*(cos(b\*x^2)\*x^2\*b-sin(b\*x^2))\*LommelS1(7/4, 1/2, b\*x^2))-1/2\*2^(1/4)/b^(5/4)\*sin(a)\*Pi^(1/2)\*(2/9/Pi^(1/2)\*x^(5/2)\*2^(3/4)\*b^(5/4)\*sin(b\*x^2)-2/9/Pi^(1/2)\*x^(9/2)\*b^(9/4)\*2^(3/4)/(b\*x^2)^(7/4)\*sin(b\*x^2)\*Lomm

$\text{elS1}(7/4, 3/2, b*x^2) - 1/2/\text{Pi}^{(1/2)} * x^{(9/2)} * b^{(9/4)} * 2^{(3/4)} / (b*x^2)^{(11/4)} * (\cos(b*x^2) * x^2 * b - \sin(b*x^2)) * \text{LommelS1}(3/4, 1/2, b*x^2)$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs.  $2(73) = 146$ .

time = 0.12, size = 158, normalized size = 1.42

$$\frac{16(bx^2)^{\frac{1}{4}} \sqrt{x} \sin(bx^2 + a) + \left( \left( \sqrt{-\sqrt{2} + 2} \Gamma\left(\frac{1}{4}, i bx^2\right) + \Gamma\left(\frac{1}{4}, -i bx^2\right) \right) + \sqrt{\sqrt{2} + 2} \left( i \Gamma\left(\frac{1}{4}, i bx^2\right) - i \Gamma\left(\frac{1}{4}, -i bx^2\right) \right) \right) \cos(a) + \left( \sqrt{\sqrt{2} + 2} \left( \Gamma\left(\frac{1}{4}, i bx^2\right) + \Gamma\left(\frac{1}{4}, -i bx^2\right) \right) + \sqrt{-\sqrt{2} + 2} \left( -i \Gamma\left(\frac{1}{4}, i bx^2\right) + i \Gamma\left(\frac{1}{4}, -i bx^2\right) \right) \right) \sin(a) \sqrt{x}}{32 (bx^2)^{\frac{1}{4}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)*cos(b*x^2+a),x, algorithm="maxima")`

[Out]  $\frac{1}{32} * (16 * (b*x^2)^{(1/4)} * \text{sqrt}(x) * \sin(b*x^2 + a) + ((\text{sqrt}(-\text{sqrt}(2) + 2) * (\text{gamma}(1/4, I*b*x^2) + \text{gamma}(1/4, -I*b*x^2)) + \text{sqrt}(\text{sqrt}(2) + 2) * (I*\text{gamma}(1/4, I*b*x^2) - I*\text{gamma}(1/4, -I*b*x^2))) * \cos(a) + (\text{sqrt}(\text{sqrt}(2) + 2) * (\text{gamma}(1/4, I*b*x^2) + \text{gamma}(1/4, -I*b*x^2)) + \text{sqrt}(-\text{sqrt}(2) + 2) * (-I*\text{gamma}(1/4, I*b*x^2) + I*\text{gamma}(1/4, -I*b*x^2))) * \sin(a) * \text{sqrt}(x)) / ((b*x^2)^{(1/4)} * b)$

**Fricas [A]**

time = 0.11, size = 56, normalized size = 0.50

$$\frac{(i b)^{\frac{3}{4}} e^{(-i a)} \Gamma\left(\frac{1}{4}, i b x^2\right) + (-i b)^{\frac{3}{4}} e^{(i a)} \Gamma\left(\frac{1}{4}, -i b x^2\right) + 8 b \sqrt{x} \sin(b x^2 + a)}{16 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)*cos(b*x^2+a),x, algorithm="fricas")`

[Out]  $\frac{1}{16} * ((I*b)^{(3/4)} * e^{(-I*a)} * \text{gamma}(1/4, I*b*x^2) + (-I*b)^{(3/4)} * e^{(I*a)} * \text{gamma}(1/4, -I*b*x^2) + 8*b*\text{sqrt}(x)*\sin(b*x^2 + a)) / b^2$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \cos(a + b x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)*cos(b*x**2+a),x)`

[Out] `Integral(x**(3/2)*cos(a + b*x**2), x)`

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)*cos(b*x^2 + a), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2} \cos(bx^2 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(a + b*x^2),x)
```

```
[Out] int(x^(3/2)*cos(a + b*x^2), x)
```

### 3.25 $\int \sqrt{x} \cos(a + bx^2) dx$

Optimal. Leaf size=81

$$-\frac{e^{ia}x^{3/2}\Gamma(\frac{3}{4}, -ibx^2)}{4(-ibx^2)^{3/4}} - \frac{e^{-ia}x^{3/2}\Gamma(\frac{3}{4}, ibx^2)}{4(ibx^2)^{3/4}}$$

[Out]  $-1/4*\exp(I*a)*x^{(3/2)*\text{GAMMA}(3/4, -I*b*x^2)/(-I*b*x^2)^{(3/4)} - 1/4*x^{(3/2)*\text{GAMMA}(3/4, I*b*x^2)/\exp(I*a)/(I*b*x^2)^{(3/4)}$

Rubi [A]

time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3471, 2250}

$$-\frac{e^{ia}x^{3/2}\Gamma(\frac{3}{4}, -ibx^2)}{4(-ibx^2)^{3/4}} - \frac{e^{-ia}x^{3/2}\Gamma(\frac{3}{4}, ibx^2)}{4(ibx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[x]*\text{Cos}[a + b*x^2], x]$

[Out]  $-1/4*(E^{(I*a)*x^{(3/2)*\text{Gamma}[3/4, (-I)*b*x^2]})/((-I)*b*x^2)^{(3/4)} - (x^{(3/2)*\text{Gamma}[3/4, I*b*x^2]})/(4*E^{(I*a)*(I*b*x^2)^{(3/4)})}$

Rule 2250

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^{(n_.)})*((e_.) + (f_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(-F^a)*((e + f*x)^{(m + 1)})/(f*n*((-b)*(c + d*x)^n*\text{Log}[F])^{((m + 1)/n)})*\text{Gamma}[(m + 1)/n, (-b)*(c + d*x)^n*\text{Log}[F]], x] /;$   $\text{FreeQ}\{F, a, b, c, d, e, f, m, n\}, x] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

Rule 3471

$\text{Int}[\text{Cos}[(c_.) + (d_.)*(x_.)^{(n_.)}]*((e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[(e*x)^m*E^{((-c)*I - d*I*x^n)}, x], x] + \text{Dist}[1/2, \text{Int}[(e*x)^m*E^{(c*I + d*I*x^n)}, x], x] /;$   $\text{FreeQ}\{c, d, e, m\}, x] \ \&\& \ \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \sqrt{x} \cos(a + bx^2) dx &= \frac{1}{2} \int e^{-ia-ibx^2} \sqrt{x} dx + \frac{1}{2} \int e^{ia+ibx^2} \sqrt{x} dx \\ &= -\frac{e^{ia}x^{3/2}\Gamma(\frac{3}{4}, -ibx^2)}{4(-ibx^2)^{3/4}} - \frac{e^{-ia}x^{3/2}\Gamma(\frac{3}{4}, ibx^2)}{4(ibx^2)^{3/4}} \end{aligned}$$

**Mathematica [A]**

time = 0.06, size = 89, normalized size = 1.10

$$\frac{x^{3/2} \left( (-ibx^2)^{3/4} \Gamma\left(\frac{3}{4}, ibx^2\right) (\cos(a) - i \sin(a)) + (ibx^2)^{3/4} \Gamma\left(\frac{3}{4}, -ibx^2\right) (\cos(a) + i \sin(a)) \right)}{4 (b^2 x^4)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*Cos[a + b\*x^2], x]

[Out] -1/4\*(x^(3/2)\*((-I)\*b\*x^2)^(3/4)\*Gamma[3/4, I\*b\*x^2]\*(Cos[a] - I\*Sin[a]) + (I\*b\*x^2)^(3/4)\*Gamma[3/4, (-I)\*b\*x^2]\*(Cos[a] + I\*Sin[a]))/(b^2\*x^4)^(3/4)

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.05, size = 290, normalized size = 3.58

method	result
meijerg	$\frac{2^{\frac{3}{4}} \cos(a) \sqrt{\pi} \left( \frac{4^{\frac{1}{4}} (b^2)^{\frac{3}{8}} \sin(bx^2)}{3\sqrt{\pi} \sqrt{x} b} + \frac{4^{\frac{1}{4}} (b^2)^{\frac{3}{8}} (\cos(bx^2)x^2 b - \sin(bx^2))}{3\sqrt{\pi} \sqrt{x} b} - \frac{x^{\frac{7}{2}} (b^2)^{\frac{3}{8}} 2^{\frac{1}{4}} b \sin(bx^2) \text{LommelS1}\left(\frac{1}{4}, \frac{3}{2}, bx^2\right)}{3\sqrt{\pi} (bx^2)^{\frac{5}{4}}} - \frac{4x^{\frac{7}{2}} (b^2)^{\frac{3}{8}}}{4x^{\frac{7}{2}} (b^2)^{\frac{3}{8}}} \right)}{4(b^2)^{\frac{3}{8}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)\*cos(b\*x^2+a), x, method=\_RETURNVERBOSE)

[Out] 1/4\*2^(3/4)/(b^2)^(3/8)\*cos(a)\*Pi^(1/2)\*(4/3/Pi^(1/2)/x^(1/2)\*2^(1/4)\*(b^2)^(3/8)/b\*sin(b\*x^2)+4/3/Pi^(1/2)/x^(1/2)\*2^(1/4)\*(b^2)^(3/8)/b\*(cos(b\*x^2)\*x^2\*b-sin(b\*x^2))-1/3/Pi^(1/2)\*x^(7/2)\*(b^2)^(3/8)\*2^(1/4)\*b/(b\*x^2)^(5/4)\*sin(b\*x^2)\*LommelS1(1/4, 3/2, b\*x^2)-4/3/Pi^(1/2)\*x^(7/2)\*(b^2)^(3/8)\*2^(1/4)\*b/(b\*x^2)^(9/4)\*(cos(b\*x^2)\*x^2\*b-sin(b\*x^2))\*LommelS1(5/4, 1/2, b\*x^2))-1/4\*2^(3/4)/b^(3/4)\*sin(a)\*Pi^(1/2)\*(4/7/Pi^(1/2)\*x^(3/2)\*2^(1/4)\*b^(3/4)\*sin(b\*x^2)-4/7/Pi^(1/2)\*x^(7/2)\*b^(7/4)\*2^(1/4)/(b\*x^2)^(5/4)\*sin(b\*x^2)\*LommelS1(5/4, 3/2, b\*x^2)-1/Pi^(1/2)\*x^(7/2)\*b^(7/4)\*2^(1/4)/(b\*x^2)^(9/4)\*(cos(b\*x^2)\*x^2\*b-sin(b\*x^2))\*LommelS1(1/4, 1/2, b\*x^2))

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(51) = 102.

time = 0.13, size = 138, normalized size = 1.70

$$\frac{(bx^2)^{\frac{1}{4}} \left( \left( \sqrt{-\sqrt{2}+2} \Gamma\left(\frac{3}{4}, ibx^2\right) + \Gamma\left(\frac{3}{4}, -ibx^2\right) \right) - \sqrt{\sqrt{2}+2} \left( i\Gamma\left(\frac{3}{4}, ibx^2\right) - i\Gamma\left(\frac{3}{4}, -ibx^2\right) \right) \right) \cos(a) - \left( \sqrt{\sqrt{2}+2} \Gamma\left(\frac{3}{4}, ibx^2\right) + \Gamma\left(\frac{3}{4}, -ibx^2\right) \right) + \sqrt{-\sqrt{2}+2} \left( i\Gamma\left(\frac{3}{4}, ibx^2\right) - i\Gamma\left(\frac{3}{4}, -ibx^2\right) \right) \sin(a)}{8b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)\*cos(b\*x^2+a), x, algorithm="maxima")

[Out] -1/8\*(b\*x^2)^(1/4)\*((sqrt(-sqrt(2) + 2))\*(gamma(3/4, I\*b\*x^2) + gamma(3/4, -I\*b\*x^2)) - sqrt(sqrt(2) + 2)\*(I\*gamma(3/4, I\*b\*x^2) - I\*gamma(3/4, -I\*b\*x^2)))

2)))\*cos(a) - (sqrt(sqrt(2) + 2)\*(gamma(3/4, I\*b\*x^2) + gamma(3/4, -I\*b\*x^2)) + sqrt(-sqrt(2) + 2)\*(I\*gamma(3/4, I\*b\*x^2) - I\*gamma(3/4, -I\*b\*x^2)))\*sin(a))/(b\*sqrt(x))

**Fricas** [A]

time = 0.10, size = 44, normalized size = 0.54

$$\frac{i(i b)^{\frac{1}{4}} e^{-i a} \Gamma\left(\frac{3}{4}, i b x^2\right) - i(-i b)^{\frac{1}{4}} e^{i a} \Gamma\left(\frac{3}{4}, -i b x^2\right)}{4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)\*cos(b\*x^2+a),x, algorithm="fricas")

[Out] 1/4\*(I\*(I\*b)^(1/4)\*e^(-I\*a)\*gamma(3/4, I\*b\*x^2) - I\*(-I\*b)^(1/4)\*e^(I\*a)\*gamma(3/4, -I\*b\*x^2))/b

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \cos(a + b x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)\*cos(b\*x\*\*2+a),x)

[Out] Integral(sqrt(x)\*cos(a + b\*x\*\*2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)\*cos(b\*x^2+a),x, algorithm="giac")

[Out] integrate(sqrt(x)\*cos(b\*x^2 + a), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x} \cos(b x^2 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)\*cos(a + b\*x^2),x)

[Out] int(x^(1/2)\*cos(a + b\*x^2), x)

$$3.26 \quad \int \frac{\cos(a+bx^2)}{\sqrt{x}} dx$$

**Optimal.** Leaf size=81

$$-\frac{e^{ia}\sqrt{x}\Gamma(\frac{1}{4},-ibx^2)}{4\sqrt[4]{-ibx^2}} - \frac{e^{-ia}\sqrt{x}\Gamma(\frac{1}{4},ibx^2)}{4\sqrt[4]{ibx^2}}$$

[Out]  $-1/4*\exp(I*a)*\text{GAMMA}(1/4,-I*b*x^2)*x^{(1/2)/(-I*b*x^2)^{(1/4)}-1/4*\text{GAMMA}(1/4,I*b*x^2)*x^{(1/2)/\exp(I*a)/(I*b*x^2)^{(1/4)}}$

**Rubi [A]**

time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3471, 2250}

$$-\frac{e^{ia}\sqrt{x}\Gamma(\frac{1}{4},-ibx^2)}{4\sqrt[4]{-ibx^2}} - \frac{e^{-ia}\sqrt{x}\Gamma(\frac{1}{4},ibx^2)}{4\sqrt[4]{ibx^2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^2]/Sqrt[x],x]

[Out]  $-1/4*(E^{(I*a)*\text{Sqrt}[x]*\text{Gamma}[1/4,(-I)*b*x^2]}/((-I)*b*x^2)^{(1/4)} - (\text{Sqrt}[x]*\text{Gamma}[1/4,I*b*x^2])/(4*E^{(I*a)*(I*b*x^2)^{(1/4)}}$

Rule 2250

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(-F^a)\*((e + f\*x)^(m + 1)/(f\*n\*((-b)\*(c + d\*x)^n\*Log[F]))^(m + 1)/n))\*Gamma[(m + 1)/n, (-b)\*(c + d\*x)^n\*Log[F]], x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

Rule 3471

Int[Cos[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/2, Int[(e\*x)^m\*E^((-c)\*I - d\*I\*x^n), x], x] + Dist[1/2, Int[(e\*x)^m\*E^(c\*I + d\*I\*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx^2)}{\sqrt{x}} dx &= \frac{1}{2} \int \frac{e^{-ia-ibx^2}}{\sqrt{x}} dx + \frac{1}{2} \int \frac{e^{ia+ibx^2}}{\sqrt{x}} dx \\ &= -\frac{e^{ia}\sqrt{x}\Gamma(\frac{1}{4},-ibx^2)}{4\sqrt[4]{-ibx^2}} - \frac{e^{-ia}\sqrt{x}\Gamma(\frac{1}{4},ibx^2)}{4\sqrt[4]{ibx^2}} \end{aligned}$$

**Mathematica [A]**

time = 0.05, size = 89, normalized size = 1.10

$$\frac{\sqrt{x} \left( \sqrt[4]{-ibx^2} \Gamma\left(\frac{1}{4}, ibx^2\right) (\cos(a) - i \sin(a)) + \sqrt[4]{ibx^2} \Gamma\left(\frac{1}{4}, -ibx^2\right) (\cos(a) + i \sin(a)) \right)}{4\sqrt[4]{b^2x^4}}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*x^2]/Sqrt[x], x]`

```
[Out] -1/4*(Sqrt[x]*((-1)*b*x^2)^(1/4)*Gamma[1/4, I*b*x^2]*(Cos[a] - I*Sin[a]) +
(I*b*x^2)^(1/4)*Gamma[1/4, (-1)*b*x^2]*(Cos[a] + I*Sin[a]))/(b^2*x^4)^(1/4)
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.06, size = 338, normalized size = 4.17

method	result
meijerg	$\frac{\cos(a) \sqrt{\pi} 2^{\frac{1}{4}} \left( \frac{6 \cdot 2^{\frac{3}{4}} (b^2)^{\frac{1}{8}} \left( \frac{8x^4 b^2}{27} + \frac{2}{3} \right) \sin(bx^2)}{\sqrt{\pi} x^{\frac{3}{2}} b} + \frac{4 \cdot 2^{\frac{3}{4}} (b^2)^{\frac{1}{8}} (\cos(bx^2) x^2 b - \sin(bx^2))}{\sqrt{\pi} x^{\frac{3}{2}} b} - \frac{16x^{\frac{9}{2}} (b^2)^{\frac{1}{8}} b^{\frac{3}{4}} \sin(bx^2) \text{LommelS1}\left(\frac{7}{4}, \frac{3}{2}, bx^2\right)}{9 \sqrt{\pi} (bx^2)^{\frac{7}{4}}} \right)}{4(b^2)^{\frac{1}{8}}}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(b*x^2+a)/x^(1/2), x, method=_RETURNVERBOSE)`

```
[Out] 1/4*cos(a)*Pi^(1/2)*2^(1/4)/(b^2)^(1/8)*(6/Pi^(1/2)/x^(3/2)*2^(3/4)*(b^2)^(1/8)
(8/27*x^4*b^2+2/3)/b*sin(b*x^2)+4/Pi^(1/2)/x^(3/2)*2^(3/4)*(b^2)^(1/8)
/b*(cos(b*x^2)*x^2*b-sin(b*x^2))-16/9/Pi^(1/2)*x^(9/2)*(b^2)^(1/8)*b^2*2^(3/4)
/(b*x^2)^(7/4)*sin(b*x^2)*LommelS1(7/4, 3/2, b*x^2)-4/Pi^(1/2)*x^(9/2)*(b^2)^(1/8)
*b^2*2^(3/4)/(b*x^2)^(11/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(3/4, 1/2, b*x^2)
-1/4*sin(a)*Pi^(1/2)*2^(1/4)/b^(1/4)*(4/5/Pi^(1/2)*x^(1/2)*2^(3/4)*b^(1/4)*sin(b*x^2)
-16/5/Pi^(1/2)*x^(1/2)*2^(3/4)*b^(1/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))-4/5/Pi^(1/2)*x^(9/2)
*b^(9/4)*2^(3/4)/(b*x^2)^(7/4)*sin(b*x^2)*LommelS1(3/4, 3/2, b*x^2)+16/5/Pi^(1/2)*x^(9/2)
*b^(9/4)*2^(3/4)/(b*x^2)^(11/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(7/4, 1/2, b*x^2)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(51) = 102.

time = 0.11, size = 135, normalized size = 1.67

$$\frac{\left( \left( \sqrt{\sqrt{2}+2} \left( \Gamma\left(\frac{1}{4}, ibx^2\right) + \Gamma\left(\frac{1}{4}, -ibx^2\right) \right) - \sqrt{-\sqrt{2}+2} \left( i \Gamma\left(\frac{1}{4}, ibx^2\right) - i \Gamma\left(\frac{1}{4}, -ibx^2\right) \right) \right) \cos(a) - \left( \sqrt{-\sqrt{2}+2} \left( \Gamma\left(\frac{1}{4}, ibx^2\right) + \Gamma\left(\frac{1}{4}, -ibx^2\right) \right) + \sqrt{\sqrt{2}+2} \left( i \Gamma\left(\frac{1}{4}, ibx^2\right) - i \Gamma\left(\frac{1}{4}, -ibx^2\right) \right) \right) \sin(a) \right) \sqrt{x}}{8(bx^2)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(b*x^2+a)/x^(1/2), x, algorithm="maxima")`



[Out]  $-1/8*((\sqrt{\sqrt{2} + 2})*(\gamma(1/4, I*b*x^2) + \gamma(1/4, -I*b*x^2)) - \sqrt{-\sqrt{2} + 2}*(I*\gamma(1/4, I*b*x^2) - I*\gamma(1/4, -I*b*x^2)))*\cos(a) - (\sqrt{-\sqrt{2} + 2}*(\gamma(1/4, I*b*x^2) + \gamma(1/4, -I*b*x^2)) + \sqrt{\sqrt{2} + 2}*(I*\gamma(1/4, I*b*x^2) - I*\gamma(1/4, -I*b*x^2)))*\sin(a))*\sqrt{x}/(b*x^2)^{(1/4)}$

**Fricas** [A]

time = 0.11, size = 44, normalized size = 0.54

$$\frac{i(i b)^{\frac{3}{4}} e^{(-i a)} \Gamma\left(\frac{1}{4}, i b x^2\right) - i(-i b)^{\frac{3}{4}} e^{(i a)} \Gamma\left(\frac{1}{4}, -i b x^2\right)}{4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)/x^(1/2),x, algorithm="fricas")`

[Out]  $1/4*(I*(I*b)^{(3/4)}*e^{(-I*a)}*\gamma(1/4, I*b*x^2) - I*(-I*b)^{(3/4)}*e^{(I*a)}*\gamma(1/4, -I*b*x^2))/b$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + b x^2)}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x**2+a)/x**(1/2),x)`

[Out] `Integral(cos(a + b*x**2)/sqrt(x), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)/x^(1/2),x, algorithm="giac")`

[Out] `integrate(cos(b*x^2 + a)/sqrt(x), x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(b x^2 + a)}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x^2)/x^(1/2),x)`

[Out] `int(cos(a + b*x^2)/x^(1/2), x)`

### 3.27 $\int \frac{\cos(a+bx^2)}{x^{3/2}} dx$

**Optimal.** Leaf size=98

$$-\frac{2 \cos(a+bx^2)}{\sqrt{x}} - \frac{ibe^{ia}x^{3/2}\Gamma(\frac{3}{4}, -ibx^2)}{(-ibx^2)^{3/4}} + \frac{ibe^{-ia}x^{3/2}\Gamma(\frac{3}{4}, ibx^2)}{(ibx^2)^{3/4}}$$

[Out]  $-I*b*\exp(I*a)*x^{(3/2)*\text{GAMMA}(3/4, -I*b*x^2)/(-I*b*x^2)^{(3/4)}+I*b*x^{(3/2)*\text{GAMMA}(3/4, I*b*x^2)/\exp(I*a)/(I*b*x^2)^{(3/4)}-2*\cos(b*x^2+a)/x^{(1/2)}$

**Rubi [A]**

time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3469, 3470, 2250}

$$-\frac{ie^{ia}bx^{3/2}\Gamma(\frac{3}{4}, -ibx^2)}{(-ibx^2)^{3/4}} + \frac{ie^{-ia}bx^{3/2}\Gamma(\frac{3}{4}, ibx^2)}{(ibx^2)^{3/4}} - \frac{2 \cos(a+bx^2)}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^2]/x^(3/2), x]

[Out]  $(-2*\text{Cos}[a + b*x^2])/ \text{Sqrt}[x] - (I*b*E^{(I*a)*x^{(3/2)*\text{Gamma}[3/4, (-I)*b*x^2]})/((-I)*b*x^2)^{(3/4)} + (I*b*x^{(3/2)*\text{Gamma}[3/4, I*b*x^2]})/(E^{(I*a)*(I*b*x^2)^{(3/4)})}$

Rule 2250

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_)^(n\_))\*((e\_) + (f\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[(-F^a)\*((e + f\*x)^(m + 1)/(f\*n\*((-b)\*(c + d\*x)^n\*Log[F])^(m + 1)/n))\*Gamma[(m + 1)/n, (-b)\*(c + d\*x)^n\*Log[F]], x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

Rule 3469

Int[Cos[(c\_) + (d\_)\*(x\_)^(n\_)]\*((e\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[(e\*x)^(m + 1)\*(Cos[c + d\*x^n]/(e\*(m + 1))), x] + Dist[d\*(n/(e^n\*(m + 1))), Int[(e\*x)^(m + n)\*Sin[c + d\*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3470

Int[((e\_)\*(x\_)^(m\_))\*Sin[(c\_) + (d\_)\*(x\_)^(n\_)], x\_Symbol] := Dist[I/2, Int[(e\*x)^m\*E^((-c)\*I - d\*I\*x^n), x], x] - Dist[I/2, Int[(e\*x)^m\*E^(c\*I + d\*I\*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos(a + bx^2)}{x^{3/2}} dx &= -\frac{2 \cos(a + bx^2)}{\sqrt{x}} - (4b) \int \sqrt{x} \sin(a + bx^2) dx \\ &= -\frac{2 \cos(a + bx^2)}{\sqrt{x}} - (2ib) \int e^{-ia-ibx^2} \sqrt{x} dx + (2ib) \int e^{ia+ibx^2} \sqrt{x} dx \\ &= -\frac{2 \cos(a + bx^2)}{\sqrt{x}} - \frac{ibe^{ia} x^{3/2} \Gamma\left(\frac{3}{4}, -ibx^2\right)}{(-ibx^2)^{3/4}} + \frac{ibe^{-ia} x^{3/2} \Gamma\left(\frac{3}{4}, ibx^2\right)}{(ibx^2)^{3/4}} \end{aligned}$$

**Mathematica [A]**

time = 0.12, size = 114, normalized size = 1.16

$$\frac{-2(b^2x^4)^{3/4} \cos(a + bx^2) + bx^2(ibx^2)^{3/4} \Gamma\left(\frac{3}{4}, -ibx^2\right) (-i \cos(a) + \sin(a)) + i(-ibx^2)^{7/4} \Gamma\left(\frac{3}{4}, ibx^2\right) (i \cos(a) + \sin(a))}{\sqrt{x} (b^2x^4)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*x^2]/x^(3/2), x]

[Out]  $(-2*(b^2*x^4)^{(3/4)}*\text{Cos}[a + b*x^2] + b*x^2*(I*b*x^2)^{(3/4)}*\text{Gamma}[3/4, (-I)*b*x^2]*((-I)*\text{Cos}[a] + \text{Sin}[a]) + I*((-I)*b*x^2)^{(7/4)}*\text{Gamma}[3/4, I*b*x^2]*(I*\text{Cos}[a] + \text{Sin}[a]))/(\text{Sqrt}[x]*(b^2*x^4)^{(3/4)})$

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.05, size = 338, normalized size = 3.45

method	result
meijerg	$\frac{\cos(a) \sqrt{\pi} 2^{\frac{3}{4}} (b^2)^{\frac{1}{8}} \left( -\frac{12 \cdot 2^{\frac{1}{4}} \left( \frac{8x^4 b^2}{21} + \frac{2}{3} \right) \sin(bx^2)}{\sqrt{\pi} x^{\frac{5}{2}} (b^2)^{\frac{1}{8}} b} - \frac{8 \cdot 2^{\frac{1}{4}} (\cos(bx^2) x^2 b - \sin(bx^2))}{\sqrt{\pi} x^{\frac{5}{2}} (b^2)^{\frac{1}{8}} b} + \frac{32x^{\frac{7}{2}} b^2 2^{\frac{1}{4}} \sin(bx^2) \text{LommelS1}\left(\frac{5}{4}, \frac{3}{2}, bx^2\right)}{7 \sqrt{\pi} (b^2)^{\frac{1}{8}} (bx^2)^{\frac{5}{4}}} + \frac{8x^{\frac{7}{2}} b^2 \cos(bx^2) \text{LommelS1}\left(\frac{5}{4}, \frac{3}{2}, bx^2\right)}{7 \sqrt{\pi} (b^2)^{\frac{1}{8}} (bx^2)^{\frac{5}{4}}} \right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b\*x^2+a)/x^(3/2), x, method=\_RETURNVERBOSE)

[Out]  $1/8*\text{cos}(a)*\text{Pi}^{(1/2)}*2^{(3/4)}*(b^2)^{(1/8)}*(-12/\text{Pi}^{(1/2)}/x^{(5/2)}*2^{(1/4)}/(b^2)^{(1/8)}*(8/21*x^4*b^2+2/3)/b*\text{sin}(b*x^2)-8/\text{Pi}^{(1/2)}/x^{(5/2)}*2^{(1/4)}/(b^2)^{(1/8)}/b*(\text{cos}(b*x^2)*x^2*b-\text{sin}(b*x^2))+32/7/\text{Pi}^{(1/2)}*x^{(7/2)}/(b^2)^{(1/8)}*b^2*2^{(1/4)}/(b*x^2)^{(5/4)}*\text{sin}(b*x^2)*\text{LommelS1}(5/4, 3/2, b*x^2)+8/\text{Pi}^{(1/2)}*x^{(7/2)}/(b^2)^{(1/8)}*b^2*2^{(1/4)}/(b*x^2)^{(9/4)}*(\text{cos}(b*x^2)*x^2*b-\text{sin}(b*x^2))*\text{LommelS1}(1/4, 1/2, b*x^2)-1/8*\text{sin}(a)*\text{Pi}^{(1/2)}*2^{(3/4)}*b^{(1/4)}*(8/3/\text{Pi}^{(1/2)}/x^{(1/2)}*2^{(1/4)}/b^{(1/4)}*\text{sin}(b*x^2)+32/3/\text{Pi}^{(1/2)}/x^{(1/2)}*2^{(1/4)}/b^{(1/4)}*(\text{cos}(b*x^2)*x^2*b-\text{sin}(b*x^2))-8/3/\text{Pi}^{(1/2)}*x^{(7/2)}*b^{(7/4)}*2^{(1/4)}/(b*x^2)^{(5/4)}*\text{sin}($

$b*x^2)*\text{LommelS1}(1/4, 3/2, b*x^2) - 32/3/\text{Pi}^{(1/2)}*x^{(7/2)}*b^{(7/4)}*2^{(1/4)}/(b*x^2)^{(9/4)}*(\cos(b*x^2)*x^2*b - \sin(b*x^2))*\text{LommelS1}(5/4, 1/2, b*x^2)$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs.  $2(66) = 132$ .

time = 0.12, size = 133, normalized size = 1.36

$$\frac{(bx^2)^{\frac{1}{4}} \left( \left( \sqrt{\sqrt{2}+2} (\Gamma(-\frac{1}{4}, ibx^2) + \Gamma(-\frac{1}{4}, -ibx^2)) + \sqrt{-\sqrt{2}+2} (i\Gamma(-\frac{1}{4}, ibx^2) - i\Gamma(-\frac{1}{4}, -ibx^2)) \right) \cos(a) + \left( \sqrt{-\sqrt{2}+2} (\Gamma(-\frac{1}{4}, ibx^2) + \Gamma(-\frac{1}{4}, -ibx^2)) + \sqrt{\sqrt{2}+2} (-i\Gamma(-\frac{1}{4}, ibx^2) + i\Gamma(-\frac{1}{4}, -ibx^2)) \right) \sin(a) \right)}{8\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)/x^(3/2), x, algorithm="maxima")

[Out]  $-1/8*(b*x^2)^{(1/4)}*((\text{sqrt}(\text{sqrt}(2) + 2)*(\text{gamma}(-1/4, I*b*x^2) + \text{gamma}(-1/4, -I*b*x^2)) + \text{sqrt}(-\text{sqrt}(2) + 2)*(I*\text{gamma}(-1/4, I*b*x^2) - I*\text{gamma}(-1/4, -I*b*x^2)))*\cos(a) + (\text{sqrt}(-\text{sqrt}(2) + 2)*(\text{gamma}(-1/4, I*b*x^2) + \text{gamma}(-1/4, -I*b*x^2)) + \text{sqrt}(\text{sqrt}(2) + 2)*(-I*\text{gamma}(-1/4, I*b*x^2) + I*\text{gamma}(-1/4, -I*b*x^2)))*\sin(a))/\text{sqrt}(x)$

**Fricas [A]**

time = 0.12, size = 56, normalized size = 0.57

$$\frac{(ib)^{\frac{1}{4}} x e^{-ia} \Gamma(\frac{3}{4}, ibx^2) + (-ib)^{\frac{1}{4}} x e^{ia} \Gamma(\frac{3}{4}, -ibx^2) - 2\sqrt{x} \cos(bx^2 + a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)/x^(3/2), x, algorithm="fricas")

[Out]  $((I*b)^{(1/4)}*x*e^{(-I*a)}*\text{gamma}(3/4, I*b*x^2) + (-I*b)^{(1/4)}*x*e^{(I*a)}*\text{gamma}(3/4, -I*b*x^2) - 2*\text{sqrt}(x)*\cos(b*x^2 + a))/x$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + bx^2)}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x\*\*2+a)/x\*\*(3/2), x)

[Out] Integral(cos(a + b\*x\*\*2)/x\*\*(3/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)/x^(3/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(bx^2 + a)}{x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)/x^(3/2),x)
```

```
[Out] int(cos(a + b*x^2)/x^(3/2), x)
```

### 3.28 $\int \frac{\cos(a+bx^2)}{x^{5/2}} dx$

**Optimal.** Leaf size=104

$$-\frac{2 \cos(a+bx^2)}{3x^{3/2}} - \frac{ibe^{ia}\sqrt{x} \Gamma(\frac{1}{4}, -ibx^2)}{3\sqrt[4]{-ibx^2}} + \frac{ibe^{-ia}\sqrt{x} \Gamma(\frac{1}{4}, ibx^2)}{3\sqrt[4]{ibx^2}}$$

[Out]  $-2/3*\cos(b*x^2+a)/x^{(3/2)}-1/3*I*b*\exp(I*a)*\text{GAMMA}(1/4,-I*b*x^2)*x^{(1/2)}/(-I*b*x^2)^{(1/4)}+1/3*I*b*\text{GAMMA}(1/4,I*b*x^2)*x^{(1/2)}/\exp(I*a)/(I*b*x^2)^{(1/4)}$

**Rubi [A]**

time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3469, 3470, 2250}

$$-\frac{ie^{ia}b\sqrt{x} \Gamma(\frac{1}{4}, -ibx^2)}{3\sqrt[4]{-ibx^2}} + \frac{ie^{-ia}b\sqrt{x} \Gamma(\frac{1}{4}, ibx^2)}{3\sqrt[4]{ibx^2}} - \frac{2 \cos(a+bx^2)}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^2]/x^(5/2), x]

[Out]  $(-2*\text{Cos}[a + b*x^2])/(3*x^{(3/2)}) - ((I/3)*b*E^{(I*a)}*\text{Sqrt}[x]*\text{Gamma}[1/4, (-I)*b*x^2])/((-I)*b*x^2)^{(1/4)} + ((I/3)*b*\text{Sqrt}[x]*\text{Gamma}[1/4, I*b*x^2])/(E^{(I*a)}*(I*b*x^2)^{(1/4)})$

Rule 2250

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-F^a)\*((e + f\*x)^(m + 1)/(f\*n\*((-b)\*(c + d\*x)^n\*Log[F])^(m + 1)/n))\*Gamma[(m + 1)/n, (-b)\*(c + d\*x)^n\*Log[F]], x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

Rule 3469

Int[Cos[(c\_.) + (d\_.)\*(x\_)]^(n\_)\*((e\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(e\*x)^(m + 1)\*(Cos[c + d\*x^n]/(e\*(m + 1))), x] + Dist[d\*(n/(e^n\*(m + 1))), Int[(e\*x)^(m + n)\*Sin[c + d\*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3470

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := Dist[I/2, Int[(e\*x)^m\*E^((-c)\*I - d\*I\*x^n), x], x] - Dist[I/2, Int[(e\*x)^m\*E^(c\*I + d\*I\*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx^2)}{x^{5/2}} dx &= -\frac{2\cos(a+bx^2)}{3x^{3/2}} - \frac{1}{3}(4b) \int \frac{\sin(a+bx^2)}{\sqrt{x}} dx \\
&= -\frac{2\cos(a+bx^2)}{3x^{3/2}} - \frac{1}{3}(2ib) \int \frac{e^{-ia-ibx^2}}{\sqrt{x}} dx + \frac{1}{3}(2ib) \int \frac{e^{ia+ibx^2}}{\sqrt{x}} dx \\
&= -\frac{2\cos(a+bx^2)}{3x^{3/2}} - \frac{ibe^{ia}\sqrt{x}\Gamma(\frac{1}{4},-ibx^2)}{3\sqrt[4]{-ibx^2}} + \frac{ibe^{-ia}\sqrt{x}\Gamma(\frac{1}{4},ibx^2)}{3\sqrt[4]{ibx^2}}
\end{aligned}$$

**Mathematica [A]**

time = 0.11, size = 117, normalized size = 1.12

$$\frac{-2\sqrt[4]{b^2x^4}\cos(a+bx^2)+bx^2\sqrt[4]{ibx^2}\Gamma(\frac{1}{4},-ibx^2)(-\cos(a)+\sin(a))+i(-ibx^2)^{5/4}\Gamma(\frac{1}{4},ibx^2)(i\cos(a)+\sin(a))}{3x^{3/2}\sqrt[4]{b^2x^4}}$$

Antiderivative was successfully verified.

**[In]** Integrate[Cos[a + b\*x^2]/x^(5/2),x]

**[Out]**  $(-2*(b^2*x^4)^{(1/4)}*\text{Cos}[a + b*x^2] + b*x^2*(I*b*x^2)^{(1/4)}*\text{Gamma}[1/4, (-I)*b*x^2]*((-I)*\text{Cos}[a] + \text{Sin}[a]) + I*((-I)*b*x^2)^{(5/4)}*\text{Gamma}[1/4, I*b*x^2]*(I*\text{Cos}[a] + \text{Sin}[a]))/(3*x^{(3/2)}*(b^2*x^4)^{(1/4)})$

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.05, size = 358, normalized size = 3.44

method	result
meijerg	$ \frac{\cos(a)\sqrt{\pi}2^{\frac{1}{4}}(b^2)^{\frac{3}{8}}\left(-\frac{42^{\frac{3}{4}}\left(\frac{8x^4b^2}{15}+\frac{2}{3}\right)\sin(bx^2)}{\sqrt{\pi}x^{\frac{7}{2}}(b^2)^{\frac{3}{8}}b}-\frac{82^{\frac{3}{4}}(-16x^4b^2+5)(\cos(bx^2)x^2b-\sin(bx^2))}{15\sqrt{\pi}x^{\frac{7}{2}}(b^2)^{\frac{3}{8}}b}+\frac{32x^{\frac{9}{2}}2^{\frac{3}{4}}b^3\sin(bx^2)\text{LommelS1}\left(\frac{3}{4},\frac{3}{2},bx^2\right)}{15\sqrt{\pi}(b^2)^{\frac{3}{8}}(bx^2)^{\frac{7}{4}}}\right)}{8} $

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(cos(b\*x^2+a)/x^(5/2),x,method=\_RETURNVERBOSE)

**[Out]**  $1/8*\cos(a)*\text{Pi}^{(1/2)}*2^{(1/4)}*(b^2)^{(3/8)}*(-4/\text{Pi}^{(1/2)}/x^{(7/2)}*2^{(3/4)}/(b^2)^{(3/8)}*(8/15*x^4*b^2+2/3)/b*\sin(b*x^2)-8/15/\text{Pi}^{(1/2)}/x^{(7/2)}*2^{(3/4)}/(b^2)^{(3/8)}/b*(-16*b^2*x^4+5)*(cos(b*x^2)*x^2*b-\sin(b*x^2))+32/15/\text{Pi}^{(1/2)}*x^{(9/2)}/(b^2)^{(3/8)}*2^{(3/4)}*b^3/(b*x^2)^{(7/4)}*\sin(b*x^2)*\text{LommelS1}(3/4,3/2,b*x^2)-128/15/\text{Pi}^{(1/2)}*x^{(9/2)}/(b^2)^{(3/8)}*2^{(3/4)}*b^3/(b*x^2)^{(11/4)}*(cos(b*x^2)*x^2*b-\sin(b*x^2))*\text{LommelS1}(7/4,1/2,b*x^2))-1/8*\sin(a)*\text{Pi}^{(1/2)}*2^{(1/4)}*b^{(3/4)}*(12/\text{Pi}^{(1/2)}/x^{(3/2)}*2^{(3/4)}/b^{(3/4)}*(32/81*x^4*b^2+2/3)*\sin(b*x^2)+32/3/\text{Pi}^{(1/2)}/x^{(3/2)}*2^{(3/4)}/b^{(3/4)}*(cos(b*x^2)*x^2*b-\sin(b*x^2))-128/27/\text{Pi}^{(1/2)}*x^{(9/2)}*b^{(9/4)}*2^{(3/4)}/(b*x^2)^{(7/4)}*\sin(b*x^2)*\text{LommelS1}(7/4,3/2,b*x^2)-32/3/\text{Pi}^{(1/2)}*x^{(9/2)}*b^{(9/4)}*2^{(3/4)}/(b*x^2)^{(11/4)}*(cos(b*x^2)*x^2*b-\sin(b*x^2))*\text{LommelS1}(3/4,1/2,b*x^2))$

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs.  $2(66) = 132$ .  
time = 0.12, size = 133, normalized size = 1.28

$$\frac{(bx^2)^{\frac{3}{2}} \left( \left( \sqrt{-\sqrt{2}+2} \Gamma(-\frac{3}{4}, ibx^2) + \Gamma(-\frac{3}{4}, -ibx^2) \right) + \sqrt{\sqrt{2}+2} \left( i\Gamma(-\frac{3}{4}, ibx^2) - i\Gamma(-\frac{3}{4}, -ibx^2) \right) \right) \cos(a) + \left( \sqrt{\sqrt{2}+2} \Gamma(-\frac{3}{4}, ibx^2) + \Gamma(-\frac{3}{4}, -ibx^2) \right) + \sqrt{-\sqrt{2}+2} \left( -i\Gamma(-\frac{3}{4}, ibx^2) + i\Gamma(-\frac{3}{4}, -ibx^2) \right) \sin(a)}{8x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)/x^(5/2),x, algorithm="maxima")

[Out]  $-1/8*(b*x^2)^{(3/4)}*((\sqrt{-\sqrt{2}+2})*(\text{gamma}(-3/4, I*b*x^2) + \text{gamma}(-3/4, -I*b*x^2)) + \sqrt{\sqrt{2}+2}*(I*\text{gamma}(-3/4, I*b*x^2) - I*\text{gamma}(-3/4, -I*b*x^2)))*\cos(a) + (\sqrt{\sqrt{2}+2}*(\text{gamma}(-3/4, I*b*x^2) + \text{gamma}(-3/4, -I*b*x^2)) + \sqrt{-\sqrt{2}+2}*(-I*\text{gamma}(-3/4, I*b*x^2) + I*\text{gamma}(-3/4, -I*b*x^2)))*\sin(a))/x^{(3/2)}$

**Fricas [A]**

time = 0.12, size = 61, normalized size = 0.59

$$\frac{(ib)^{\frac{3}{4}} x^2 e^{-ia} \Gamma(\frac{1}{4}, ibx^2) + (-ib)^{\frac{3}{4}} x^2 e^{ia} \Gamma(\frac{1}{4}, -ibx^2) - 2\sqrt{x} \cos(bx^2 + a)}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)/x^(5/2),x, algorithm="fricas")

[Out]  $1/3*((I*b)^{(3/4)}*x^2*e^{-I*a}*\text{gamma}(1/4, I*b*x^2) + (-I*b)^{(3/4)}*x^2*e^{I*a})*\text{gamma}(1/4, -I*b*x^2) - 2*\sqrt{x}*\cos(b*x^2 + a))/x^2$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + bx^2)}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x\*\*2+a)/x\*\*(5/2),x)

[Out] Integral(cos(a + b\*x\*\*2)/x\*\*(5/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)/x^(5/2),x, algorithm="giac")



[Out] integrate(cos(b\*x^2 + a)/x^(5/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(bx^2 + a)}{x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^2)/x^(5/2),x)

[Out] int(cos(a + b\*x^2)/x^(5/2), x)

### 3.29 $\int x^{5/2} \cos^2(a + bx^2) dx$

**Optimal.** Leaf size=132

$$\frac{x^{7/2}}{7} - \frac{3ie^{2ia}x^{3/2}\Gamma(\frac{3}{4}, -2ibx^2)}{64 2^{3/4}b(-ibx^2)^{3/4}} + \frac{3ie^{-2ia}x^{3/2}\Gamma(\frac{3}{4}, 2ibx^2)}{64 2^{3/4}b(ibx^2)^{3/4}} + \frac{x^{3/2} \sin(2(a + bx^2))}{8b}$$

[Out] 1/7\*x^(7/2)-3/128\*I\*exp(2\*I\*a)\*x^(3/2)\*GAMMA(3/4,-2\*I\*b\*x^2)\*2^(1/4)/b/(-I\*b\*x^2)^(3/4)+3/128\*I\*x^(3/2)\*GAMMA(3/4,2\*I\*b\*x^2)\*2^(1/4)/b/exp(2\*I\*a)/(I\*b\*x^2)^(3/4)+1/8\*x^(3/2)\*sin(2\*b\*x^2+2\*a)/b

**Rubi [A]**

time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3483, 3485, 3467, 3470, 2250}

$$-\frac{3ie^{2ia}x^{3/2}\Gamma(\frac{3}{4}, -2ibx^2)}{64 2^{3/4}b(-ibx^2)^{3/4}} + \frac{3ie^{-2ia}x^{3/2}\Gamma(\frac{3}{4}, 2ibx^2)}{64 2^{3/4}b(ibx^2)^{3/4}} + \frac{x^{3/2} \sin(2(a + bx^2))}{8b} + \frac{x^{7/2}}{7}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)\*Cos[a + b\*x^2]^2,x]

[Out] x^(7/2)/7 - (((3\*I)/64)\*E^((2\*I)\*a)\*x^(3/2)\*Gamma[3/4, (-2\*I)\*b\*x^2])/(2^(3/4)\*b\*((-I)\*b\*x^2)^(3/4)) + (((3\*I)/64)\*x^(3/2)\*Gamma[3/4, (2\*I)\*b\*x^2])/(2^(3/4)\*b\*E^((2\*I)\*a)\*(I\*b\*x^2)^(3/4)) + (x^(3/2)\*Sin[2\*(a + b\*x^2)])/(8\*b)

Rule 2250

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-F^a)\*((e + f\*x)^(m + 1)/(f\*n\*((-b)\*(c + d\*x)^n\*Log[F])^(m + 1)/n))\*Gamma[(m + 1)/n, (-b)\*(c + d\*x)^n\*Log[F]], x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

Rule 3467

Int[Cos[(c\_.) + (d\_.)\*(x\_)]^(n\_)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[e^(n - 1)\*(e\*x)^(m - n + 1)\*(Sin[c + d\*x^n]/(d\*n)), x] - Dist[e^n\*((m - n + 1)/(d\*n)), Int[(e\*x)^(m - n)\*Sin[c + d\*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

Rule 3470

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := Dist[I/2, Int[(e\*x)^m\*E^((-c)\*I - d\*I\*x^n), x], x] - Dist[I/2, Int[(e\*x)^m\*E^(c\*I + d\*I\*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]

Rule 3483

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*Cos[c + d\*(x^(k\*n)/e^n])]^p, x], x, (e\*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]

Rule 3485

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandTrigReduce[(e\*x)^m, (a + b\*Cos[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int x^{5/2} \cos^2(a + bx^2) dx &= 2 \text{Subst} \left( \int x^6 \cos^2(a + bx^4) dx, x, \sqrt{x} \right) \\
 &= 2 \text{Subst} \left( \int \left( \frac{x^6}{2} + \frac{1}{2} x^6 \cos(2a + 2bx^4) \right) dx, x, \sqrt{x} \right) \\
 &= \frac{x^{7/2}}{7} + \text{Subst} \left( \int x^6 \cos(2a + 2bx^4) dx, x, \sqrt{x} \right) \\
 &= \frac{x^{7/2}}{7} + \frac{x^{3/2} \sin(2(a + bx^2))}{8b} - \frac{3 \text{Subst} \left( \int x^2 \sin(2a + 2bx^4) dx, x, \sqrt{x} \right)}{8b} \\
 &= \frac{x^{7/2}}{7} + \frac{x^{3/2} \sin(2(a + bx^2))}{8b} - \frac{(3i) \text{Subst} \left( \int e^{-2ia - 2ibx^4} x^2 dx, x, \sqrt{x} \right)}{16b} + \frac{(3i) \text{Subst} \left( \int e^{2ia + 2ibx^4} x^2 dx, x, \sqrt{x} \right)}{16b} \\
 &= \frac{x^{7/2}}{7} - \frac{3ie^{2ia} x^{3/2} \Gamma\left(\frac{3}{4}, -2ibx^2\right)}{64 2^{3/4} b (-ibx^2)^{3/4}} + \frac{3ie^{-2ia} x^{3/2} \Gamma\left(\frac{3}{4}, 2ibx^2\right)}{64 2^{3/4} b (ibx^2)^{3/4}} + \frac{x^{3/2} \sin(2(a + bx^2))}{8b}
 \end{aligned}$$

Mathematica [A]

time = 0.27, size = 142, normalized size = 1.08

$$\frac{bx^{11/2} \left( 21\sqrt{2} (ibx^2)^{3/4} \Gamma\left(\frac{3}{4}, -2ibx^2\right) (-i \cos(2a) + \sin(2a)) + 21\sqrt{2} (-ibx^2)^{3/4} \Gamma\left(\frac{3}{4}, 2ibx^2\right) (i \cos(2a) + \sin(2a)) + 16(b^2x^4)^{3/4} (8bx^2 + 7 \sin(2(a + bx^2))) \right)}{896 (b^2x^4)^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)\*Cos[a + b\*x^2]^2,x]

[Out] (b\*x^(11/2)\*(21\*2^(1/4)\*(I\*b\*x^2)^(3/4)\*Gamma[3/4, (-2\*I)\*b\*x^2]\*((-I)\*Cos[2\*a] + Sin[2\*a]) + 21\*2^(1/4)\*((-I)\*b\*x^2)^(3/4)\*Gamma[3/4, (2\*I)\*b\*x^2]\*(I\*Cos[2\*a] + Sin[2\*a]) + 16\*(b^2\*x^4)^(3/4)\*(8\*b\*x^2 + 7\*Sin[2\*(a + b\*x^2)]))/(896\*(b^2\*x^4)^(7/4))

**Maple [F]**

time = 0.12, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} (\cos^2 (b x^2 + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*cos(b\*x^2+a)^2,x)

[Out] int(x^(5/2)\*cos(b\*x^2+a)^2,x)

**Maxima [A]**

time = 0.12, size = 171, normalized size = 1.30

$$\frac{256 b^2 x^4 + 224 b x^2 \sin(2 b x^2 + 2 a) + 21 \cdot 2^{1/4} (b x^2)^{3/4} \left( \left( \sqrt{\sqrt{2} + 2} \Gamma\left(\frac{3}{4}, 2i b x^2\right) + \Gamma\left(\frac{3}{4}, -2i b x^2\right) \right) + \sqrt{-\sqrt{2} + 2} \left( i \Gamma\left(\frac{3}{4}, 2i b x^2\right) - i \Gamma\left(\frac{3}{4}, -2i b x^2\right) \right) \right) \cos(2 a) + \left( \sqrt{-\sqrt{2} + 2} \Gamma\left(\frac{3}{4}, 2i b x^2\right) + \Gamma\left(\frac{3}{4}, -2i b x^2\right) \right) + \sqrt{\sqrt{2} + 2} \left( -i \Gamma\left(\frac{3}{4}, 2i b x^2\right) + i \Gamma\left(\frac{3}{4}, -2i b x^2\right) \right) \sin(2 a)}{1792 \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*cos(b\*x^2+a)^2,x, algorithm="maxima")

[Out] 1/1792\*(256\*b^2\*x^4 + 224\*b\*x^2\*sin(2\*b\*x^2 + 2\*a) + 21\*2^(1/4)\*(b\*x^2)^(1/4)\*((sqrt(sqrt(2) + 2)\*(gamma(3/4, 2\*I\*b\*x^2) + gamma(3/4, -2\*I\*b\*x^2)) + sqrt(-sqrt(2) + 2)\*(I\*gamma(3/4, 2\*I\*b\*x^2) - I\*gamma(3/4, -2\*I\*b\*x^2)))\*cos(2\*a) + (sqrt(-sqrt(2) + 2)\*(gamma(3/4, 2\*I\*b\*x^2) + gamma(3/4, -2\*I\*b\*x^2)) + sqrt(sqrt(2) + 2)\*(-I\*gamma(3/4, 2\*I\*b\*x^2) + I\*gamma(3/4, -2\*I\*b\*x^2)))\*sin(2\*a)))/(b^2\*sqrt(x))

**Fricas [A]**

time = 0.13, size = 78, normalized size = 0.59

$$\frac{21 (2i b)^{\frac{1}{4}} e^{(-2i a)} \Gamma\left(\frac{3}{4}, 2i b x^2\right) + 21 (-2i b)^{\frac{1}{4}} e^{(2i a)} \Gamma\left(\frac{3}{4}, -2i b x^2\right) + 32 (4 b^2 x^3 + 7 b x \cos(b x^2 + a) \sin(b x^2 + a)) \sqrt{x}}{896 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*cos(b\*x^2+a)^2,x, algorithm="fricas")

[Out] 1/896\*(21\*(2\*I\*b)^(1/4)\*e^(-2\*I\*a)\*gamma(3/4, 2\*I\*b\*x^2) + 21\*(-2\*I\*b)^(1/4)\*e^(2\*I\*a)\*gamma(3/4, -2\*I\*b\*x^2) + 32\*(4\*b^2\*x^3 + 7\*b\*x\*cos(b\*x^2 + a)\*sin(b\*x^2 + a))\*sqrt(x))/b^2

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} \cos^2 (a + b x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)\*cos(b\*x\*\*2+a)\*\*2,x)

[Out] Integral(x\*\*(5/2)\*cos(a + b\*x\*\*2)\*\*2, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)\*cos(b\*x^2+a)^2,x, algorithm="giac")

[Out] integrate(x^(5/2)\*cos(b\*x^2 + a)^2, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2} \cos(bx^2 + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*cos(a + b\*x^2)^2,x)

[Out] int(x^(5/2)\*cos(a + b\*x^2)^2, x)

### 3.30 $\int x^{3/2} \cos^2(a + bx^2) dx$

**Optimal.** Leaf size=132

$$\frac{x^{5/2}}{5} - \frac{ie^{2ia} \sqrt{x} \Gamma(\frac{1}{4}, -2ibx^2)}{64\sqrt[4]{2} b\sqrt[4]{-ibx^2}} + \frac{ie^{-2ia} \sqrt{x} \Gamma(\frac{1}{4}, 2ibx^2)}{64\sqrt[4]{2} b\sqrt[4]{ibx^2}} + \frac{\sqrt{x} \sin(2(a + bx^2))}{8b}$$

[Out] 1/5\*x^(5/2)-1/128\*I\*exp(2\*I\*a)\*GAMMA(1/4,-2\*I\*b\*x^2)\*x^(1/2)\*2^(3/4)/b/(-I\*b\*x^2)^(1/4)+1/128\*I\*GAMMA(1/4,2\*I\*b\*x^2)\*x^(1/2)\*2^(3/4)/b/exp(2\*I\*a)/(I\*b\*x^2)^(1/4)+1/8\*sin(2\*b\*x^2+2\*a)\*x^(1/2)/b

**Rubi [A]**

time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3483, 3485, 3467, 3436, 2239}

$$-\frac{ie^{2ia} \sqrt{x} \Gamma(\frac{1}{4}, -2ibx^2)}{64\sqrt[4]{2} b\sqrt[4]{-ibx^2}} + \frac{ie^{-2ia} \sqrt{x} \Gamma(\frac{1}{4}, 2ibx^2)}{64\sqrt[4]{2} b\sqrt[4]{ibx^2}} + \frac{\sqrt{x} \sin(2(a + bx^2))}{8b} + \frac{x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)\*Cos[a + b\*x^2]^2,x]

[Out] x^(5/2)/5 - ((I/64)\*E^((2\*I)\*a)\*Sqrt[x]\*Gamma[1/4, (-2\*I)\*b\*x^2])/(2^(1/4)\*b\*((-I)\*b\*x^2)^(1/4)) + ((I/64)\*Sqrt[x]\*Gamma[1/4, (2\*I)\*b\*x^2])/(2^(1/4)\*b\*E^((2\*I)\*a)\*(I\*b\*x^2)^(1/4)) + (Sqrt[x]\*Sin[2\*(a + b\*x^2)])/(8\*b)

Rule 2239

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] :> Simp[(-F^a)\*(c + d\*x)\*(Gamma[1/n, (-b)\*(c + d\*x)^n\*Log[F]]/(d\*n\*((-b)\*(c + d\*x)^n\*Log[F]))^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

Rule 3436

Int[Sin[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_))^(n\_)], x\_Symbol] :> Dist[I/2, Int[E^((-c)\*I - d\*I\*(e + f\*x)^n), x], x] - Dist[I/2, Int[E^(c\*I + d\*I\*(e + f\*x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]

Rule 3467

Int[Cos[(c\_.) + (d\_.)\*(x\_)]^n\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[e^(n-1)\*(e\*x)^(m-n+1)\*(Sin[c + d\*x^n]/(d\*n)), x] - Dist[e^n\*((m-n+1)/(d\*n)), Int[(e\*x)^(m-n)\*Sin[c + d\*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m+1]

Rule 3483

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*((e_.)*(x_)^(m_.), x
_Symbol] := With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1)
*(a + b*Cos[c + d*(x^(k*n)/e^n)])^p, x], x, (e*x)^(1/k)], x] /; FreeQ[{a,
b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

### Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*((e_.)*(x_)^(m_.), x
_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^{3/2} \cos^2(a + bx^2) dx &= 2 \text{Subst}\left(\int x^4 \cos^2(a + bx^4) dx, x, \sqrt{x}\right) \\
&= 2 \text{Subst}\left(\int \left(\frac{x^4}{2} + \frac{1}{2}x^4 \cos(2a + 2bx^4)\right) dx, x, \sqrt{x}\right) \\
&= \frac{x^{5/2}}{5} + \text{Subst}\left(\int x^4 \cos(2a + 2bx^4) dx, x, \sqrt{x}\right) \\
&= \frac{x^{5/2}}{5} + \frac{\sqrt{x} \sin(2(a + bx^2))}{8b} - \frac{\text{Subst}\left(\int \sin(2a + 2bx^4) dx, x, \sqrt{x}\right)}{8b} \\
&= \frac{x^{5/2}}{5} + \frac{\sqrt{x} \sin(2(a + bx^2))}{8b} - \frac{i \text{Subst}\left(\int e^{-2ia - 2ibx^4} dx, x, \sqrt{x}\right)}{16b} + \frac{i \text{Subst}\left(\int e^{2ia + 2ibx^4} dx, x, \sqrt{x}\right)}{16b} \\
&= \frac{x^{5/2}}{5} - \frac{ie^{2ia} \sqrt{x} \Gamma\left(\frac{1}{4}, -2ibx^2\right)}{64\sqrt[4]{2} b^4 \sqrt{-ibx^2}} + \frac{ie^{-2ia} \sqrt{x} \Gamma\left(\frac{1}{4}, 2ibx^2\right)}{64\sqrt[4]{2} b^4 \sqrt{ibx^2}} + \frac{\sqrt{x} \sin(2(a + bx^2))}{8b}
\end{aligned}$$

### Mathematica [A]

time = 0.22, size = 142, normalized size = 1.08

$$\frac{bx^{9/2} \left( 5 \sqrt[4]{ibx^2} \Gamma\left(\frac{1}{4}, -2ibx^2\right) (-i \cos(2a) + \sin(2a)) + 5 \sqrt[4]{-ibx^2} \Gamma\left(\frac{1}{4}, 2ibx^2\right) (i \cos(2a) + \sin(2a)) + 16 \sqrt[4]{b^2 x^4} (8bx^2 + 5 \sin(2(a + bx^2))) \right)}{640 (b^2 x^4)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Cos[a + b*x^2]^2, x]
```

```
[Out] (b*x^(9/2)*(5*2^(3/4)*(I*b*x^2)^(1/4)*Gamma[1/4, (-2*I)*b*x^2]*((-I)*Cos[2*
a] + Sin[2*a]) + 5*2^(3/4)*((-I)*b*x^2)^(1/4)*Gamma[1/4, (2*I)*b*x^2]*(I*Co
s[2*a] + Sin[2*a]) + 16*(b^2*x^4)^(1/4)*(8*b*x^2 + 5*Sin[2*(a + b*x^2)])))/
(640*(b^2*x^4)^(5/4))
```

**Maple [F]**

time = 0.10, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} (\cos^2 (b x^2 + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*cos(b\*x^2+a)^2,x)

[Out] int(x^(3/2)\*cos(b\*x^2+a)^2,x)

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(87) = 174.

time = 0.12, size = 183, normalized size = 1.39

$$\frac{2^{\frac{1}{2}} (16 \cdot 2^{\frac{1}{2}} (8 b x^{\frac{3}{2}} + 5 \sqrt{x} \sin(2 b x^2 + 2 a)) (b x^2)^{\frac{1}{4}} + 5 \left( (\sqrt{-\sqrt{2} + 2} \Gamma(\frac{1}{4}, 2 i b x^2) + \Gamma(\frac{1}{4}, -2 i b x^2)) - \sqrt{\sqrt{2} + 2} (-i \Gamma(\frac{1}{4}, 2 i b x^2) + i \Gamma(\frac{1}{4}, -2 i b x^2)) \right) \cos(2 a) + (\sqrt{\sqrt{2} + 2} \Gamma(\frac{1}{4}, 2 i b x^2) + \Gamma(\frac{1}{4}, -2 i b x^2)) - \sqrt{-\sqrt{2} + 2} (i \Gamma(\frac{1}{4}, 2 i b x^2) - i \Gamma(\frac{1}{4}, -2 i b x^2)) \sin(2 a)) \sqrt{x}}{1280 (b x^2)^{\frac{1}{4}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*cos(b\*x^2+a)^2,x, algorithm="maxima")

[Out] 1/1280\*2^(3/4)\*(16\*2^(1/4)\*(8\*b\*x^(5/2) + 5\*sqrt(x)\*sin(2\*b\*x^2 + 2\*a))\*(b\*x^2)^(1/4) + 5\*((sqrt(-sqrt(2) + 2)\*(gamma(1/4, 2\*I\*b\*x^2) + gamma(1/4, -2\*I\*b\*x^2)) - sqrt(sqrt(2) + 2)\*(-I\*gamma(1/4, 2\*I\*b\*x^2) + I\*gamma(1/4, -2\*I\*b\*x^2)))\*cos(2\*a) + (sqrt(sqrt(2) + 2)\*(gamma(1/4, 2\*I\*b\*x^2) + gamma(1/4, -2\*I\*b\*x^2)) - sqrt(-sqrt(2) + 2)\*(I\*gamma(1/4, 2\*I\*b\*x^2) - I\*gamma(1/4, -2\*I\*b\*x^2)))\*sin(2\*a))\*sqrt(x))/(b\*x^2)^(1/4)\*b

**Fricas [A]**

time = 0.11, size = 77, normalized size = 0.58

$$\frac{5(2i b)^{\frac{3}{4}} e^{(-2i a)} \Gamma(\frac{1}{4}, 2i b x^2) + 5(-2i b)^{\frac{3}{4}} e^{(2i a)} \Gamma(\frac{1}{4}, -2i b x^2) + 32(4b^2x^2 + 5b \cos(bx^2 + a) \sin(bx^2 + a)) \sqrt{x}}{640b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*cos(b\*x^2+a)^2,x, algorithm="fricas")

[Out] 1/640\*(5\*(2\*I\*b)^(3/4)\*e^(-2\*I\*a)\*gamma(1/4, 2\*I\*b\*x^2) + 5\*(-2\*I\*b)^(3/4)\*e^(2\*I\*a)\*gamma(1/4, -2\*I\*b\*x^2) + 32\*(4\*b^2\*x^2 + 5\*b\*cos(b\*x^2 + a)\*sin(b\*x^2 + a))\*sqrt(x))/b^2

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \cos^2 (a + b x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x\*\*(3/2)\*cos(b\*x\*\*2+a)\*\*2,x)

[Out] Integral(x\*\*(3/2)\*cos(a + b\*x\*\*2)\*\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*cos(b\*x^2+a)^2,x, algorithm="giac")

[Out] integrate(x^(3/2)\*cos(b\*x^2 + a)^2, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2} \cos(bx^2 + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*cos(a + b\*x^2)^2,x)

[Out] int(x^(3/2)\*cos(a + b\*x^2)^2, x)

### 3.31 $\int \sqrt{x} \cos^2(a + bx^2) dx$

Optimal. Leaf size=100

$$\frac{x^{3/2}}{3} - \frac{e^{2ia}x^{3/2}\Gamma(\frac{3}{4}, -2ibx^2)}{8 \cdot 2^{3/4}(-ibx^2)^{3/4}} - \frac{e^{-2ia}x^{3/2}\Gamma(\frac{3}{4}, 2ibx^2)}{8 \cdot 2^{3/4}(ibx^2)^{3/4}}$$

[Out] 1/3\*x^(3/2)-1/16\*exp(2\*I\*a)\*x^(3/2)\*GAMMA(3/4,-2\*I\*b\*x^2)\*2^(1/4)/(-I\*b\*x^2)^(3/4)-1/16\*x^(3/2)\*GAMMA(3/4,2\*I\*b\*x^2)\*2^(1/4)/exp(2\*I\*a)/(I\*b\*x^2)^(3/4)

Rubi [A]

time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3483, 3485, 3471, 2250}

$$-\frac{e^{2ia}x^{3/2}\Gamma(\frac{3}{4}, -2ibx^2)}{8 \cdot 2^{3/4}(-ibx^2)^{3/4}} - \frac{e^{-2ia}x^{3/2}\Gamma(\frac{3}{4}, 2ibx^2)}{8 \cdot 2^{3/4}(ibx^2)^{3/4}} + \frac{x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]\*Cos[a + b\*x^2]^2,x]

[Out] x^(3/2)/3 - (E^((2\*I)\*a)\*x^(3/2)\*Gamma[3/4, (-2\*I)\*b\*x^2])/(8\*2^(3/4)\*((-I)\*b\*x^2)^(3/4)) - (x^(3/2)\*Gamma[3/4, (2\*I)\*b\*x^2])/(8\*2^(3/4)\*E^((2\*I)\*a)\*(I\*b\*x^2)^(3/4))

Rule 2250

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-F^a)\*((e + f\*x)^(m + 1)/(f\*n\*((-b)\*(c + d\*x)^n\*Log[F])^(m + 1)/n))\*Gamma[(m + 1)/n, (-b)\*(c + d\*x)^n\*Log[F]], x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

Rule 3471

Int[Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)]\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/2, Int[(e\*x)^m\*E^((-c)\*I - d\*I\*x^n), x], x] + Dist[1/2, Int[(e\*x)^m\*E^(c\*I + d\*I\*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]

Rule 3483

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)]\*(b\_.))^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*Cos[c + d\*(x^(k\*n)/e^n]])^p, x], x, (e\*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]

Rule 3485

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_)\*((e\_.)\*(x\_)^(m\_.), x  
\_Symbol] :> Int[ExpandTrigReduce[(e\*x)^m, (a + b\*Cos[c + d\*x^n])^p, x], x]  
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \sqrt{x} \cos^2(a + bx^2) dx &= 2\text{Subst}\left(\int x^2 \cos^2(a + bx^4) dx, x, \sqrt{x}\right) \\
 &= 2\text{Subst}\left(\int \left(\frac{x^2}{2} + \frac{1}{2}x^2 \cos(2a + 2bx^4)\right) dx, x, \sqrt{x}\right) \\
 &= \frac{x^{3/2}}{3} + \text{Subst}\left(\int x^2 \cos(2a + 2bx^4) dx, x, \sqrt{x}\right) \\
 &= \frac{x^{3/2}}{3} + \frac{1}{2}\text{Subst}\left(\int e^{-2ia-2ibx^4} x^2 dx, x, \sqrt{x}\right) + \frac{1}{2}\text{Subst}\left(\int e^{2ia+2ibx^4} x^2 dx, x, \sqrt{x}\right) \\
 &= \frac{x^{3/2}}{3} - \frac{e^{2ia} x^{3/2} \Gamma\left(\frac{3}{4}, -2ibx^2\right)}{8 \cdot 2^{3/4} (-ibx^2)^{3/4}} - \frac{e^{-2ia} x^{3/2} \Gamma\left(\frac{3}{4}, 2ibx^2\right)}{8 \cdot 2^{3/4} (ibx^2)^{3/4}}
 \end{aligned}$$

Mathematica [A]

time = 0.15, size = 122, normalized size = 1.22

$$\frac{x^{3/2} \left( 16(b^2 x^4)^{3/4} - 3\sqrt{2} (-ibx^2)^{3/4} \Gamma\left(\frac{3}{4}, 2ibx^2\right) (\cos(2a) - i \sin(2a)) - 3\sqrt{2} (ibx^2)^{3/4} \Gamma\left(\frac{3}{4}, -2ibx^2\right) (\cos(2a) + i \sin(2a)) \right)}{48 (b^2 x^4)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*Cos[a + b\*x^2]^2,x]

[Out] (x^(3/2)\*(16\*(b^2\*x^4)^(3/4) - 3\*2^(1/4)\*((-I)\*b\*x^2)^(3/4)\*Gamma[3/4, (2\*I)\*b\*x^2]\*(Cos[2\*a] - I\*Sin[2\*a]) - 3\*2^(1/4)\*(I\*b\*x^2)^(3/4)\*Gamma[3/4, (-2\*I)\*b\*x^2]\*(Cos[2\*a] + I\*Sin[2\*a]))/(48\*(b^2\*x^4)^(3/4))

Maple [F]

time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{x} (\cos^2(bx^2 + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)\*cos(b\*x^2+a)^2,x)

[Out] int(x^(1/2)\*cos(b\*x^2+a)^2,x)

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 154 vs.  $2(62) = 124$ .  
time = 0.12, size = 154, normalized size = 1.54

$$\frac{32bx^2 - 3 \cdot 2^{\frac{1}{2}}(bx^2)^{\frac{1}{2}} \left( \left( \sqrt{-\sqrt{2} + 2} \left( \Gamma\left(\frac{3}{4}, 2ibx^2\right) + \Gamma\left(\frac{3}{4}, -2ibx^2\right) \right) + \sqrt{\sqrt{2} + 2} \left( -i\Gamma\left(\frac{3}{4}, 2ibx^2\right) + i\Gamma\left(\frac{3}{4}, -2ibx^2\right) \right) \right) \cos(2a) - \left( \sqrt{\sqrt{2} + 2} \left( \Gamma\left(\frac{3}{4}, 2ibx^2\right) + \Gamma\left(\frac{3}{4}, -2ibx^2\right) \right) - \sqrt{-\sqrt{2} + 2} \left( -i\Gamma\left(\frac{3}{4}, 2ibx^2\right) + i\Gamma\left(\frac{3}{4}, -2ibx^2\right) \right) \right) \sin(2a)}{96b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)\*cos(b\*x^2+a)^2,x, algorithm="maxima")

[Out]  $\frac{1}{96} * (32 * b * x^2 - 3 * 2^{(1/4)} * (b * x^2)^{(1/4)} * ((\sqrt{-\sqrt{2} + 2}) * (\gamma(3/4, 2 * I * b * x^2) + \gamma(3/4, -2 * I * b * x^2)) + \sqrt{\sqrt{2} + 2}) * (-I * \gamma(3/4, 2 * I * b * x^2) + I * \gamma(3/4, -2 * I * b * x^2))) * \cos(2 * a) - (\sqrt{\sqrt{2} + 2}) * (\gamma(3/4, 2 * I * b * x^2) + \gamma(3/4, -2 * I * b * x^2)) - \sqrt{-\sqrt{2} + 2}) * (-I * \gamma(3/4, 2 * I * b * x^2) + I * \gamma(3/4, -2 * I * b * x^2))) * \sin(2 * a)) / (b * \sqrt{x})$

**Fricas [A]**

time = 0.12, size = 50, normalized size = 0.50

$$\frac{16bx^{\frac{3}{2}} + 3i(2ib)^{\frac{1}{4}}e^{(-2ia)}\Gamma\left(\frac{3}{4}, 2ibx^2\right) - 3i(-2ib)^{\frac{1}{4}}e^{(2ia)}\Gamma\left(\frac{3}{4}, -2ibx^2\right)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)\*cos(b\*x^2+a)^2,x, algorithm="fricas")

[Out]  $\frac{1}{48} * (16 * b * x^{(3/2)} + 3 * I * (2 * I * b)^{(1/4)} * e^{(-2 * I * a)} * \gamma(3/4, 2 * I * b * x^2) - 3 * I * (-2 * I * b)^{(1/4)} * e^{(2 * I * a)} * \gamma(3/4, -2 * I * b * x^2)) / b$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \cos^2(a + bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)\*cos(b\*x\*\*2+a)\*\*2,x)

[Out] Integral(sqrt(x)\*cos(a + b\*x\*\*2)\*\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)\*cos(b\*x^2+a)^2,x, algorithm="giac")

[Out] integrate(sqrt(x)\*cos(b\*x^2 + a)^2, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x} \cos (b x^2 + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)\*cos(a + b\*x^2)^2,x)

[Out] int(x^(1/2)\*cos(a + b\*x^2)^2, x)

$$3.32 \quad \int \frac{\cos^2(a+bx^2)}{\sqrt{x}} dx$$

Optimal. Leaf size=96

$$\sqrt{x} - \frac{e^{2ia}\sqrt{x}\Gamma(\frac{1}{4}, -2ibx^2)}{8\sqrt[4]{2}\sqrt[4]{-ibx^2}} - \frac{e^{-2ia}\sqrt{x}\Gamma(\frac{1}{4}, 2ibx^2)}{8\sqrt[4]{2}\sqrt[4]{ibx^2}}$$

[Out]  $x^{(1/2)} - 1/16 * \exp(2*I*a) * \text{GAMMA}(1/4, -2*I*b*x^2) * x^{(1/2)} * 2^{(3/4)} / (-I*b*x^2)^{(1/4)} - 1/16 * \text{GAMMA}(1/4, 2*I*b*x^2) * x^{(1/2)} * 2^{(3/4)} / \exp(2*I*a) / (I*b*x^2)^{(1/4)}$

Rubi [A]

time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3483, 3439, 3437, 2239}

$$-\frac{e^{2ia}\sqrt{x}\Gamma(\frac{1}{4}, -2ibx^2)}{8\sqrt[4]{2}\sqrt[4]{-ibx^2}} - \frac{e^{-2ia}\sqrt{x}\Gamma(\frac{1}{4}, 2ibx^2)}{8\sqrt[4]{2}\sqrt[4]{ibx^2}} + \sqrt{x}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^2]^2/Sqrt[x], x]

[Out]  $\text{Sqrt}[x] - (E^{((2*I)*a)} * \text{Sqrt}[x] * \text{Gamma}[1/4, (-2*I)*b*x^2]) / (8*2^{(1/4)} * ((-I)*b*x^2)^{(1/4)}) - (\text{Sqrt}[x] * \text{Gamma}[1/4, (2*I)*b*x^2]) / (8*2^{(1/4)} * E^{((2*I)*a)} * (I*b*x^2)^{(1/4)})$

Rule 2239

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[(-F^a)\*(c + d\*x)\*(Gamma[1/n, (-b)\*(c + d\*x)^n\*Log[F]]/(d\*n\*((-b)\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

Rule 3437

Int[Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_))^(n\_)], x\_Symbol] := Dist[1/2, Int[E^((-c)\*I - d\*I\*(e + f\*x)^n), x], x] + Dist[1/2, Int[E^(c\*I + d\*I\*(e + f\*x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]

Rule 3439

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_))^(n\_)])\*(b\_.)^(p\_), x\_Symbol] := Int[ExpandTrigReduce[(a + b\*Cos[c + d\*(e + f\*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]

Rule 3483

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*((e_.)*(x_)^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1)
*(a + b*Cos[c + d*(x^(k*n)/e^n])]^p, x], x, (e*x)^(1/k)], x] /; FreeQ[{a,
b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a + bx^2)}{\sqrt{x}} dx &= 2 \text{Subst} \left( \int \cos^2(a + bx^4) dx, x, \sqrt{x} \right) \\
&= 2 \text{Subst} \left( \int \left( \frac{1}{2} + \frac{1}{2} \cos(2a + 2bx^4) \right) dx, x, \sqrt{x} \right) \\
&= \sqrt{x} + \text{Subst} \left( \int \cos(2a + 2bx^4) dx, x, \sqrt{x} \right) \\
&= \sqrt{x} + \frac{1}{2} \text{Subst} \left( \int e^{-2ia - 2ibx^4} dx, x, \sqrt{x} \right) + \frac{1}{2} \text{Subst} \left( \int e^{2ia + 2ibx^4} dx, x, \sqrt{x} \right) \\
&= \sqrt{x} - \frac{e^{2ia} \sqrt{x} \Gamma\left(\frac{1}{4}, -2ibx^2\right)}{8\sqrt[4]{2} \sqrt[4]{-ibx^2}} - \frac{e^{-2ia} \sqrt{x} \Gamma\left(\frac{1}{4}, 2ibx^2\right)}{8\sqrt[4]{2} \sqrt[4]{ibx^2}}
\end{aligned}$$

**Mathematica [A]**

time = 0.15, size = 120, normalized size = 1.25

$$\frac{\sqrt{x} \left( -16\sqrt[4]{b^2x^4} + 2^{3/4}\sqrt[4]{-ibx^2} \Gamma\left(\frac{1}{4}, 2ibx^2\right) (\cos(2a) - i \sin(2a)) + 2^{3/4}\sqrt[4]{ibx^2} \Gamma\left(\frac{1}{4}, -2ibx^2\right) (\cos(2a) + i \sin(2a)) \right)}{16\sqrt[4]{b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2/Sqrt[x], x]
```

```
[Out] -1/16*(Sqrt[x]*(-16*(b^2*x^4)^(1/4) + 2^(3/4)*((-I)*b*x^2)^(1/4)*Gamma[1/4,
(2*I)*b*x^2]*(Cos[2*a] - I*Sin[2*a]) + 2^(3/4)*(I*b*x^2)^(1/4)*Gamma[1/4,
(-2*I)*b*x^2]*(Cos[2*a] + I*Sin[2*a]))/(b^2*x^4)^(1/4)
```

**Maple [F]**

time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(bx^2 + a)}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^2/x^(1/2), x)
```

```
[Out] int(cos(b*x^2+a)^2/x^(1/2), x)
```

**Maxima [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs.  $2(60) = 120$ .  
time = 0.11, size = 159, normalized size = 1.66

$$\frac{2^{\frac{3}{4}} \left( \left( \left( \sqrt{\sqrt{2} + 2} \Gamma\left(\frac{1}{4}, 2i b x^2\right) + \Gamma\left(\frac{1}{4}, -2i b x^2\right) - \sqrt{-\sqrt{2} + 2} \left( i \Gamma\left(\frac{1}{4}, 2i b x^2\right) - i \Gamma\left(\frac{1}{4}, -2i b x^2\right) \right) \right) \cos(2a) - \left( \sqrt{-\sqrt{2} + 2} \left( \Gamma\left(\frac{1}{4}, 2i b x^2\right) + \Gamma\left(\frac{1}{4}, -2i b x^2\right) \right) + \sqrt{\sqrt{2} + 2} \left( i \Gamma\left(\frac{1}{4}, 2i b x^2\right) - i \Gamma\left(\frac{1}{4}, -2i b x^2\right) \right) \right) \sin(2a) \right) \sqrt{x} - 16 \cdot 2^{\frac{1}{4}} (b x^2)^{\frac{1}{4}} \sqrt{x}}{32 (b x^2)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)^2/x^(1/2),x, algorithm="maxima")

[Out]  $-1/32 \cdot 2^{3/4} \cdot \left( \left( \left( \sqrt{\sqrt{2} + 2} \cdot \left( \Gamma\left(\frac{1}{4}, 2i b x^2\right) + \Gamma\left(\frac{1}{4}, -2i b x^2\right) \right) - \sqrt{-\sqrt{2} + 2} \cdot \left( i \Gamma\left(\frac{1}{4}, 2i b x^2\right) - i \Gamma\left(\frac{1}{4}, -2i b x^2\right) \right) \right) \cdot \cos(2a) - \left( \sqrt{-\sqrt{2} + 2} \cdot \left( \Gamma\left(\frac{1}{4}, 2i b x^2\right) + \Gamma\left(\frac{1}{4}, -2i b x^2\right) \right) + \sqrt{\sqrt{2} + 2} \cdot \left( i \Gamma\left(\frac{1}{4}, 2i b x^2\right) - i \Gamma\left(\frac{1}{4}, -2i b x^2\right) \right) \right) \cdot \sin(2a) \right) \cdot \sqrt{x} - 16 \cdot 2^{1/4} \cdot (b x^2)^{1/4} \cdot \sqrt{x} \right) / (b x^2)^{1/4}$

**Fricas [A]**

time = 0.12, size = 50, normalized size = 0.52

$$\frac{i (2i b)^{\frac{3}{4}} e^{(-2i a)} \Gamma\left(\frac{1}{4}, 2i b x^2\right) - i (-2i b)^{\frac{3}{4}} e^{(2i a)} \Gamma\left(\frac{1}{4}, -2i b x^2\right) + 16 b \sqrt{x}}{16 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)^2/x^(1/2),x, algorithm="fricas")

[Out]  $1/16 \cdot \left( i \cdot (2i b)^{3/4} \cdot e^{(-2i a)} \cdot \Gamma\left(\frac{1}{4}, 2i b x^2\right) - i \cdot (-2i b)^{3/4} \cdot e^{(2i a)} \cdot \Gamma\left(\frac{1}{4}, -2i b x^2\right) + 16 \cdot b \cdot \sqrt{x} \right) / b$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a + b x^2)}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x\*\*2+a)\*\*2/x\*\*(1/2),x)

[Out] Integral(cos(a + b\*x\*\*2)\*\*2/sqrt(x), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)^2/x^(1/2),x, algorithm="giac")



[Out] integrate(cos(b\*x^2 + a)^2/sqrt(x), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(bx^2 + a)^2}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^2)^2/x^(1/2), x)

[Out] int(cos(a + b\*x^2)^2/x^(1/2), x)

### 3.33 $\int \frac{\cos^2(a+bx^2)}{x^{3/2}} dx$

**Optimal.** Leaf size=117

$$-\frac{1}{\sqrt{x}} - \frac{\cos(2(a+bx^2))}{\sqrt{x}} - \frac{ibe^{2ia}x^{3/2}\Gamma(\frac{3}{4}, -2ibx^2)}{2^{3/4}(-ibx^2)^{3/4}} + \frac{ibe^{-2ia}x^{3/2}\Gamma(\frac{3}{4}, 2ibx^2)}{2^{3/4}(ibx^2)^{3/4}}$$

[Out]  $-1/2*I*b*\exp(2*I*a)*x^{(3/2)}*GAMMA(3/4, -2*I*b*x^2)*2^{(1/4)} / (-I*b*x^2)^{(3/4)} + 1/2*I*b*x^{(3/2)}*GAMMA(3/4, 2*I*b*x^2)*2^{(1/4)} / \exp(2*I*a) / (I*b*x^2)^{(3/4)} - 1/x^{(1/2)} - \cos(2*b*x^2+2*a)/x^{(1/2)}$

**Rubi [A]**

time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3483, 3485, 3469, 3470, 2250}

$$-\frac{ie^{2ia}bx^{3/2}\Gamma(\frac{3}{4}, -2ibx^2)}{2^{3/4}(-ibx^2)^{3/4}} + \frac{ie^{-2ia}bx^{3/2}\Gamma(\frac{3}{4}, 2ibx^2)}{2^{3/4}(ibx^2)^{3/4}} - \frac{\cos(2(a+bx^2))}{\sqrt{x}} - \frac{1}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^2]^2/x^(3/2), x]

[Out]  $-(1/\text{Sqrt}[x]) - \text{Cos}[2*(a + b*x^2)]/\text{Sqrt}[x] - (I*b*E^{((2*I)*a)}*x^{(3/2)}*\Gamma[3/4, (-2*I)*b*x^2])/(2^{(3/4)}*((-I)*b*x^2)^{(3/4)}) + (I*b*x^{(3/2)}*\Gamma[3/4, (2*I)*b*x^2])/(2^{(3/4)}*E^{((2*I)*a)}*(I*b*x^2)^{(3/4)})$

Rule 2250

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-F^a)\*((e + f\*x)^(m + 1)/(f\*n\*((-b)\*(c + d\*x)^n\*Log[F])^(m + 1)/n))\*Gamma[(m + 1)/n, (-b)\*(c + d\*x)^n\*Log[F]], x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d\*e - c\*f, 0]

Rule 3469

Int[Cos[(c\_.) + (d\_.)\*(x\_)]^(n\_)\*((e\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(e\*x)^(m + 1)\*(Cos[c + d\*x^n]/(e\*(m + 1))), x] + Dist[d\*(n/(e^n\*(m + 1))), Int[(e\*x)^(m + n)\*Sin[c + d\*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3470

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := Dist[I/2, Int[(e\*x)^m\*E^{(-c)\*I - d\*I\*x^n}, x], x] - Dist[I/2, Int[(e\*x)^m\*E^{(c\*I + d\*I\*x^n)}, x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]

Rule 3483

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*Cos[c + d\*(x^(k\*n)/e^n])]^p, x], x, (e\*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]

Rule 3485

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandTrigReduce[(e\*x)^m, (a + b\*Cos[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^2(a + bx^2)}{x^{3/2}} dx &= 2\text{Subst}\left(\int \frac{\cos^2(a + bx^4)}{x^2} dx, x, \sqrt{x}\right) \\
 &= 2\text{Subst}\left(\int \left(\frac{1}{2x^2} + \frac{\cos(2a + 2bx^4)}{2x^2}\right) dx, x, \sqrt{x}\right) \\
 &= -\frac{1}{\sqrt{x}} + \text{Subst}\left(\int \frac{\cos(2a + 2bx^4)}{x^2} dx, x, \sqrt{x}\right) \\
 &= -\frac{1}{\sqrt{x}} - \frac{\cos(2(a + bx^2))}{\sqrt{x}} - (8b)\text{Subst}\left(\int x^2 \sin(2a + 2bx^4) dx, x, \sqrt{x}\right) \\
 &= -\frac{1}{\sqrt{x}} - \frac{\cos(2(a + bx^2))}{\sqrt{x}} - (4ib)\text{Subst}\left(\int e^{-2ia-2ibx^4} x^2 dx, x, \sqrt{x}\right) + (4ib)\text{Subst}\left(\int e^{-2ia-2ibx^4} x^2 dx, x, \sqrt{x}\right) \\
 &= -\frac{1}{\sqrt{x}} - \frac{\cos(2(a + bx^2))}{\sqrt{x}} - \frac{ibe^{2ia}x^{3/2}\Gamma\left(\frac{3}{4}, -2ibx^2\right)}{2^{3/4}(-ibx^2)^{3/4}} + \frac{ibe^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4}, 2ibx^2\right)}{2^{3/4}(ibx^2)^{3/4}}
 \end{aligned}$$

Mathematica [A]

time = 0.23, size = 137, normalized size = 1.17

$$\frac{-4(b^2x^4)^{3/4} \cos^2(a + bx^2) + \sqrt[4]{2} bx^2 (ibx^2)^{3/4} \Gamma\left(\frac{3}{4}, -2ibx^2\right) (-i \cos(2a) + \sin(2a)) + i\sqrt[4]{2} (-ibx^2)^{7/4} \Gamma\left(\frac{3}{4}, 2ibx^2\right) (i \cos(2a) + \sin(2a))}{2\sqrt{x} (b^2x^4)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*x^2]^2/x^(3/2), x]

[Out] (-4\*(b^2\*x^4)^(3/4)\*Cos[a + b\*x^2]^2 + 2^(1/4)\*b\*x^2\*(I\*b\*x^2)^(3/4)\*Gamma[3/4, (-2\*I)\*b\*x^2]\*((-I)\*Cos[2\*a] + Sin[2\*a]) + I\*2^(1/4)\*((-I)\*b\*x^2)^(7/4)\*Gamma[3/4, (2\*I)\*b\*x^2]\*(I\*Cos[2\*a] + Sin[2\*a]))/(2\*Sqrt[x]\*(b^2\*x^4)^(3/4))

**Maple [F]**

time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(bx^2 + a)}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b\*x^2+a)^2/x^(3/2),x)

[Out] int(cos(b\*x^2+a)^2/x^(3/2),x)

**Maxima [A]**

time = 0.13, size = 143, normalized size = 1.22

$$\frac{2^i (bx^2)^i \left( \left( \sqrt{\sqrt{2} + 2} \Gamma(-\frac{1}{4}, 2i bx^2) + \Gamma(-\frac{1}{4}, -2i bx^2) \right) + \sqrt{-\sqrt{2} + 2} \left( i \Gamma(-\frac{1}{4}, 2i bx^2) - i \Gamma(-\frac{1}{4}, -2i bx^2) \right) \right) \cos(2a) + \left( \sqrt{-\sqrt{2} + 2} \Gamma(-\frac{1}{4}, 2i bx^2) + \Gamma(-\frac{1}{4}, -2i bx^2) \right) + \sqrt{\sqrt{2} + 2} \left( -i \Gamma(-\frac{1}{4}, 2i bx^2) + i \Gamma(-\frac{1}{4}, -2i bx^2) \right) \sin(2a) \right) + 16}{16 \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)^2/x^(3/2),x, algorithm="maxima")

[Out] -1/16\*(2^(1/4)\*(b\*x^2)^(1/4)\*((sqrt(sqrt(2) + 2)\*(gamma(-1/4, 2\*I\*b\*x^2) + gamma(-1/4, -2\*I\*b\*x^2)) + sqrt(-sqrt(2) + 2)\*(I\*gamma(-1/4, 2\*I\*b\*x^2) - I\*gamma(-1/4, -2\*I\*b\*x^2)))\*cos(2\*a) + (sqrt(-sqrt(2) + 2)\*(gamma(-1/4, 2\*I\*b\*x^2) + gamma(-1/4, -2\*I\*b\*x^2)) + sqrt(sqrt(2) + 2)\*(-I\*gamma(-1/4, 2\*I\*b\*x^2) + I\*gamma(-1/4, -2\*I\*b\*x^2)))\*sin(2\*a)) + 16)/sqrt(x)

**Fricas [A]**

time = 0.11, size = 59, normalized size = 0.50

$$\frac{(2i b)^{\frac{1}{4}} x e^{(-2i a)} \Gamma\left(\frac{3}{4}, 2i b x^2\right) + (-2i b)^{\frac{1}{4}} x e^{(2i a)} \Gamma\left(\frac{3}{4}, -2i b x^2\right) - 4 \sqrt{x} \cos(bx^2 + a)^2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)^2/x^(3/2),x, algorithm="fricas")

[Out] 1/2\*((2\*I\*b)^(1/4)\*x\*e^(-2\*I\*a)\*gamma(3/4, 2\*I\*b\*x^2) + (-2\*I\*b)^(1/4)\*x\*e^(2\*I\*a)\*gamma(3/4, -2\*I\*b\*x^2) - 4\*sqrt(x)\*cos(b\*x^2 + a)^2)/x

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a + bx^2)}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x\*\*2+a)\*\*2/x\*\*(3/2),x)

[Out] Integral(cos(a + b\*x\*\*2)\*\*2/x\*\*(3/2), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b\*x^2+a)^2/x^(3/2),x, algorithm="giac")

[Out] integrate(cos(b\*x^2 + a)^2/x^(3/2), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(bx^2 + a)^2}{x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^2)^2/x^(3/2),x)

[Out] int(cos(a + b\*x^2)^2/x^(3/2), x)

### 3.34 $\int \frac{\cos^2(a+bx^2)}{x^{5/2}} dx$

**Optimal.** Leaf size=116

$$-\frac{2 \cos^2(a+bx^2)}{3x^{3/2}} - \frac{ibe^{2ia} \sqrt{x} \Gamma(\frac{1}{4}, -2ibx^2)}{3\sqrt[4]{2} \sqrt[4]{-ibx^2}} + \frac{ibe^{-2ia} \sqrt{x} \Gamma(\frac{1}{4}, 2ibx^2)}{3\sqrt[4]{2} \sqrt[4]{ibx^2}}$$

[Out]  $-2/3*\cos(b*x^2+a)^2/x^{(3/2)}-1/6*I*b*\exp(2*I*a)*\text{GAMMA}(1/4,-2*I*b*x^2)*x^{(1/2)}$   
 $*2^{(3/4)/(-I*b*x^2)^{(1/4)}+1/6*I*b*\text{GAMMA}(1/4,2*I*b*x^2)*x^{(1/2)}*2^{(3/4)}/\exp$   
 $(2*I*a)/(I*b*x^2)^{(1/4)}$

**Rubi [A]**

time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {3483, 3475, 4669, 3454, 3436, 2239}

$$-\frac{ie^{2ia}b\sqrt{x} \Gamma(\frac{1}{4}, -2ibx^2)}{3\sqrt[4]{2} \sqrt[4]{-ibx^2}} + \frac{ie^{-2ia}b\sqrt{x} \Gamma(\frac{1}{4}, 2ibx^2)}{3\sqrt[4]{2} \sqrt[4]{ibx^2}} - \frac{2 \cos^2(a+bx^2)}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^2]^2/x^(5/2), x]

[Out]  $(-2*\text{Cos}[a + b*x^2]^2)/(3*x^{(3/2)}) - ((I/3)*b*E^{((2*I)*a)*\text{Sqrt}[x]*\text{Gamma}[1/4,$   
 $(-2*I)*b*x^2])/(2^{(1/4)}*((-I)*b*x^2)^{(1/4)}) + ((I/3)*b*\text{Sqrt}[x]*\text{Gamma}[1/4,$   
 $(2*I)*b*x^2])/(2^{(1/4)}*E^{((2*I)*a)*(I*b*x^2)^{(1/4)})}$

Rule 2239

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_))), x\_Symbol] := Simp[(-F^a)\*(c + d\*x)\*(Gamma[1/n, (-b)\*(c + d\*x)^n\*Log[F]]/(d\*n\*((-b)\*(c + d\*x)^n\*Log[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

Rule 3436

Int[Sin[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_)^(n\_))], x\_Symbol] := Dist[I/2, Int[E^((-c)\*I - d\*I\*(e + f\*x)^n), x], x] - Dist[I/2, Int[E^(c\*I + d\*I\*(e + f\*x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]

Rule 3454

Int[((a\_.) + (b\_.)\*Sin[u\_])^(p\_.), x\_Symbol] := Int[(a + b\*SIN[ExpandToSum[u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rule 3475

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_), x_Symbol] := Simp[x^(m +
1)*(Cos[a + b*x^n]^(p/(m + 1))), x] + Dist[b*n*(p/(m + 1)), Int[Cos[a + b*x^n
]^(p - 1)*Sin[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
+ n, 0] && NeQ[n, 1] && IntegerQ[n]
```

### Rule 3483

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] := With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + d*(x^(k*n)/e^n)]^(p), x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

### Rule 4669

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] := Dist[1/2^p, Int[u*SIN[2
*v]^(p), x], x] /; EqQ[w, v] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a + bx^2)}{x^{5/2}} dx &= 2 \text{Subst} \left( \int \frac{\cos^2(a + bx^4)}{x^4} dx, x, \sqrt{x} \right) \\
&= -\frac{2 \cos^2(a + bx^2)}{3x^{3/2}} - \frac{1}{3}(16b) \text{Subst} \left( \int \cos(a + bx^4) \sin(a + bx^4) dx, x, \sqrt{x} \right) \\
&= -\frac{2 \cos^2(a + bx^2)}{3x^{3/2}} - \frac{1}{3}(8b) \text{Subst} \left( \int \sin(2(a + bx^4)) dx, x, \sqrt{x} \right) \\
&= -\frac{2 \cos^2(a + bx^2)}{3x^{3/2}} - \frac{1}{3}(8b) \text{Subst} \left( \int \sin(2a + 2bx^4) dx, x, \sqrt{x} \right) \\
&= -\frac{2 \cos^2(a + bx^2)}{3x^{3/2}} - \frac{1}{3}(4ib) \text{Subst} \left( \int e^{-2ia - 2ibx^4} dx, x, \sqrt{x} \right) + \frac{1}{3}(4ib) \text{Subst} \left( \int e^{2ia + 2ibx^4} dx, x, \sqrt{x} \right) \\
&= -\frac{2 \cos^2(a + bx^2)}{3x^{3/2}} - \frac{ibe^{2ia} \sqrt{x} \Gamma(\frac{1}{4}, -2ibx^2)}{3\sqrt[4]{2} \sqrt[4]{-ibx^2}} + \frac{ibe^{-2ia} \sqrt{x} \Gamma(\frac{1}{4}, 2ibx^2)}{3\sqrt[4]{2} \sqrt[4]{ibx^2}}
\end{aligned}$$

### Mathematica [A]

time = 0.24, size = 137, normalized size = 1.18

$$\frac{-4\sqrt[4]{b^2x^4} \cos^2(a + bx^2) + 2^{3/4}bx^2\sqrt[4]{ibx^2} \text{Gamma}(\frac{1}{4}, -2ibx^2) (-i \cos(2a) + \sin(2a)) + i2^{3/4}(-ibx^2)^{5/4} \text{Gamma}(\frac{1}{4}, 2ibx^2) (i \cos(2a) + \sin(2a))}{6x^{3/2}\sqrt[4]{b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2/x^(5/2), x]
```

[Out]  $(-4*(b^2*x^4)^{(1/4)}*\cos[a + b*x^2]^2 + 2^{(3/4)}*b*x^2*(I*b*x^2)^{(1/4)}*\Gamma[1/4, (-2*I)*b*x^2]*((-I)*\cos[2*a] + \sin[2*a]) + I*2^{(3/4)}*((-I)*b*x^2)^{(5/4)}*\Gamma[1/4, (2*I)*b*x^2]*(I*\cos[2*a] + \sin[2*a]))/(6*x^{(3/2)}*(b^2*x^4)^{(1/4)})$

**Maple [F]**

time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(bx^2 + a)}{x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x^2+a)^2/x^(5/2),x)`

[Out] `int(cos(b*x^2+a)^2/x^(5/2),x)`

**Maxima [A]**

time = 0.13, size = 146, normalized size = 1.26

$$\frac{3 \cdot 2^{3/4} (bx^2)^{3/4} \left( \left( \sqrt{-\sqrt{2}+2} \Gamma(-\frac{3}{4}, 2i bx^2) + \Gamma(-\frac{3}{4}, -2i bx^2) \right) - \sqrt{\sqrt{2}+2} \left( -i \Gamma(-\frac{3}{4}, 2i bx^2) + i \Gamma(-\frac{3}{4}, -2i bx^2) \right) \right) \cos(2a) + \left( \sqrt{\sqrt{2}+2} \Gamma(-\frac{3}{4}, 2i bx^2) + \Gamma(-\frac{3}{4}, -2i bx^2) \right) - \sqrt{-\sqrt{2}+2} \left( i \Gamma(-\frac{3}{4}, 2i bx^2) - i \Gamma(-\frac{3}{4}, -2i bx^2) \right) \sin(2a) + 16}{48 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^2/x^(5/2),x, algorithm="maxima")`

[Out]  $-1/48*(3*2^{(3/4)}*(b*x^2)^{(3/4)}*((\sqrt{-\sqrt{2}+2}*(\gamma(-3/4, 2*I*b*x^2) + \gamma(-3/4, -2*I*b*x^2)) - \sqrt{\sqrt{2}+2}*(-I*\gamma(-3/4, 2*I*b*x^2) + I*\gamma(-3/4, -2*I*b*x^2))))*\cos(2*a) + (\sqrt{\sqrt{2}+2}*(\gamma(-3/4, 2*I*b*x^2) + \gamma(-3/4, -2*I*b*x^2)) - \sqrt{-\sqrt{2}+2}*(I*\gamma(-3/4, 2*I*b*x^2) - I*\gamma(-3/4, -2*I*b*x^2))))*\sin(2*a) + 16)/x^{(3/2)}$

**Fricas [A]**

time = 0.11, size = 63, normalized size = 0.54

$$\frac{(2i b)^{3/4} x^2 e^{(-2i a)} \Gamma(\frac{1}{4}, 2i bx^2) + (-2i b)^{3/4} x^2 e^{(2i a)} \Gamma(\frac{1}{4}, -2i bx^2) - 4 \sqrt{x} \cos(bx^2 + a)^2}{6 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^2/x^(5/2),x, algorithm="fricas")`

[Out]  $1/6*((2*I*b)^{(3/4)}*x^2*e^{(-2*I*a)}*\gamma(1/4, 2*I*b*x^2) + (-2*I*b)^{(3/4)}*x^2*e^{(2*I*a)}*\gamma(1/4, -2*I*b*x^2) - 4*\sqrt{x}*\cos(b*x^2 + a)^2)/x^2$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a + bx^2)}{x^{5/2}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x**2+a)**2/x**(5/2),x)`

[Out] `Integral(cos(a + b*x**2)**2/x**(5/2), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x^2+a)^2/x^(5/2),x, algorithm="giac")`

[Out] `integrate(cos(b*x^2 + a)^2/x^(5/2), x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(bx^2 + a)^2}{x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x^2)^2/x^(5/2),x)`

[Out] `int(cos(a + b*x^2)^2/x^(5/2), x)`

### 3.35 $\int \cos\left(a + \frac{b}{x}\right) dx$

Optimal. Leaf size=31

$$x \cos\left(a + \frac{b}{x}\right) + b \operatorname{CosIntegral}\left(\frac{b}{x}\right) \sin(a) + b \cos(a) \operatorname{Si}\left(\frac{b}{x}\right)$$

[Out] x\*cos(a+b/x)+b\*cos(a)\*Si(b/x)+b\*Ci(b/x)\*sin(a)

**Rubi [A]**

time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {3443, 3378, 3384, 3380, 3383}

$$b \sin(a) \operatorname{CosIntegral}\left(\frac{b}{x}\right) + b \cos(a) \operatorname{Si}\left(\frac{b}{x}\right) + x \cos\left(a + \frac{b}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b/x], x]

[Out] x\*Cos[a + b/x] + b\*CosIntegral[b/x]\*Sin[a] + b\*Cos[a]\*SinIntegral[b/x]

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c
+ d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3443

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/(n\*f), Subst[Int[x^(1/n - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, (e + f\*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned} \int \cos\left(a + \frac{b}{x}\right) dx &= -\text{Subst}\left(\int \frac{\cos(a + bx)}{x^2} dx, x, \frac{1}{x}\right) \\ &= x \cos\left(a + \frac{b}{x}\right) + b \text{Subst}\left(\int \frac{\sin(a + bx)}{x} dx, x, \frac{1}{x}\right) \\ &= x \cos\left(a + \frac{b}{x}\right) + (b \cos(a)) \text{Subst}\left(\int \frac{\sin(bx)}{x} dx, x, \frac{1}{x}\right) + (b \sin(a)) \text{Subst}\left(\int \frac{\cos(bx)}{x} dx, x, \frac{1}{x}\right) \\ &= x \cos\left(a + \frac{b}{x}\right) + b \text{Ci}\left(\frac{b}{x}\right) \sin(a) + b \cos(a) \text{Si}\left(\frac{b}{x}\right) \end{aligned}$$

Mathematica [A]

time = 0.02, size = 31, normalized size = 1.00

$$x \cos\left(a + \frac{b}{x}\right) + b \text{CosIntegral}\left(\frac{b}{x}\right) \sin(a) + b \cos(a) \text{Si}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b/x], x]

[Out] x\*Cos[a + b/x] + b\*CosIntegral[b/x]\*Sin[a] + b\*Cos[a]\*SinIntegral[b/x]

Maple [A]

time = 0.12, size = 39, normalized size = 1.26

method	result
derivativedivides	$-b \left( -\frac{\cos\left(a + \frac{b}{x}\right)x}{b} - \text{sinIntegral}\left(\frac{b}{x}\right) \cos(a) - \text{cosineIntegral}\left(\frac{b}{x}\right) \sin(a) \right)$
default	$-b \left( -\frac{\cos\left(a + \frac{b}{x}\right)x}{b} - \text{sinIntegral}\left(\frac{b}{x}\right) \cos(a) - \text{cosineIntegral}\left(\frac{b}{x}\right) \sin(a) \right)$
risch	$\frac{ib \exp\text{Integral}\left(1, -\frac{ib}{x}\right) e^{ia}}{2} - \frac{\pi \text{csgn}\left(\frac{b}{x}\right) e^{-ia} b}{2} + \text{sinIntegral}\left(\frac{b}{x}\right) e^{-ia} b - \frac{i \exp\text{Integral}\left(1, -\frac{ib}{x}\right) e^{-ia} b}{2} + x \cos\left(a + \frac{b}{x}\right)$

meijerg	$\frac{\sqrt{\pi} \cos(a) \sqrt{b^2} \left( \frac{4x b^2 \cos\left(\frac{\sqrt{b^2}}{x}\right) - 4 \operatorname{sinIntegral}\left(\frac{\sqrt{b^2}}{x}\right)}{(b^2)^{\frac{3}{2}} \sqrt{\pi}} - \frac{1}{\sqrt{\pi}} \right)}{4} + \frac{\sqrt{\pi} \sin(a) b \left( \frac{4\gamma - 4 - 4 \ln(x) + 4 \ln(b)}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}} \right)}{4}$
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Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b/x),x,method=_RETURNVERBOSE)`

[Out] `-b*(-cos(a+b/x)/b*x-Si(b/x)*cos(a)-Ci(b/x)*sin(a))`

**Maxima** [C] Result contains complex when optimal does not.

time = 0.32, size = 57, normalized size = 1.84

$$\frac{1}{2} \left( \left( -i \operatorname{Ei}\left(\frac{ib}{x}\right) + i \operatorname{Ei}\left(-\frac{ib}{x}\right) \right) \cos(a) + \left( \operatorname{Ei}\left(\frac{ib}{x}\right) + \operatorname{Ei}\left(-\frac{ib}{x}\right) \right) \sin(a) \right) b + x \cos\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x),x, algorithm="maxima")`

[Out] `1/2*((-I*Ei(I*b/x) + I*Ei(-I*b/x))*cos(a) + (Ei(I*b/x) + Ei(-I*b/x))*sin(a))*b + x*cos((a*x + b)/x)`

**Fricas** [A]

time = 0.40, size = 45, normalized size = 1.45

$$b \cos(a) \operatorname{Si}\left(\frac{b}{x}\right) + x \cos\left(\frac{ax+b}{x}\right) + \frac{1}{2} \left( b \operatorname{Ci}\left(\frac{b}{x}\right) + b \operatorname{Ci}\left(-\frac{b}{x}\right) \right) \sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x),x, algorithm="fricas")`

[Out] `b*cos(a)*sin_integral(b/x) + x*cos((a*x + b)/x) + 1/2*(b*cos_integral(b/x) + b*cos_integral(-b/x))*sin(a)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(a + \frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x),x)`

[Out] `Integral(cos(a + b/x), x)`

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(31) = 62.

time = 0.41, size = 132, normalized size = 4.26

$$\frac{ab^2 \operatorname{Ci}\left(-a + \frac{ax+b}{x}\right) \sin(a) - ab^2 \cos(a) \operatorname{Si}\left(a - \frac{ax+b}{x}\right) - \frac{(ax+b)b^2 \operatorname{Ci}\left(-a + \frac{ax+b}{x}\right) \sin(a)}{x} + \frac{(ax+b)b^2 \cos(a) \operatorname{Si}\left(a - \frac{ax+b}{x}\right)}{x} - b^2 \cos\left(\frac{ax+b}{x}\right)}{\left(a - \frac{ax+b}{x}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x),x, algorithm="giac")

[Out] (a\*b^2\*cos\_integral(-a + (a\*x + b)/x)\*sin(a) - a\*b^2\*cos(a)\*sin\_integral(a - (a\*x + b)/x) - (a\*x + b)\*b^2\*cos\_integral(-a + (a\*x + b)/x)\*sin(a)/x + (a\*x + b)\*b^2\*cos(a)\*sin\_integral(a - (a\*x + b)/x)/x - b^2\*cos((a\*x + b)/x))/((a - (a\*x + b)/x)\*b)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.03

$$\int \cos\left(a + \frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b/x),x)

[Out] int(cos(a + b/x), x)

$$3.36 \quad \int \frac{\cos\left(a + \frac{b}{x}\right)}{x} dx$$

Optimal. Leaf size=20

$$-\cos(a)\text{CosIntegral}\left(\frac{b}{x}\right) + \sin(a)\text{Si}\left(\frac{b}{x}\right)$$

[Out] -Ci(b/x)\*cos(a)+Si(b/x)\*sin(a)

Rubi [A]

time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3459, 3457, 3456}

$$\sin(a)\text{Si}\left(\frac{b}{x}\right) - \cos(a)\text{CosIntegral}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b/x]/x,x]

[Out] -(Cos[a]\*CosIntegral[b/x]) + Sin[a]\*SinIntegral[b/x]

Rule 3456

Int[Sin[(d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] := Simp[SinIntegral[d\*x^n]/n, x] / ; FreeQ[{d, n}, x]

Rule 3457

Int[Cos[(d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] := Simp[CosIntegral[d\*x^n]/n, x] / ; FreeQ[{d, n}, x]

Rule 3459

Int[Cos[(c\_) + (d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] := Dist[Cos[c], Int[Cos[d\*x^n]/x, x], x] - Dist[Sin[c], Int[Sin[d\*x^n]/x, x], x] / ; FreeQ[{c, d, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(a + \frac{b}{x}\right)}{x} dx &= \cos(a) \int \frac{\cos\left(\frac{b}{x}\right)}{x} dx - \sin(a) \int \frac{\sin\left(\frac{b}{x}\right)}{x} dx \\ &= -\cos(a)\text{Ci}\left(\frac{b}{x}\right) + \sin(a)\text{Si}\left(\frac{b}{x}\right) \end{aligned}$$

**Mathematica [A]**

time = 0.03, size = 20, normalized size = 1.00

$$-\cos(a)\operatorname{CosIntegral}\left(\frac{b}{x}\right) + \sin(a)\operatorname{Si}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b/x]/x,x]``[Out] -(Cos[a]*CosIntegral[b/x]) + Sin[a]*SinIntegral[b/x]`**Maple [A]**

time = 0.07, size = 21, normalized size = 1.05

method	result
derivativedivides	$-\operatorname{cosineIntegral}\left(\frac{b}{x}\right)\cos(a) + \operatorname{sinIntegral}\left(\frac{b}{x}\right)\sin(a)$
default	$-\operatorname{cosineIntegral}\left(\frac{b}{x}\right)\cos(a) + \operatorname{sinIntegral}\left(\frac{b}{x}\right)\sin(a)$
risch	$-\frac{ie^{-ia}\pi\operatorname{csgn}\left(\frac{b}{x}\right)}{2} + ie^{-ia}\operatorname{sinIntegral}\left(\frac{b}{x}\right) + \frac{e^{-ia}\operatorname{expIntegral}\left(1,-\frac{ib}{x}\right)}{2} + \frac{e^{ia}\operatorname{expIntegral}\left(1,-\frac{ib}{x}\right)}{2}$
meijerg	$-\frac{\sqrt{\pi}\cos(a)\left(\frac{2\gamma-2\ln(x)+\ln(b^2)}{\sqrt{\pi}} - \frac{2\gamma}{\sqrt{\pi}} - \frac{2\ln(2)}{\sqrt{\pi}} - \frac{2\ln\left(\frac{b}{2x}\right)}{\sqrt{\pi}} + \frac{2\operatorname{cosineIntegral}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{2} + \operatorname{sinIntegral}\left(\frac{b}{x}\right)\sin(a)$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(a+b/x)/x,x,method=_RETURNVERBOSE)``[Out] -Ci(b/x)*cos(a)+Si(b/x)*sin(a)`**Maxima [C]** Result contains complex when optimal does not.

time = 0.33, size = 43, normalized size = 2.15

$$-\frac{1}{2}\left(\operatorname{Ei}\left(\frac{ib}{x}\right) + \operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\cos(a) - \frac{1}{2}\left(i\operatorname{Ei}\left(\frac{ib}{x}\right) - i\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(a+b/x)/x,x, algorithm="maxima")``[Out] -1/2*(Ei(I*b/x) + Ei(-I*b/x))*cos(a) - 1/2*(I*Ei(I*b/x) - I*Ei(-I*b/x))*sin(a)`**Fricas [A]**

time = 0.38, size = 28, normalized size = 1.40

$$-\frac{1}{2}\left(\operatorname{Ci}\left(\frac{b}{x}\right) + \operatorname{Ci}\left(-\frac{b}{x}\right)\right)\cos(a) + \sin(a)\operatorname{Si}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x,x, algorithm="fricas")

[Out]  $-1/2*(\cos\_integral(b/x) + \cos\_integral(-b/x))*\cos(a) + \sin(a)*\sin\_integral(b/x)$

**Sympy** [A]

time = 0.56, size = 15, normalized size = 0.75

$$\sin(a) \operatorname{Si}\left(\frac{b}{x}\right) - \cos(a) \operatorname{Ci}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x,x)

[Out]  $\sin(a)*\operatorname{Si}(b/x) - \cos(a)*\operatorname{Ci}(b/x)$

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.  
time = 0.45, size = 41, normalized size = 2.05

$$-\frac{b \cos(a) \operatorname{Ci}\left(-a + \frac{ax+b}{x}\right) + b \sin(a) \operatorname{Si}\left(a - \frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x,x, algorithm="giac")

[Out]  $-(b*\cos(a)*\cos\_integral(-a + (a*x + b)/x) + b*\sin(a)*\sin\_integral(a - (a*x + b)/x))/b$

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.05

$$\sin(a) \operatorname{sinint}\left(\frac{b}{x}\right) - \cos(a) \operatorname{cosint}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b/x)/x,x)

[Out]  $\sin(a)*\operatorname{sinint}(b/x) - \cos(a)*\operatorname{cosint}(b/x)$



$$3.37 \quad \int \frac{\cos\left(a + \frac{b}{x}\right)}{x^2} dx$$

Optimal. Leaf size=13

$$-\frac{\sin\left(a + \frac{b}{x}\right)}{b}$$

[Out] -sin(a+b/x)/b

Rubi [A]

time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3461, 2717}

$$-\frac{\sin\left(a + \frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b/x]/x^2,x]

[Out] -(Sin[a + b/x]/b)

Rule 2717

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3461

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(a + \frac{b}{x}\right)}{x^2} dx &= -\text{Subst}\left(\int \cos(a + bx) dx, x, \frac{1}{x}\right) \\ &= -\frac{\sin\left(a + \frac{b}{x}\right)}{b} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 13, normalized size = 1.00

$$-\frac{\sin\left(a + \frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b/x]/x^2,x]

[Out] -(Sin[a + b/x]/b)

**Maple [A]**

time = 0.04, size = 14, normalized size = 1.08

method	result	size
derivativdivides	$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$	14
default	$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$	14
risch	$-\frac{\sin\left(\frac{ax+b}{x}\right)}{b}$	16
norman	$-\frac{2 \tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}$	34
meijerg	$-\frac{\cos(a)\sin\left(\frac{b}{x}\right)}{b} + \frac{\sqrt{\pi} \sin(a)\left(\frac{1}{\sqrt{\pi}} - \frac{\cos\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{b}$	39

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b/x)/x^2,x,method=\_RETURNVERBOSE)

[Out] -sin(a+b/x)/b

**Maxima [A]**

time = 0.29, size = 13, normalized size = 1.00

$$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x^2,x, algorithm="maxima")

[Out] -sin(a + b/x)/b

**Fricas [A]**

time = 0.39, size = 15, normalized size = 1.15

$$-\frac{\sin\left(\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x^2,x, algorithm="fricas")

[Out]  $-\sin((a*x + b)/x)/b$

**Sympy [A]**

time = 0.34, size = 15, normalized size = 1.15

$$\begin{cases} -\frac{\sin\left(a+\frac{b}{x}\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cos(a)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x)/x**2,x)`

[Out] `Piecewise((-sin(a + b/x)/b, Ne(b, 0)), (-cos(a)/x, True))`

**Giac [A]**

time = 0.41, size = 15, normalized size = 1.15

$$-\frac{\sin\left(\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x)/x^2,x, algorithm="giac")`

[Out]  $-\sin((a*x + b)/x)/b$

**Mupad [B]**

time = 0.26, size = 13, normalized size = 1.00

$$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b/x)/x^2,x)`

[Out]  $-\sin(a + b/x)/b$

$$3.38 \quad \int \frac{\cos\left(a + \frac{b}{x}\right)}{x^3} dx$$

Optimal. Leaf size=30

$$-\frac{\cos\left(a + \frac{b}{x}\right)}{b^2} - \frac{\sin\left(a + \frac{b}{x}\right)}{bx}$$

[Out]  $-\cos(a+b/x)/b^2 - \sin(a+b/x)/b/x$

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3461, 3377, 2718}

$$-\frac{\cos\left(a + \frac{b}{x}\right)}{b^2} - \frac{\sin\left(a + \frac{b}{x}\right)}{bx}$$

Antiderivative was successfully verified.

[In] `Int[Cos[a + b/x]/x^3,x]`

[Out] `-(Cos[a + b/x]/b^2) - Sin[a + b/x]/(b*x)`

Rule 2718

`Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 3377

`Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 3461

`Int[((a_.) + Cos[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*(x_.)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))`

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(a + \frac{b}{x}\right)}{x^3} dx &= -\text{Subst}\left(\int x \cos(a + bx) dx, x, \frac{1}{x}\right) \\ &= -\frac{\sin\left(a + \frac{b}{x}\right)}{bx} + \frac{\text{Subst}\left(\int \sin(a + bx) dx, x, \frac{1}{x}\right)}{b} \\ &= -\frac{\cos\left(a + \frac{b}{x}\right)}{b^2} - \frac{\sin\left(a + \frac{b}{x}\right)}{bx} \end{aligned}$$

**Mathematica [A]**

time = 0.03, size = 29, normalized size = 0.97

$$-\frac{x \cos\left(a + \frac{b}{x}\right) + b \sin\left(a + \frac{b}{x}\right)}{b^2 x}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b/x]/x^3,x]``[Out] -((x*Cos[a + b/x] + b*Sin[a + b/x])/(b^2*x))`**Maple [A]**

time = 0.07, size = 42, normalized size = 1.40

method	result	size
risch	$-\frac{\cos\left(\frac{ax+b}{x}\right)}{b^2} - \frac{\sin\left(\frac{ax+b}{x}\right)}{bx}$	35
derivativedivides	$-\frac{\cos\left(a + \frac{b}{x}\right) + \left(a + \frac{b}{x}\right) \sin\left(a + \frac{b}{x}\right) - a \sin\left(a + \frac{b}{x}\right)}{b^2}$	42
default	$-\frac{\cos\left(a + \frac{b}{x}\right) + \left(a + \frac{b}{x}\right) \sin\left(a + \frac{b}{x}\right) - a \sin\left(a + \frac{b}{x}\right)}{b^2}$	42
norman	$\frac{2x^2 \left(\tan^2\left(\frac{a}{2} + \frac{b}{2x}\right)\right) - 2x \tan\left(\frac{a}{2} + \frac{b}{2x}\right)}{b^2 \left(1 + \tan^2\left(\frac{a}{2} + \frac{b}{2x}\right)\right) x^2}$	61
meijerg	$-\frac{2\sqrt{\pi} \cos(a) \left(-\frac{1}{2\sqrt{\pi}} + \frac{\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}} + \frac{b \sin\left(\frac{b}{x}\right)}{2\sqrt{\pi} x}\right)}{b^2} + \frac{2\sqrt{\pi} \sin(a) \left(-\frac{b \cos\left(\frac{b}{x}\right)}{2\sqrt{\pi} x} + \frac{\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}}\right)}{b^2}$	81

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(a+b/x)/x^3,x,method=_RETURNVERBOSE)``[Out] -1/b^2*(cos(a+b/x)+(a+b/x)*sin(a+b/x)-a*sin(a+b/x))`**Maxima [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.

time = 0.32, size = 51, normalized size = 1.70

$$-\frac{\left(\Gamma\left(2, \frac{ib}{x}\right) + \Gamma\left(2, -\frac{ib}{x}\right)\right) \cos(a) - \left(i \Gamma\left(2, \frac{ib}{x}\right) - i \Gamma\left(2, -\frac{ib}{x}\right)\right) \sin(a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x^3,x, algorithm="maxima")

[Out]  $-1/2*((\text{gamma}(2, I*b/x) + \text{gamma}(2, -I*b/x))*\cos(a) - (I*\text{gamma}(2, I*b/x) - I*\text{gamma}(2, -I*b/x))*\sin(a))/b^2$

**Fricas** [A]

time = 0.34, size = 33, normalized size = 1.10

$$-\frac{x \cos\left(\frac{ax+b}{x}\right) + b \sin\left(\frac{ax+b}{x}\right)}{b^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x^3,x, algorithm="fricas")

[Out]  $-(x*\cos((a*x + b)/x) + b*\sin((a*x + b)/x))/(b^2*x)$

**Sympy** [A]

time = 0.47, size = 31, normalized size = 1.03

$$\begin{cases} -\frac{\sin\left(a+\frac{b}{x}\right)}{bx} - \frac{\cos\left(a+\frac{b}{x}\right)}{b^2} & \text{for } b \neq 0 \\ -\frac{\cos(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x\*\*3,x)

[Out] Piecewise((-sin(a + b/x)/(b\*x) - cos(a + b/x)/b\*\*2, Ne(b, 0)), (-cos(a)/(2\*x\*\*2), True))

**Giac** [A]

time = 0.45, size = 49, normalized size = 1.63

$$\frac{a \sin\left(\frac{ax+b}{x}\right) - \frac{(ax+b) \sin\left(\frac{ax+b}{x}\right)}{x} - \cos\left(\frac{ax+b}{x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x^3,x, algorithm="giac")

[Out]  $(a*\sin((a*x + b)/x) - (a*x + b)*\sin((a*x + b)/x)/x - \cos((a*x + b)/x))/b^2$

**Mupad** [B]

time = 0.28, size = 30, normalized size = 1.00

$$-\frac{\cos\left(a + \frac{b}{x}\right)}{b^2} - \frac{\sin\left(a + \frac{b}{x}\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b/x)/x^3,x)

[Out]  $-\cos(a + b/x)/b^2 - \sin(a + b/x)/(b*x)$

$$3.39 \quad \int \frac{\cos\left(a + \frac{b}{x}\right)}{x^4} dx$$

Optimal. Leaf size=46

$$-\frac{2 \cos\left(a + \frac{b}{x}\right)}{b^2 x} + \frac{2 \sin\left(a + \frac{b}{x}\right)}{b^3} - \frac{\sin\left(a + \frac{b}{x}\right)}{b x^2}$$

[Out]  $-2*\cos(a+b/x)/b^2/x+2*\sin(a+b/x)/b^3-\sin(a+b/x)/b/x^2$

Rubi [A]

time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3461, 3377, 2717}

$$\frac{2 \sin\left(a + \frac{b}{x}\right)}{b^3} - \frac{2 \cos\left(a + \frac{b}{x}\right)}{b^2 x} - \frac{\sin\left(a + \frac{b}{x}\right)}{b x^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b/x]/x^4,x]

[Out]  $(-2*\cos[a + b/x])/(b^2*x) + (2*\sin[a + b/x])/b^3 - \sin[a + b/x]/(b*x^2)$

Rule 2717

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /;  
FreeQ[{c, d}, x]

Rule 3377

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(-(c + d\*x)^m)\*(Cos[e + f\*x]/f), x] + Dist[d\*(m/f), Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3461

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(a + \frac{b}{x}\right)}{x^4} dx &= -\text{Subst}\left(\int x^2 \cos(a + bx) dx, x, \frac{1}{x}\right) \\
&= -\frac{\sin\left(a + \frac{b}{x}\right)}{bx^2} + \frac{2\text{Subst}\left(\int x \sin(a + bx) dx, x, \frac{1}{x}\right)}{b} \\
&= -\frac{2 \cos\left(a + \frac{b}{x}\right)}{b^2x} - \frac{\sin\left(a + \frac{b}{x}\right)}{bx^2} + \frac{2\text{Subst}\left(\int \cos(a + bx) dx, x, \frac{1}{x}\right)}{b^2} \\
&= -\frac{2 \cos\left(a + \frac{b}{x}\right)}{b^2x} + \frac{2 \sin\left(a + \frac{b}{x}\right)}{b^3} - \frac{\sin\left(a + \frac{b}{x}\right)}{bx^2}
\end{aligned}$$

**Mathematica [A]**

time = 0.00, size = 46, normalized size = 1.00

$$-\frac{2 \cos\left(a + \frac{b}{x}\right)}{b^2x} + \frac{2 \sin\left(a + \frac{b}{x}\right)}{b^3} - \frac{\sin\left(a + \frac{b}{x}\right)}{bx^2}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b/x]/x^4, x]``[Out] (-2*Cos[a + b/x])/(b^2*x) + (2*Sin[a + b/x])/b^3 - Sin[a + b/x]/(b*x^2)`**Maple [A]**

time = 0.09, size = 92, normalized size = 2.00

method	result
risch	$-\frac{2 \cos\left(\frac{ax+b}{x}\right)}{b^2x} - \frac{(b^2-2x^2) \sin\left(\frac{ax+b}{x}\right)}{b^3x^2}$
norman	$-\frac{2x^2}{b^2} + \frac{4x^3 \tan\left(\frac{a}{2} + \frac{b}{2x}\right)}{b^3} + \frac{2x^2 \left(\tan^2\left(\frac{a}{2} + \frac{b}{2x}\right)\right)}{b^2} - \frac{2x \tan\left(\frac{a}{2} + \frac{b}{2x}\right)}{b}$ $\frac{\left(1 + \tan^2\left(\frac{a}{2} + \frac{b}{2x}\right)\right)x^3}{b^3}$
derivativedivides	$-\frac{a^2 \sin\left(a + \frac{b}{x}\right) - 2a \left(\cos\left(a + \frac{b}{x}\right) + \left(a + \frac{b}{x}\right) \sin\left(a + \frac{b}{x}\right)\right) + \left(a + \frac{b}{x}\right)^2 \sin\left(a + \frac{b}{x}\right) - 2 \sin\left(a + \frac{b}{x}\right) + 2 \cos\left(a + \frac{b}{x}\right) \left(a + \frac{b}{x}\right)}{b^3}$
default	$-\frac{a^2 \sin\left(a + \frac{b}{x}\right) - 2a \left(\cos\left(a + \frac{b}{x}\right) + \left(a + \frac{b}{x}\right) \sin\left(a + \frac{b}{x}\right)\right) + \left(a + \frac{b}{x}\right)^2 \sin\left(a + \frac{b}{x}\right) - 2 \sin\left(a + \frac{b}{x}\right) + 2 \cos\left(a + \frac{b}{x}\right) \left(a + \frac{b}{x}\right)}{b^3}$
meijerg	$-\frac{4\sqrt{\pi} \cos(a) \sqrt{b^2} \left( \frac{(b^2)^{\frac{3}{2}} \cos\left(\frac{b}{x}\right)}{2\sqrt{\pi} x b^2} - \frac{(b^2)^{\frac{3}{2}} \left(-\frac{3b^2}{2x^2} + 3\right) \sin\left(\frac{b}{x}\right)}{6\sqrt{\pi} b^3} \right)}{b^4} + \frac{4\sqrt{\pi} \sin(a) \left( -\frac{1}{2\sqrt{\pi}} + \frac{\left(-\frac{b^2}{2x^2} + 1\right) \cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}} \right)}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(a+b/x)/x^4, x, method=_RETURNVERBOSE)``[Out] -1/b^3*(a^2*sin(a+b/x)-2*a*(cos(a+b/x)+(a+b/x)*sin(a+b/x))+(a+b/x)^2*sin(a+b/x)-2*sin(a+b/x)+2*cos(a+b/x)*(a+b/x))`



**Maxima [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.  
time = 0.31, size = 50, normalized size = 1.09

$$\frac{(i\Gamma(3, \frac{ib}{x}) - i\Gamma(3, -\frac{ib}{x}))\cos(a) + (\Gamma(3, \frac{ib}{x}) + \Gamma(3, -\frac{ib}{x}))\sin(a)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x^4,x, algorithm="maxima")

[Out] 1/2\*((I\*gamma(3, I\*b/x) - I\*gamma(3, -I\*b/x))\*cos(a) + (gamma(3, I\*b/x) + gamma(3, -I\*b/x))\*sin(a))/b^3

**Fricas [A]**

time = 0.35, size = 43, normalized size = 0.93

$$-\frac{2bx\cos\left(\frac{ax+b}{x}\right) + (b^2 - 2x^2)\sin\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x^4,x, algorithm="fricas")

[Out] -(2\*b\*x\*cos((a\*x + b)/x) + (b^2 - 2\*x^2)\*sin((a\*x + b)/x))/(b^3\*x^2)

**Sympy [A]**

time = 0.66, size = 46, normalized size = 1.00

$$\begin{cases} -\frac{\sin\left(\frac{a+b}{x}\right)}{bx^2} - \frac{2\cos\left(\frac{a+b}{x}\right)}{b^2x} + \frac{2\sin\left(\frac{a+b}{x}\right)}{b^3} & \text{for } b \neq 0 \\ -\frac{\cos(a)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x\*\*4,x)

[Out] Piecewise((-sin(a + b/x)/(b\*x\*\*2) - 2\*cos(a + b/x)/(b\*\*2\*x) + 2\*sin(a + b/x)/b\*\*3, Ne(b, 0)), (-cos(a)/(3\*x\*\*3), True))

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(46) = 92.

time = 0.42, size = 107, normalized size = 2.33

$$-\frac{a^2\sin\left(\frac{ax+b}{x}\right) - 2a\cos\left(\frac{ax+b}{x}\right) - \frac{2(ax+b)a\sin\left(\frac{ax+b}{x}\right)}{x} + \frac{2(ax+b)\cos\left(\frac{ax+b}{x}\right)}{x} + \frac{(ax+b)^2\sin\left(\frac{ax+b}{x}\right)}{x^2} - 2\sin\left(\frac{ax+b}{x}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x)/x^4,x, algorithm="giac")

[Out]  $-(a^2 \sin((a*x + b)/x) - 2*a*\cos((a*x + b)/x) - 2*(a*x + b)*a*\sin((a*x + b)/x)/x + 2*(a*x + b)*\cos((a*x + b)/x)/x + (a*x + b)^2*\sin((a*x + b)/x)/x^2 - 2*\sin((a*x + b)/x))/b^3$

**Mupad [B]**

time = 0.37, size = 47, normalized size = 1.02

$$\frac{2 \sin\left(a + \frac{b}{x}\right)}{b^3} - \frac{b^2 \sin\left(a + \frac{b}{x}\right) + 2bx \cos\left(a + \frac{b}{x}\right)}{b^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b/x)/x^4,x)`

[Out]  $(2*\sin(a + b/x))/b^3 - (b^2*\sin(a + b/x) + 2*b*x*\cos(a + b/x))/(b^3*x^2)$

### 3.40 $\int \cos\left(a + \frac{b}{x^2}\right) dx$

Optimal. Leaf size=79

$$x \cos\left(a + \frac{b}{x^2}\right) + \sqrt{b} \sqrt{2\pi} \cos(a) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) + \sqrt{b} \sqrt{2\pi} \text{FresnelC}\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) \sin(a)$$

[Out] x\*cos(a+b/x^2)+cos(a)\*FresnelS(b^(1/2)\*2^(1/2)/Pi^(1/2)/x)\*b^(1/2)\*2^(1/2)\*Pi^(1/2)+FresnelC(b^(1/2)\*2^(1/2)/Pi^(1/2)/x)\*sin(a)\*b^(1/2)\*2^(1/2)\*Pi^(1/2)

**Rubi** [A]

time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {3441, 3469, 3434, 3433, 3432}

$$\sqrt{2\pi} \sqrt{b} \sin(a) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b}}{x}\right) + \sqrt{2\pi} \sqrt{b} \cos(a) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) + x \cos\left(a + \frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b/x^2], x]

[Out] x\*Cos[a + b/x^2] + Sqrt[b]\*Sqrt[2\*Pi]\*Cos[a]\*FresnelS[(Sqrt[b]\*Sqrt[2/Pi])/x] + Sqrt[b]\*Sqrt[2\*Pi]\*FresnelC[(Sqrt[b]\*Sqrt[2/Pi])/x]\*Sin[a]

Rule 3432

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] :> Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

Rule 3433

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] :> Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

Rule 3434

Int[Sin[(c\_) + (d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] :> Dist[Sin[c], Int[Cos[d\*(e + f\*x)^2], x], x] + Dist[Cos[c], Int[Sin[d\*(e + f\*x)^2], x], x] /; FreeQ[{c, d, e, f}, x]

Rule 3441

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(a + b*Cos[c + d/x^n])^p/x^2, x], x, 1/(e + f*x)], x] /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] && EqQ[n, -2]
```

### Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)^(m + 1)*
(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(e*x)^(m + n)*
Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \cos\left(a + \frac{b}{x^2}\right) dx &= -\text{Subst}\left(\int \frac{\cos(a + bx^2)}{x^2} dx, x, \frac{1}{x}\right) \\ &= x \cos\left(a + \frac{b}{x^2}\right) + (2b) \text{Subst}\left(\int \sin(a + bx^2) dx, x, \frac{1}{x}\right) \\ &= x \cos\left(a + \frac{b}{x^2}\right) + (2b \cos(a)) \text{Subst}\left(\int \sin(bx^2) dx, x, \frac{1}{x}\right) + (2b \sin(a)) \text{Subst}\left(\int \cos(bx^2) dx, x, \frac{1}{x}\right) \\ &= x \cos\left(a + \frac{b}{x^2}\right) + \sqrt{b} \sqrt{2\pi} \cos(a) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) + \sqrt{b} \sqrt{2\pi} C\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) \sin(a) \end{aligned}$$

### Mathematica [A]

time = 0.09, size = 80, normalized size = 1.01

$$x \cos(a) \cos\left(\frac{b}{x^2}\right) + \sqrt{b} \sqrt{2\pi} \left( \cos(a) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) + \text{FresnelC}\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) \sin(a) \right) - x \sin(a) \sin\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b/x^2], x]
```

```
[Out] x*Cos[a]*Cos[b/x^2] + Sqrt[b]*Sqrt[2*Pi]*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x] + FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a]) - x*Sin[a]*Sin[b/x^2]
```

### Maple [A]

time = 0.11, size = 57, normalized size = 0.72

method	result
derivativedivides	$x \cos\left(a + \frac{b}{x^2}\right) + \sqrt{b} \sqrt{2} \sqrt{\pi} \left( \cos(a) S\left(\frac{\sqrt{b} \sqrt{2}}{\sqrt{\pi} x}\right) + \sin(a) \text{FresnelC}\left(\frac{\sqrt{b} \sqrt{2}}{\sqrt{\pi} x}\right) \right)$
default	$x \cos\left(a + \frac{b}{x^2}\right) + \sqrt{b} \sqrt{2} \sqrt{\pi} \left( \cos(a) S\left(\frac{\sqrt{b} \sqrt{2}}{\sqrt{\pi} x}\right) + \sin(a) \text{FresnelC}\left(\frac{\sqrt{b} \sqrt{2}}{\sqrt{\pi} x}\right) \right)$
risch	$\frac{ie^{-ia} b \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{ib}}{x}\right)}{2\sqrt{ib}} - \frac{ie^{ia} b \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{-ib}}{x}\right)}{2\sqrt{-ib}} + x \cos\left(\frac{ax^2+b}{x^2}\right)$
meijerg	$-\frac{\sqrt{\pi} \cos(a) \sqrt{2} (b^2)^{\frac{1}{4}} \left( -\frac{4x \sqrt{2} \cos\left(\frac{b}{x^2}\right)}{\sqrt{\pi} (b^2)^{\frac{1}{4}}} - \frac{8\sqrt{b} s\left(\frac{\sqrt{b} \sqrt{2}}{\sqrt{\pi} x}\right)}{(b^2)^{\frac{1}{4}}} \right)}{8} + \frac{\sqrt{\pi} \sin(a) \sqrt{2} \sqrt{b} \left( -\frac{4\sqrt{2} x}{\sqrt{b}} \right)}{\sqrt{b}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b/x^2),x,method=_RETURNVERBOSE)`

[Out]  $x \cos\left(a + \frac{b}{x^2}\right) + b^{1/2} 2^{1/2} \pi^{1/2} \left( \cos(a) \text{FresnelS}\left(\frac{b^{1/2} 2^{1/2}}{\pi^{1/2} x}\right) + \sin(a) \text{FresnelC}\left(\frac{b^{1/2} 2^{1/2}}{\pi^{1/2} x}\right) \right)$

**Maxima** [C] Result contains complex when optimal does not.

time = 0.35, size = 127, normalized size = 1.61

$$\frac{\sqrt{2} \left( 2\sqrt{2} b x^2 \sqrt{\frac{1}{x^4}} \cos\left(\frac{ax^2+b}{x^2}\right) + \left( (i+1) \sqrt{\pi} \left( \operatorname{erf}\left(\sqrt{\frac{ib}{x^2}}\right) - 1 \right) - (i-1) \sqrt{\pi} \left( \operatorname{erf}\left(\sqrt{\frac{-ib}{x^2}}\right) - 1 \right) \right) \cos(a) + \left( -(i-1) \sqrt{\pi} \left( \operatorname{erf}\left(\sqrt{\frac{ib}{x^2}}\right) - 1 \right) + (i+1) \sqrt{\pi} \left( \operatorname{erf}\left(\sqrt{\frac{-ib}{x^2}}\right) - 1 \right) \right) \sin(a) \right) b \left(\frac{b}{x^2}\right)^{\frac{1}{4}} \sqrt{x^4}}{4bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x^2),x, algorithm="maxima")`

[Out]  $\frac{1}{4} \sqrt{2} * (2 \sqrt{2} * b * x^2 * \sqrt{x^4}) * \cos\left(\frac{a * x^2 + b}{x^2}\right) + \left( (i+1) * \sqrt{\pi} * \left( \operatorname{erf}\left(\sqrt{\frac{ib}{x^2}}\right) - 1 \right) - (i-1) * \sqrt{\pi} * \left( \operatorname{erf}\left(\sqrt{\frac{-ib}{x^2}}\right) - 1 \right) \right) * \cos(a) + \left( -(i-1) * \sqrt{\pi} * \left( \operatorname{erf}\left(\sqrt{\frac{ib}{x^2}}\right) - 1 \right) + (i+1) * \sqrt{\pi} * \left( \operatorname{erf}\left(\sqrt{\frac{-ib}{x^2}}\right) - 1 \right) \right) * \sin(a) * b * \left(\frac{b}{x^2}\right)^{\frac{1}{4}} * \sqrt{x^4} / (b * x)$

**Fricas** [A]

time = 0.35, size = 73, normalized size = 0.92

$$\sqrt{2} \pi \sqrt{\frac{b}{\pi}} \cos(a) S\left(\frac{\sqrt{2} \sqrt{\frac{b}{\pi}}}{x}\right) + \sqrt{2} \pi \sqrt{\frac{b}{\pi}} C\left(\frac{\sqrt{2} \sqrt{\frac{b}{\pi}}}{x}\right) \sin(a) + x \cos\left(\frac{ax^2+b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x^2),x, algorithm="fricas")`

[Out]  $\sqrt{2}\pi\sqrt{b/\pi}\cos(a)\text{fresnel\_sin}(\sqrt{2}\sqrt{b/\pi}/x) + \sqrt{2}\pi\sqrt{b/\pi}\text{fresnel\_cos}(\sqrt{2}\sqrt{b/\pi}/x)\sin(a) + x\cos((a*x^2 + b)/x^2)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x**2),x)`

[Out] `Integral(cos(a + b/x**2), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x^2),x, algorithm="giac")`

[Out] `integrate(cos(a + b/x^2), x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b/x^2),x)`

[Out] `int(cos(a + b/x^2), x)`

$$3.41 \quad \int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x} dx$$

Optimal. Leaf size=25

$$-\frac{1}{2} \cos(a) \operatorname{CosIntegral}\left(\frac{b}{x^2}\right) + \frac{1}{2} \sin(a) \operatorname{Si}\left(\frac{b}{x^2}\right)$$

[Out] -1/2\*Ci(b/x^2)\*cos(a)+1/2\*Si(b/x^2)\*sin(a)

Rubi [A]

time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3459, 3457, 3456}

$$\frac{1}{2} \sin(a) \operatorname{Si}\left(\frac{b}{x^2}\right) - \frac{1}{2} \cos(a) \operatorname{CosIntegral}\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b/x^2]/x,x]

[Out] -1/2\*(Cos[a]\*CosIntegral[b/x^2]) + (Sin[a]\*SinIntegral[b/x^2])/2

Rule 3456

Int[Sin[(d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] :> Simp[SinIntegral[d\*x^n]/n, x] / ; FreeQ[{d, n}, x]

Rule 3457

Int[Cos[(d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] :> Simp[CosIntegral[d\*x^n]/n, x] / ; FreeQ[{d, n}, x]

Rule 3459

Int[Cos[(c\_) + (d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] :> Dist[Cos[c], Int[Cos[d\*x^n]/x, x], x] - Dist[Sin[c], Int[Sin[d\*x^n]/x, x], x] / ; FreeQ[{c, d, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x} dx &= \cos(a) \int \frac{\cos\left(\frac{b}{x^2}\right)}{x} dx - \sin(a) \int \frac{\sin\left(\frac{b}{x^2}\right)}{x} dx \\ &= -\frac{1}{2} \cos(a) \operatorname{Ci}\left(\frac{b}{x^2}\right) + \frac{1}{2} \sin(a) \operatorname{Si}\left(\frac{b}{x^2}\right) \end{aligned}$$

**Mathematica [A]**

time = 0.03, size = 24, normalized size = 0.96

$$\frac{1}{2} \left( -\cos(a) \operatorname{CosIntegral} \left( \frac{b}{x^2} \right) + \sin(a) \operatorname{Si} \left( \frac{b}{x^2} \right) \right)$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b/x^2]/x,x]``[Out] (-Cos[a]*CosIntegral[b/x^2]) + Sin[a]*SinIntegral[b/x^2])/2`**Maple [A]**

time = 0.07, size = 22, normalized size = 0.88

method	result
derivativedivides	$-\frac{\operatorname{cosineIntegral}\left(\frac{b}{x^2}\right) \cos(a)}{2} + \frac{\operatorname{sinIntegral}\left(\frac{b}{x^2}\right) \sin(a)}{2}$
default	$-\frac{\operatorname{cosineIntegral}\left(\frac{b}{x^2}\right) \cos(a)}{2} + \frac{\operatorname{sinIntegral}\left(\frac{b}{x^2}\right) \sin(a)}{2}$
risch	$-\frac{ie^{-ia} \pi \operatorname{csgn}\left(\frac{b}{x^2}\right)}{4} + \frac{ie^{-ia} \operatorname{sinIntegral}\left(\frac{b}{x^2}\right)}{2} + \frac{e^{-ia} \operatorname{expIntegral}\left(1, -\frac{ib}{x^2}\right)}{4} + \frac{e^{ia} \operatorname{expIntegral}\left(1, -\frac{ib}{x^2}\right)}{4}$
meijerg	$-\frac{\sqrt{\pi} \cos(a) \left( \frac{2\gamma - 4 \ln(x) + \ln(b^2)}{\sqrt{\pi}} - \frac{2\gamma}{\sqrt{\pi}} - \frac{2 \ln(2)}{\sqrt{\pi}} - \frac{2 \ln\left(\frac{b}{2x^2}\right)}{\sqrt{\pi}} + \frac{2 \operatorname{cosineIntegral}\left(\frac{b}{x^2}\right)}{\sqrt{\pi}} \right)}{4} + \frac{\operatorname{sinIntegral}\left(\frac{b}{x^2}\right) \sin(a)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(a+b/x^2)/x,x,method=_RETURNVERBOSE)``[Out] -1/2*Ci(b/x^2)*cos(a)+1/2*Si(b/x^2)*sin(a)`**Maxima [C]** Result contains complex when optimal does not.

time = 0.31, size = 43, normalized size = 1.72

$$-\frac{1}{4} \left( \operatorname{Ei} \left( \frac{ib}{x^2} \right) + \operatorname{Ei} \left( -\frac{ib}{x^2} \right) \right) \cos(a) - \frac{1}{4} \left( i \operatorname{Ei} \left( \frac{ib}{x^2} \right) - i \operatorname{Ei} \left( -\frac{ib}{x^2} \right) \right) \sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(a+b/x^2)/x,x, algorithm="maxima")``[Out] -1/4*(Ei(I*b/x^2) + Ei(-I*b/x^2))*cos(a) - 1/4*(I*Ei(I*b/x^2) - I*Ei(-I*b/x^2))*sin(a)`**Fricas [A]**

time = 0.37, size = 29, normalized size = 1.16

$$-\frac{1}{4} \left( \operatorname{Ci} \left( \frac{b}{x^2} \right) + \operatorname{Ci} \left( -\frac{b}{x^2} \right) \right) \cos(a) + \frac{1}{2} \sin(a) \operatorname{Si} \left( \frac{b}{x^2} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x^2)/x,x, algorithm="fricas")

[Out]  $-1/4*(\cos\_integral(b/x^2) + \cos\_integral(-b/x^2))*\cos(a) + 1/2*\sin(a)*\sin\_integral(b/x^2)$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x\*\*2)/x,x)

[Out] Integral(cos(a + b/x\*\*2)/x, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x^2)/x,x, algorithm="giac")

[Out] integrate(cos(a + b/x^2)/x, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.04

$$\frac{\sin(a) \operatorname{sinint}\left(\frac{b}{x^2}\right)}{2} - \frac{\cos(a) \operatorname{cosint}\left(\frac{b}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b/x^2)/x,x)

[Out]  $(\sin(a)*\operatorname{sinint}(b/x^2))/2 - (\cos(a)*\operatorname{cosint}(b/x^2))/2$

$$3.42 \quad \int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^2} dx$$

Optimal. Leaf size=74

$$-\frac{\sqrt{\frac{\pi}{2}} \cos(a) \operatorname{FresnelC}\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} + \frac{\sqrt{\frac{\pi}{2}} S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) \sin(a)}{\sqrt{b}}$$

[Out]  $-1/2*\cos(a)*\operatorname{FresnelC}(b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)}/x)*2^{(1/2)}*\pi^{(1/2)}/b^{(1/2)}+1/2*\operatorname{FresnelS}(b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)}/x)*\sin(a)*2^{(1/2)}*\pi^{(1/2)}/b^{(1/2)}$

**Rubi [A]**

time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3465, 3435, 3433, 3432}

$$\frac{\sqrt{\frac{\pi}{2}} \sin(a) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}} \cos(a) \operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b}}{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[a + b/x^2]/x^2, x]$

[Out]  $-\left(\left(\operatorname{Sqrt}[\pi/2]*\operatorname{Cos}[a]*\operatorname{FresnelC}\left[\left(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\pi]\right)/x\right]\right)/\operatorname{Sqrt}[b]\right) + \left(\operatorname{Sqrt}[\pi/2]*\operatorname{FresnelS}\left[\left(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\pi]\right)/x\right]*\operatorname{Sin}[a]\right)/\operatorname{Sqrt}[b]$

Rule 3432

$\operatorname{Int}[\operatorname{Sin}[(d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[\pi/2]/(f*\operatorname{Rt}[d, 2]))*\operatorname{FresnelS}[\operatorname{Sqrt}[2/\pi]*\operatorname{Rt}[d, 2]*(e + f*x)], x] /; \operatorname{FreeQ}\{d, e, f\}, x]$

Rule 3433

$\operatorname{Int}[\operatorname{Cos}[(d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[\pi/2]/(f*\operatorname{Rt}[d, 2]))*\operatorname{FresnelC}[\operatorname{Sqrt}[2/\pi]*\operatorname{Rt}[d, 2]*(e + f*x)], x] /; \operatorname{FreeQ}\{d, e, f\}, x]$

Rule 3435

$\operatorname{Int}[\operatorname{Cos}[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[c], \operatorname{Int}[\operatorname{Cos}[d*(e + f*x)^2], x], x] - \operatorname{Dist}[\operatorname{Sin}[c], \operatorname{Int}[\operatorname{Sin}[d*(e + f*x)^2], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x]$

Rule 3465

`Int[Cos[(a_.) + (b_.)*(x_)^(n_)]*(x_)^(m_.), x_Symbol] := Dist[2/n, Subst[Int[Cos[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n/2 - 1]`

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^2} dx &= -\text{Subst}\left(\int \cos(a + bx^2) dx, x, \frac{1}{x}\right) \\ &= -\left(\cos(a)\text{Subst}\left(\int \cos(bx^2) dx, x, \frac{1}{x}\right)\right) + \sin(a)\text{Subst}\left(\int \sin(bx^2) dx, x, \frac{1}{x}\right) \\ &= -\frac{\sqrt{\frac{\pi}{2}} \cos(a) C\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} + \frac{\sqrt{\frac{\pi}{2}} S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) \sin(a)}{\sqrt{b}} \end{aligned}$$

Mathematica [A]

time = 0.07, size = 62, normalized size = 0.84

$$-\frac{\sqrt{\frac{\pi}{2}} \left( \cos(a) \text{FresnelC}\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) - S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) \sin(a) \right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[a + b/x^2]/x^2, x]`

[Out] `-((Sqrt[Pi/2]*(Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x] - FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a]))/Sqrt[b])`

Maple [A]

time = 0.07, size = 48, normalized size = 0.65

method	result	size
derivativedivides	$-\frac{\sqrt{2} \sqrt{\pi} \left( \cos(a) \text{FresnelC}\left(\frac{\sqrt{b} \sqrt{2}}{\sqrt{\pi} x}\right) - \sin(a) S\left(\frac{\sqrt{b} \sqrt{2}}{\sqrt{\pi} x}\right) \right)}{2\sqrt{b}}$	48

default	$\frac{\sqrt{2} \sqrt{\pi} \left( \cos(a) \operatorname{FresnelC} \left( \frac{\sqrt{b} \sqrt{2}}{\sqrt{\pi} x} \right) - \sin(a) \operatorname{S} \left( \frac{\sqrt{b} \sqrt{2}}{\sqrt{\pi} x} \right) \right)}{2\sqrt{b}}$	48
meijerg	$\frac{\cos(a) \operatorname{FresnelC} \left( \frac{\sqrt{b} \sqrt{2}}{\sqrt{\pi} x} \right) \sqrt{2} \sqrt{\pi}}{2\sqrt{b}} + \frac{\operatorname{S} \left( \frac{\sqrt{b} \sqrt{2}}{\sqrt{\pi} x} \right) \sin(a) \sqrt{2} \sqrt{\pi}}{2\sqrt{b}}$	56
risch	$\frac{e^{-ia} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{ib}}{x} \right)}{4\sqrt{ib}} - \frac{e^{ia} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{-ib}}{x} \right)}{4\sqrt{-ib}}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b/x^2)/x^2,x,method=_RETURNVERBOSE)`

[Out]  $-1/2*2^{(1/2)}*Pi^{(1/2)}/b^{(1/2)}*(\cos(a)*\operatorname{FresnelC}(b^{(1/2)}*2^{(1/2)}/Pi^{(1/2)}/x) - \sin(a)*\operatorname{FresnelS}(b^{(1/2)}*2^{(1/2)}/Pi^{(1/2)}/x))$

**Maxima** [C] Result contains complex when optimal does not.

time = 0.33, size = 98, normalized size = 1.32

$$\frac{\sqrt{2} \sqrt{x^4} \left( \left( -(i-1) \sqrt{\pi} \left( \operatorname{erf} \left( \frac{\sqrt{ib}}{x^2} \right) - 1 \right) + (i+1) \sqrt{\pi} \left( \operatorname{erf} \left( \frac{\sqrt{-ib}}{x^2} \right) - 1 \right) \right) \cos(a) + \left( -(i+1) \sqrt{\pi} \left( \operatorname{erf} \left( \frac{\sqrt{ib}}{x^2} \right) - 1 \right) + (i-1) \sqrt{\pi} \left( \operatorname{erf} \left( \frac{\sqrt{-ib}}{x^2} \right) - 1 \right) \right) \sin(a) \right) \left( \frac{x^2}{2} \right)^{\frac{1}{4}}}{8bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x^2)/x^2,x, algorithm="maxima")`

[Out]  $-1/8*\sqrt{2}*((-I - 1)*\sqrt{\pi}*(\operatorname{erf}(\sqrt{I*b/x^2}) - 1) + (I + 1)*\sqrt{\pi}*(\operatorname{erf}(\sqrt{-I*b/x^2}) - 1))*\cos(a) + (-I + 1)*\sqrt{\pi}*(\operatorname{erf}(\sqrt{I*b/x^2}) - 1) + (I - 1)*\sqrt{\pi}*(\operatorname{erf}(\sqrt{-I*b/x^2}) - 1))*\sin(a))*\sqrt{x^4}*(b^2/x^4)^{(1/4)}/(b*x)$

**Fricas** [A]

time = 0.36, size = 65, normalized size = 0.88

$$\frac{\sqrt{2} \pi \sqrt{\frac{b}{\pi}} \cos(a) C \left( \frac{\sqrt{2} \sqrt{\frac{b}{\pi}}}{x} \right) - \sqrt{2} \pi \sqrt{\frac{b}{\pi}} S \left( \frac{\sqrt{2} \sqrt{\frac{b}{\pi}}}{x} \right) \sin(a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x^2)/x^2,x, algorithm="fricas")`

[Out]  $-1/2*(\sqrt{2}*\pi*\sqrt{b/\pi}*\cos(a)*\operatorname{fresnel\_cos}(\sqrt{2}*\sqrt{b/\pi}/x) - \sqrt{2}*\pi*\sqrt{b/\pi}*\operatorname{fresnel\_sin}(\sqrt{2}*\sqrt{b/\pi}/x)*\sin(a))/b$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x\*\*2)/x\*\*2,x)

[Out] Integral(cos(a + b/x\*\*2)/x\*\*2, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x^2)/x^2,x, algorithm="giac")

[Out] integrate(cos(a + b/x^2)/x^2, x)

**Mupad [B]**

time = 0.41, size = 55, normalized size = 0.74

$$\frac{\sqrt{2} \sqrt{\pi} S\left(\frac{\sqrt{2} \sqrt{b}}{x \sqrt{\pi}}\right) \sin(a)}{2 \sqrt{b}} - \frac{\sqrt{2} \sqrt{\pi} C\left(\frac{\sqrt{2} \sqrt{b}}{x \sqrt{\pi}}\right) \cos(a)}{2 \sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b/x^2)/x^2,x)

[Out]  $(2^{(1/2)} \pi^{(1/2)} \text{fresnels}((2^{(1/2)} b^{(1/2)})/(x \pi^{(1/2)})) \sin(a))/(2 b^{(1/2)}) - (2^{(1/2)} \pi^{(1/2)} \text{fresnelc}((2^{(1/2)} b^{(1/2)})/(x \pi^{(1/2)})) \cos(a))/(2 b^{(1/2)})$

### 3.43

$$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^3} dx$$

Optimal. Leaf size=15

$$-\frac{\sin\left(a + \frac{b}{x^2}\right)}{2b}$$

[Out] -1/2\*sin(a+b/x^2)/b

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3461, 2717}

$$-\frac{\sin\left(a + \frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b/x^2]/x^3,x]

[Out] -1/2\*Sin[a + b/x^2]/b

Rule 2717

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3461

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^3} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \cos(a + bx) dx, x, \frac{1}{x^2}\right)\right) \\ &= -\frac{\sin\left(a + \frac{b}{x^2}\right)}{2b} \end{aligned}$$

Mathematica [A]

time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\sin\left(a + \frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b/x^2]/x^3,x]

[Out] -1/2\*Sin[a + b/x^2]/b

**Maple [A]**

time = 0.04, size = 14, normalized size = 0.93

method	result	size
derivativdivides	$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$	14
default	$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$	14
risch	$-\frac{\sin\left(\frac{ax^2+b}{x^2}\right)}{2b}$	18
norman	$-\frac{\tan\left(\frac{a}{2}+\frac{b}{2x^2}\right)}{b\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x^2}\right)\right)}$	34
meijerg	$-\frac{\cos(a)\sin\left(\frac{b}{x^2}\right)}{2b} + \frac{\sqrt{\pi}\sin(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{2b}$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b/x^2)/x^3,x,method=\_RETURNVERBOSE)

[Out] -1/2\*sin(a+b/x^2)/b

**Maxima [A]**

time = 0.28, size = 13, normalized size = 0.87

$$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x^2)/x^3,x, algorithm="maxima")

[Out] -1/2\*sin(a + b/x^2)/b

**Fricas [A]**

time = 0.35, size = 17, normalized size = 1.13

$$-\frac{\sin\left(\frac{ax^2+b}{x^2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x^2)/x^3,x, algorithm="fricas")

[Out]  $-1/2*\sin((a*x^2 + b)/x^2)/b$

**Sympy [A]**

time = 0.67, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\sin\left(a + \frac{b}{x^2}\right)}{2b} & \text{for } b \neq 0 \\ -\frac{\cos(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x**2)/x**3,x)`

[Out] `Piecewise((-sin(a + b/x**2)/(2*b), Ne(b, 0)), (-cos(a)/(2*x**2), True))`

**Giac [A]**

time = 0.45, size = 17, normalized size = 1.13

$$-\frac{\sin\left(\frac{ax^2+b}{x^2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b/x^2)/x^3,x, algorithm="giac")`

[Out]  $-1/2*\sin((a*x^2 + b)/x^2)/b$

**Mupad [B]**

time = 0.27, size = 13, normalized size = 0.87

$$-\frac{\sin\left(a + \frac{b}{x^2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b/x^2)/x^3,x)`

[Out]  $-\sin(a + b/x^2)/(2*b)$



$$3.44 \quad \int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^4} dx$$

**Optimal.** Leaf size=97

$$\frac{\sqrt{\frac{\pi}{2}} \cos(a) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) \sin(a)}{2b^{3/2}} - \frac{\sin\left(a + \frac{b}{x^2}\right)}{2bx}$$

[Out]  $-1/2*\sin(a+b/x^2)/b/x+1/4*\cos(a)*\text{FresnelS}(b^{(1/2)}*2^{(1/2)}/\text{Pi}^{(1/2)}/x)*2^{(1/2)}/2*\text{Pi}^{(1/2)}/b^{(3/2)}+1/4*\text{FresnelC}(b^{(1/2)}*2^{(1/2)}/\text{Pi}^{(1/2)}/x)*\sin(a)*2^{(1/2)}/\text{Pi}^{(1/2)}/b^{(3/2)}$

**Rubi [A]**

time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {3491, 3467, 3434, 3433, 3432}

$$\frac{\sqrt{\frac{\pi}{2}} \sin(a) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{b}}{x}\right)}{2b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \cos(a) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} - \frac{\sin\left(a + \frac{b}{x^2}\right)}{2bx}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[a + b/x^2]/x^4, x]$

[Out]  $(\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a]*\text{FresnelS}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}])/x])/(2*b^{(3/2)}) + (\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[(\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}])/x]*\text{Sin}[a])/(2*b^{(3/2)}) - \text{Sin}[a + b/x^2]/(2*b*x)$

**Rule 3432**

$\text{Int}[\text{Sin}[(d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]/(f*\text{Rt}[d, 2]))*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)], x] /; \text{FreeQ}\{d, e, f, x\}$

**Rule 3433**

$\text{Int}[\text{Cos}[(d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2]/(f*\text{Rt}[d, 2]))*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*\text{Rt}[d, 2]*(e + f*x)], x] /; \text{FreeQ}\{d, e, f, x\}$

**Rule 3434**

$\text{Int}[\text{Sin}[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^{2}], x\_Symbol] \rightarrow \text{Dist}[\text{Sin}[c], \text{Int}[\text{Cos}[d*(e + f*x)^2], x], x] + \text{Dist}[\text{Cos}[c], \text{Int}[\text{Sin}[d*(e + f*x)^2], x], x] /$

; FreeQ[{c, d, e, f}, x]

### Rule 3467

Int[Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)]\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[e^(n - 1)\*(e\*x)^(m - n + 1)\*(Sin[c + d\*x^n]/(d\*n)), x] - Dist[e^n\*((m - n + 1)/(d\*n)), Int[(e\*x)^(m - n)\*Sin[c + d\*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]

### Rule 3491

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)]\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := -Subst[Int[(a + b\*Cos[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]

### Rubi steps

$$\begin{aligned} \int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^4} dx &= -\text{Subst}\left(\int x^2 \cos(a + bx^2) dx, x, \frac{1}{x}\right) \\ &= -\frac{\sin\left(a + \frac{b}{x^2}\right)}{2bx} + \frac{\text{Subst}\left(\int \sin(a + bx^2) dx, x, \frac{1}{x}\right)}{2b} \\ &= -\frac{\sin\left(a + \frac{b}{x^2}\right)}{2bx} + \frac{\cos(a)\text{Subst}\left(\int \sin(bx^2) dx, x, \frac{1}{x}\right)}{2b} + \frac{\sin(a)\text{Subst}\left(\int \cos(bx^2) dx, x, \frac{1}{x}\right)}{2b} \\ &= \frac{\sqrt{\frac{\pi}{2}} \cos(a) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} C\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) \sin(a)}{2b^{3/2}} - \frac{\sin\left(a + \frac{b}{x^2}\right)}{2bx} \end{aligned}$$

### Mathematica [A]

time = 0.11, size = 88, normalized size = 0.91

$$\frac{\sqrt{2\pi} x \cos(a) S\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) + \sqrt{2\pi} x \text{FresnelC}\left(\frac{\sqrt{b} \sqrt{\frac{2}{\pi}}}{x}\right) \sin(a) - 2\sqrt{b} \sin\left(a + \frac{b}{x^2}\right)}{4b^{3/2}x}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b/x^2]/x^4, x]

[Out] (Sqrt[2\*Pi]\*x\*Cos[a]\*FresnelS[(Sqrt[b]\*Sqrt[2/Pi])/x] + Sqrt[2\*Pi]\*x\*FresnelC[(Sqrt[b]\*Sqrt[2/Pi])/x]\*Sin[a] - 2\*Sqrt[b]\*Sin[a + b/x^2])/(4\*b^(3/2)\*x)

**Maple [A]**

time = 0.09, size = 64, normalized size = 0.66

method	result
derivativedivides	$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx} + \frac{\sqrt{2}\sqrt{\pi}\left(\cos(a)\operatorname{S}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}x}\right) + \sin(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}x}\right)\right)}{4b^{\frac{3}{2}}}$
default	$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx} + \frac{\sqrt{2}\sqrt{\pi}\left(\cos(a)\operatorname{S}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}x}\right) + \sin(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}x}\right)\right)}{4b^{\frac{3}{2}}}$
risch	$\frac{i e^{-ia} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{ib}}{x}\right)}{8b\sqrt{ib}} - \frac{i e^{ia} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{-ib}}{x}\right)}{8b\sqrt{-ib}} - \frac{\sin\left(\frac{ax^2+b}{x^2}\right)}{2bx}$
meijerg	$-\frac{\sqrt{\pi} \cos(a) \sqrt{2} (b^2)^{\frac{1}{4}} \left( \frac{\sqrt{2} (b^2)^{\frac{3}{4}} \sin\left(\frac{b}{x^2}\right)}{2\sqrt{\pi}xb} - \frac{(b^2)^{\frac{3}{4}} \operatorname{S}\left(\frac{\sqrt{b}\sqrt{2}}{\sqrt{\pi}x}\right)}{2b^{\frac{3}{2}}} \right)}{2b^2} + \frac{\sqrt{\pi} \sin(a) \sqrt{2} \left( -\frac{\sqrt{2}\sqrt{b}}{2\sqrt{\pi}} \right)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** `int(cos(a+b/x^2)/x^4,x,method=_RETURNVERBOSE)`**[Out]** `-1/2*sin(a+b/x^2)/b/x+1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)`**Maxima [C]** Result contains complex when optimal does not.

time = 0.34, size = 74, normalized size = 0.76

$$\frac{\sqrt{2}(x^4)^{\frac{3}{2}}\left(\left(-i+1\right)\Gamma\left(\frac{3}{2},\frac{ib}{x^2}\right)+\left(i-1\right)\Gamma\left(\frac{3}{2},-\frac{ib}{x^2}\right)\right)\cos(a)+\left(\left(i-1\right)\Gamma\left(\frac{3}{2},\frac{ib}{x^2}\right)-\left(i+1\right)\Gamma\left(\frac{3}{2},-\frac{ib}{x^2}\right)\right)\sin(a)\left(\frac{b^2}{x^4}\right)^{\frac{3}{4}}}{8b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** `integrate(cos(a+b/x^2)/x^4,x,algorithm="maxima")`**[Out]** `1/8*sqrt(2)*(x^4)^(3/2)*((-I+1)*gamma(3/2,I*b/x^2)+(I-1)*gamma(3/2,-I*b/x^2))*cos(a)+((I-1)*gamma(3/2,I*b/x^2)-(I+1)*gamma(3/2,-I*b/x^2))*sin(a)*(b^2/x^4)^(3/4)/(b^3*x^3)`**Fricas [A]**

time = 0.37, size = 84, normalized size = 0.87

$$\frac{\sqrt{2}\pi x \sqrt{\frac{b}{\pi}} \cos(a) \operatorname{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right) + \sqrt{2}\pi x \sqrt{\frac{b}{\pi}} \operatorname{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right) \sin(a) - 2b \sin\left(\frac{ax^2+b}{x^2}\right)}{4b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x^2)/x^4,x, algorithm="fricas")

[Out]  $\frac{1}{4} * (\sqrt{2} * \pi * x * \sqrt{b/\pi}) * \cos(a) * \text{fresnel\_sin}(\sqrt{2} * \sqrt{b/\pi}/x) + \sqrt{2} * \pi * x * \sqrt{b/\pi} * \text{fresnel\_cos}(\sqrt{2} * \sqrt{b/\pi}/x) * \sin(a) - 2 * b * \sin((a * x^2 + b)/x^2)) / (b^2 * x)$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x\*\*2)/x\*\*4,x)

[Out] Integral(cos(a + b/x\*\*2)/x\*\*4, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b/x^2)/x^4,x, algorithm="giac")

[Out] integrate(cos(a + b/x^2)/x^4, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b/x^2)/x^4,x)

[Out] int(cos(a + b/x^2)/x^4, x)

$$3.45 \quad \int \frac{\cos^2(\sqrt{x})}{\sqrt{x}} dx$$

Optimal. Leaf size=19

$$\sqrt{x} + \cos(\sqrt{x}) \sin(\sqrt{x})$$

[Out]  $\cos(x^{1/2})\sin(x^{1/2})+x^{1/2}$

Rubi [A]

time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3461, 2715, 8}

$$\sqrt{x} + \sin(\sqrt{x}) \cos(\sqrt{x})$$

Antiderivative was successfully verified.

[In] `Int[Cos[Sqrt[x]]^2/Sqrt[x],x]`

[Out] `Sqrt[x] + Cos[Sqrt[x]]*Sin[Sqrt[x]]`

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2715

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1)/(d*n), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3461

`Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*COS[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))`

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(\sqrt{x})}{\sqrt{x}} dx &= 2\text{Subst}\left(\int \cos^2(x) dx, x, \sqrt{x}\right) \\ &= \cos(\sqrt{x}) \sin(\sqrt{x}) + \text{Subst}\left(\int 1 dx, x, \sqrt{x}\right) \\ &= \sqrt{x} + \cos(\sqrt{x}) \sin(\sqrt{x}) \end{aligned}$$

**Mathematica [A]**

time = 0.02, size = 18, normalized size = 0.95

$$\sqrt{x} + \frac{1}{2} \sin(2\sqrt{x})$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[Sqrt[x]]^2/Sqrt[x], x]``[Out] Sqrt[x] + Sin[2*Sqrt[x]]/2`**Maple [A]**

time = 0.05, size = 14, normalized size = 0.74

method	result	size
derivativdivides	$\cos(\sqrt{x}) \sin(\sqrt{x}) + \sqrt{x}$	14
default	$\cos(\sqrt{x}) \sin(\sqrt{x}) + \sqrt{x}$	14

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x^(1/2))^2/x^(1/2), x, method=_RETURNVERBOSE)``[Out] cos(x^(1/2))*sin(x^(1/2))+x^(1/2)`**Maxima [A]**

time = 0.28, size = 12, normalized size = 0.63

$$\sqrt{x} + \frac{1}{2} \sin(2\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x^(1/2))^2/x^(1/2), x, algorithm="maxima")``[Out] sqrt(x) + 1/2*sin(2*sqrt(x))`**Fricas [A]**

time = 0.39, size = 13, normalized size = 0.68

$$\cos(\sqrt{x}) \sin(\sqrt{x}) + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="fricas")`

[Out] `cos(sqrt(x))*sin(sqrt(x)) + sqrt(x)`

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

time = 0.08, size = 39, normalized size = 2.05

$$\sqrt{x} \sin^2(\sqrt{x}) + \sqrt{x} \cos^2(\sqrt{x}) + \sin(\sqrt{x}) \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x**(1/2))**2/x**(1/2),x)`

[Out] `sqrt(x)*sin(sqrt(x))**2 + sqrt(x)*cos(sqrt(x))**2 + sin(sqrt(x))*cos(sqrt(x))`

**Giac** [A]

time = 0.40, size = 12, normalized size = 0.63

$$\sqrt{x} + \frac{1}{2} \sin(2\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="giac")`

[Out] `sqrt(x) + 1/2*sin(2*sqrt(x))`

**Mupad** [B]

time = 0.37, size = 12, normalized size = 0.63

$$\frac{\sin(2\sqrt{x})}{2} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x^(1/2))^2/x^(1/2),x)`

[Out] `sin(2*x^(1/2))/2 + x^(1/2)`

$$3.46 \quad \int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$$

Optimal. Leaf size=8

$$2 \sin(\sqrt{x})$$

[Out] 2\*sin(x^(1/2))

Rubi [A]

time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3461, 2717}

$$2 \sin(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Cos[Sqrt[x]]/Sqrt[x],x]

[Out] 2\*Sin[Sqrt[x]]

Rule 2717

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /;  
FreeQ[{c, d}, x]

Rule 3461

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx &= 2 \text{Subst} \left( \int \cos(x) dx, x, \sqrt{x} \right) \\ &= 2 \sin(\sqrt{x}) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$2 \sin(\sqrt{x})$$



Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sin[Sqrt[x]]
```

**Maple** [A]

time = 0.02, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$2 \sin(\sqrt{x})$	7
default	$2 \sin(\sqrt{x})$	7
meijerg	$2 \sin(\sqrt{x})$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*sin(x^(1/2))
```

**Maxima** [A]

time = 0.28, size = 6, normalized size = 0.75

$$2 \sin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sin(sqrt(x))
```

**Fricas** [A]

time = 0.42, size = 6, normalized size = 0.75

$$2 \sin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sin(sqrt(x))
```

**Sympy** [A]

time = 0.09, size = 7, normalized size = 0.88

$$2 \sin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sin(sqrt(x))
```

**Giac [A]**

time = 0.43, size = 6, normalized size = 0.75

$$2 \sin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sin(sqrt(x))
```

**Mupad [B]**

time = 0.03, size = 6, normalized size = 0.75

$$2 \sin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))/x^(1/2),x)
```

```
[Out] 2*sin(x^(1/2))
```

### 3.47 $\int \cos(\sqrt{x}) dx$

Optimal. Leaf size=22

$$2 \cos(\sqrt{x}) + 2\sqrt{x} \sin(\sqrt{x})$$

[Out] 2\*cos(x^(1/2))+2\*sin(x^(1/2))\*x^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3443, 3377, 2718}

$$2\sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Cos[Sqrt[x]], x]

[Out] 2\*Cos[Sqrt[x]] + 2\*Sqrt[x]\*Sin[Sqrt[x]]

Rule 2718

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := Simp[(-(c + d\*x)^m)\*(Cos[e + f\*x]/f), x] + Dist[d\*(m/f), Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3443

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_.))]^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/(n\*f), Subst[Int[x^(1/n - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, (e + f\*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned} \int \cos(\sqrt{x}) dx &= 2\text{Subst}\left(\int x \cos(x) dx, x, \sqrt{x}\right) \\ &= 2\sqrt{x} \sin(\sqrt{x}) - 2\text{Subst}\left(\int \sin(x) dx, x, \sqrt{x}\right) \\ &= 2 \cos(\sqrt{x}) + 2\sqrt{x} \sin(\sqrt{x}) \end{aligned}$$

**Mathematica [A]**

time = 0.01, size = 22, normalized size = 1.00

$$2 \cos(\sqrt{x}) + 2\sqrt{x} \sin(\sqrt{x})$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[Sqrt[x]],x]``[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]`**Maple [A]**

time = 0.02, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$2 \cos(\sqrt{x}) + 2 \sin(\sqrt{x}) \sqrt{x}$	17
default	$2 \cos(\sqrt{x}) + 2 \sin(\sqrt{x}) \sqrt{x}$	17
meijerg	$4\sqrt{\pi} \left( -\frac{1}{2\sqrt{\pi}} + \frac{\cos(\sqrt{x})}{2\sqrt{\pi}} + \frac{\sqrt{x} \sin(\sqrt{x})}{2\sqrt{\pi}} \right)$	33

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x^(1/2)),x,method=_RETURNVERBOSE)``[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)`**Maxima [A]**

time = 0.28, size = 16, normalized size = 0.73

$$2 \sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x^(1/2)),x, algorithm="maxima")``[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))`**Fricas [A]**

time = 0.38, size = 16, normalized size = 0.73

$$2 \sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x^(1/2)),x, algorithm="fricas")``[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))`

**Sympy [A]**

time = 0.09, size = 20, normalized size = 0.91

$$2\sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

**Giac [A]**

time = 0.49, size = 16, normalized size = 0.73

$$2\sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

**Mupad [B]**

time = 0.28, size = 16, normalized size = 0.73

$$2 \cos(\sqrt{x}) + 2\sqrt{x} \sin(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
```

```
[Out] 2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

### 3.48 $\int \cos^2(\sqrt{x}) dx$

Optimal. Leaf size=36

$$\frac{x}{2} + \frac{1}{2} \cos^2(\sqrt{x}) + \sqrt{x} \cos(\sqrt{x}) \sin(\sqrt{x})$$

[Out] 1/2\*x+1/2\*cos(x^(1/2))^2+cos(x^(1/2))\*sin(x^(1/2))\*x^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {3443, 3391, 30}

$$\frac{x}{2} + \frac{1}{2} \cos^2(\sqrt{x}) + \sqrt{x} \sin(\sqrt{x}) \cos(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Cos[Sqrt[x]]^2,x]

[Out] x/2 + Cos[Sqrt[x]]^2/2 + Sqrt[x]\*Cos[Sqrt[x]]\*Sin[Sqrt[x]]

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3391

Int[((c\_) + (d\_)\*(x\_))\*((b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[d\*((b\*SIN[e + f\*x])^n/(f^2\*n^2)), x] + (Dist[b^2\*((n - 1)/n), Int[(c + d\*x)\*(b\*SIN[e + f\*x])^(n - 2), x], x] - Simp[b\*(c + d\*x)\*Cos[e + f\*x]\*((b\*SIN[e + f\*x])^(n - 1)/(f\*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 3443

Int[((a\_) + Cos[(c\_) + (d\_)\*((e\_) + (f\_)\*(x\_))]^(n\_)]\*(b\_)^(p\_), x\_Symbol] := Dist[1/(n\*f), Subst[Int[x^(1/n - 1)\*(a + b\*COS[c + d\*x])^p, x], x, (e + f\*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned}
\int \cos^2(\sqrt{x}) dx &= 2\text{Subst}\left(\int x \cos^2(x) dx, x, \sqrt{x}\right) \\
&= \frac{1}{2} \cos^2(\sqrt{x}) + \sqrt{x} \cos(\sqrt{x}) \sin(\sqrt{x}) + \text{Subst}\left(\int x dx, x, \sqrt{x}\right) \\
&= \frac{x}{2} + \frac{1}{2} \cos^2(\sqrt{x}) + \sqrt{x} \cos(\sqrt{x}) \sin(\sqrt{x})
\end{aligned}$$

**Mathematica [A]**

time = 0.02, size = 31, normalized size = 0.86

$$\frac{1}{4}(\cos(2\sqrt{x}) + 2(x + \sqrt{x} \sin(2\sqrt{x})))$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[Sqrt[x]]^2,x]``[Out] (Cos[2*Sqrt[x]] + 2*(x + Sqrt[x]*Sin[2*Sqrt[x]]))/4`**Maple [A]**

time = 0.03, size = 34, normalized size = 0.94

method	result	size
derivativedivides	$2\sqrt{x} \left( \frac{\cos(\sqrt{x}) \sin(\sqrt{x})}{2} + \frac{\sqrt{x}}{2} \right) - \frac{x}{2} - \frac{(\sin^2(\sqrt{x}))}{2}$	34
default	$2\sqrt{x} \left( \frac{\cos(\sqrt{x}) \sin(\sqrt{x})}{2} + \frac{\sqrt{x}}{2} \right) - \frac{x}{2} - \frac{(\sin^2(\sqrt{x}))}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(x^(1/2))^2,x,method=_RETURNVERBOSE)``[Out] 2*x^(1/2)*(1/2*cos(x^(1/2))*sin(x^(1/2))+1/2*x^(1/2))-1/2*x-1/2*sin(x^(1/2))^2`**Maxima [A]**

time = 0.30, size = 23, normalized size = 0.64

$$\frac{1}{2} \sqrt{x} \sin(2\sqrt{x}) + \frac{1}{2} x + \frac{1}{4} \cos(2\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(x^(1/2))^2,x, algorithm="maxima")`

[Out]  $1/2*\sqrt{x}*\sin(2*\sqrt{x}) + 1/2*x + 1/4*\cos(2*\sqrt{x})$

**Fricas** [A]

time = 0.37, size = 24, normalized size = 0.67

$$\sqrt{x} \cos(\sqrt{x}) \sin(\sqrt{x}) + \frac{1}{2} \cos(\sqrt{x})^2 + \frac{1}{2} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x^(1/2))^2,x, algorithm="fricas")`

[Out]  $\sqrt{x}*\cos(\sqrt{x})*\sin(\sqrt{x}) + 1/2*\cos(\sqrt{x})^2 + 1/2*x$

**Sympy** [A]

time = 0.08, size = 51, normalized size = 1.42

$$\sqrt{x} \sin(\sqrt{x}) \cos(\sqrt{x}) + \frac{x \sin^2(\sqrt{x})}{2} + \frac{x \cos^2(\sqrt{x})}{2} + \frac{\cos^2(\sqrt{x})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x**(1/2))**2,x)`

[Out]  $\sqrt{x}*\sin(\sqrt{x})*\cos(\sqrt{x}) + x*\sin(\sqrt{x})^2/2 + x*\cos(\sqrt{x})^2/2 + \cos(\sqrt{x})^2/2$

**Giac** [A]

time = 0.42, size = 23, normalized size = 0.64

$$\frac{1}{2} \sqrt{x} \sin(2 \sqrt{x}) + \frac{1}{2} x + \frac{1}{4} \cos(2 \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x^(1/2))^2,x, algorithm="giac")`

[Out]  $1/2*\sqrt{x}*\sin(2*\sqrt{x}) + 1/2*x + 1/4*\cos(2*\sqrt{x})$

**Mupad** [B]

time = 0.35, size = 23, normalized size = 0.64

$$\frac{x}{2} - \frac{\sin(\sqrt{x})^2}{2} + \frac{\sqrt{x} \sin(2 \sqrt{x})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x^(1/2))^2,x)`

[Out]  $x/2 - \sin(x^(1/2))^2/2 + (x^(1/2)*\sin(2*x^(1/2)))/2$



### 3.49 $\int x^{3/2} \cos(a + b\sqrt[3]{x}) dx$

**Optimal.** Leaf size=235

$$\frac{135135\sqrt{x} \cos(a + b\sqrt[3]{x})}{32b^6} - \frac{3861x^{7/6} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2} + \frac{405405\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b}\right)}{64b^{15/2}}$$

[Out]  $-3861/8*x^{(7/6)}*\cos(a+b*x^{(1/3)})/b^4+39/2*x^{(11/6)}*\cos(a+b*x^{(1/3)})/b^2-405405/64*x^{(1/6)}*\sin(a+b*x^{(1/3)})/b^7+27027/16*x^{(5/6)}*\sin(a+b*x^{(1/3)})/b^5-429/4*x^{(3/2)}*\sin(a+b*x^{(1/3)})/b^3+3*x^{(13/6)}*\sin(a+b*x^{(1/3)})/b+405405/128*\cos(a)*\text{FresnelS}(x^{(1/6)}*b^{(1/2)}*2^{(1/2)}/\text{Pi}^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(15/2)}+405405/128*\text{FresnelC}(x^{(1/6)}*b^{(1/2)}*2^{(1/2)}/\text{Pi}^{(1/2)})*\sin(a)*2^{(1/2)}*\text{Pi}^{(1/2)}/b^{(15/2)}+135135/32*\cos(a+b*x^{(1/3)})*x^{(1/2)}/b^6$

**Rubi [A]**

time = 0.27, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {3497, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{405405\sqrt{\frac{\pi}{2}} \sin(a) \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt[3]{x}\right)}{64b^{15/2}} + \frac{405405\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[3]{x}\right)}{64b^{15/2}} - \frac{405405\sqrt{x} \sin(a + b\sqrt[3]{x})}{64b^7} + \frac{135135\sqrt{x} \cos(a + b\sqrt[3]{x})}{32b^6} + \frac{27027x^{5/6} \sin(a + b\sqrt[3]{x})}{16b^5} - \frac{3861x^{7/6} \cos(a + b\sqrt[3]{x})}{8b^4} - \frac{429x^{3/2} \sin(a + b\sqrt[3]{x})}{4b^3} + \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2} + \frac{3x^{13/6} \sin(a + b\sqrt[3]{x})}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(3/2)}*\text{Cos}[a + b*x^{(1/3)}], x]$

[Out]  $(135135*\text{Sqrt}[x]*\text{Cos}[a + b*x^{(1/3)}])/(32*b^6) - (3861*x^{(7/6)}*\text{Cos}[a + b*x^{(1/3)}])/(8*b^4) + (39*x^{(11/6)}*\text{Cos}[a + b*x^{(1/3)}])/(2*b^2) + (405405*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[a]*\text{FresnelS}[\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*x^{(1/6)}])/(64*b^{(15/2)}) + (405405*\text{Sqrt}[\text{Pi}/2]*\text{FresnelC}[\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*x^{(1/6)}]*\text{Sin}[a])/(64*b^{(15/2)}) - (405405*x^{(1/6)}*\text{Sin}[a + b*x^{(1/3)}])/(64*b^7) + (27027*x^{(5/6)}*\text{Sin}[a + b*x^{(1/3)}])/(16*b^5) - (429*x^{(3/2)}*\text{Sin}[a + b*x^{(1/3)}])/(4*b^3) + (3*x^{(13/6)}*\text{Sin}[a + b*x^{(1/3)}])/b$

Rule 3377

$\text{Int}[(c + d*x)^m*\sin[e + f*x], x\_Symbol] \rightarrow \text{Simp}[-(c + d*x)^m*(\text{Cos}[e + f*x]/f), x] + \text{Dist}[d*(m/f), \text{Int}[(c + d*x)^{m-1}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{GtQ}[m, 0]$

Rule 3385

$\text{Int}[\sin[\text{Pi}/2 + (e + f*x)/\text{Sqrt}[c + d*x]], x\_Symbol] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[f*(x^2/d)], x], x, \text{Sqrt}[c + d*x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

#### Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

#### Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

#### Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

#### Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol
] := Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

#### Rubi steps

$$\begin{aligned}
\int x^{3/2} \cos(a + b\sqrt[3]{x}) dx &= 3 \text{Subst} \left( \int x^{13/2} \cos(a + bx) dx, x, \sqrt[3]{x} \right) \\
&= \frac{3x^{13/6} \sin(a + b\sqrt[3]{x})}{b} - \frac{39 \text{Subst}(\int x^{11/2} \sin(a + bx) dx, x, \sqrt[3]{x})}{2b} \\
&= \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2} + \frac{3x^{13/6} \sin(a + b\sqrt[3]{x})}{b} - \frac{429 \text{Subst}(\int x^{9/2} \cos(a + bx) dx, x, \sqrt[3]{x})}{4b^2} \\
&= \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2} - \frac{429x^{3/2} \sin(a + b\sqrt[3]{x})}{4b^3} + \frac{3x^{13/6} \sin(a + b\sqrt[3]{x})}{b} + \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2} \\
&= -\frac{3861x^{7/6} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2} - \frac{429x^{3/2} \sin(a + b\sqrt[3]{x})}{4b^3} \\
&= -\frac{3861x^{7/6} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2} + \frac{27027x^{5/6} \sin(a + b\sqrt[3]{x})}{16b^5} \\
&= \frac{135135\sqrt{x} \cos(a + b\sqrt[3]{x})}{32b^6} - \frac{3861x^{7/6} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2} \\
&= \frac{135135\sqrt{x} \cos(a + b\sqrt[3]{x})}{32b^6} - \frac{3861x^{7/6} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2} \\
&= \frac{135135\sqrt{x} \cos(a + b\sqrt[3]{x})}{32b^6} - \frac{3861x^{7/6} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2} \\
&= \frac{135135\sqrt{x} \cos(a + b\sqrt[3]{x})}{32b^6} - \frac{3861x^{7/6} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2} \\
&= \frac{135135\sqrt{x} \cos(a + b\sqrt[3]{x})}{32b^6} - \frac{3861x^{7/6} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{39x^{11/6} \cos(a + b\sqrt[3]{x})}{2b^2}
\end{aligned}$$

**Mathematica [A]**

time = 0.39, size = 165, normalized size = 0.70

$$\frac{405405\sqrt{2\pi} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[3]{x}\right) + 405405\sqrt{2\pi} \text{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[3]{x}\right) \sin(a) + 6\sqrt{b} \sqrt[3]{x} (26(3465b\sqrt[3]{x} - 396b^2x + 16b^2x^{5/3})) \cos(a + b\sqrt[3]{x}) + (-135135 + 36036b^2x^{2/3} - 2288b^4x^{4/3} + 64b^6x^2) \sin(a + b\sqrt[3]{x})}{128b^{15/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[x^(3/2)*Cos[a + b*x^(1/3)], x]`

```

[Out] (405405*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)] + 405405*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a] + 6*Sqrt[b]*x^(1/6)*(26*(3465*b*x^(1/3) - 396*b^3*x + 16*b^5*x^(5/3))*Cos[a + b*x^(1/3)] + (-135135 + 36036*b^2*x^(2/3) - 2288*b^4*x^(4/3) + 64*b^6*x^2)*Sin[a + b*x^(1/3)])/(128*b^(15/2))

```

Maple [A]

time = 0.08, size = 196, normalized size = 0.83

method	result
derivativedivides	$\frac{3x^{\frac{13}{6}} \sin\left(a+bx^{\frac{1}{3}}\right)}{b} - \left( \frac{39}{99} \left( \frac{x^{\frac{11}{6}} \cos\left(a+bx^{\frac{1}{3}}\right)}{2b} + \frac{11x^{\frac{3}{2}} \sin\left(a+bx^{\frac{1}{3}}\right)}{4b} \right) - \frac{35}{99} \left( \frac{x^{\frac{7}{6}} \cos\left(a+bx^{\frac{1}{3}}\right)}{2b} + \frac{7x^{\frac{5}{6}} \sin\left(a+bx^{\frac{1}{3}}\right)}{4b} \right) \right)$

	$39 \left( -\frac{x^{\frac{11}{6}} \cos\left(a+b x^{\frac{1}{3}}\right)}{2b} + \frac{11x^{\frac{3}{2}} \sin\left(a+b x^{\frac{1}{3}}\right)}{4b} - \left( -\frac{x^{\frac{7}{6}} \cos\left(a+b x^{\frac{1}{3}}\right)}{2b} + \frac{7x^{\frac{5}{6}} \sin\left(a+b x^{\frac{1}{3}}\right)}{4b} - \dots \right) \right)$
default	$\frac{3x^{\frac{13}{6}} \sin\left(a+b x^{\frac{1}{3}}\right)}{b} - \dots$
meijerg	$192\sqrt{2} \cos(a) \sqrt{\pi} \left( \frac{\sqrt{x} \sqrt{2} (b^2)^{\frac{15}{4}} \left( 3120x^{\frac{4}{3}} b^4 - 77220x^{\frac{2}{3}} b^2 + 675675 \right) \cos\left(b x^{\frac{1}{3}}\right)}{61440\sqrt{\pi} b^6} - \frac{x^{\frac{1}{6}} \sqrt{2} (b^2)^{\frac{15}{4}} \left( -960x^2 b^6 + 34 \dots \right)}{(b^2)^{\frac{15}{4}}} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*cos(a+b*x^(1/3)),x,method=_RETURNVERBOSE)`

[Out] `3*x^(13/6)*sin(a+b*x^(1/3))/b-39/b*(-1/2/b*x^(11/6)*cos(a+b*x^(1/3))+11/2/b*(1/2/b*x^(3/2)*sin(a+b*x^(1/3))-9/2/b*(-1/2/b*x^(7/6)*cos(a+b*x^(1/3))+7/2/b*(1/2/b*x^(5/6)*sin(a+b*x^(1/3))-5/2/b*(-1/2/b*x^(1/2)*cos(a+b*x^(1/3))+3/2/b*(1/2*x^(1/6)*sin(a+b*x^(1/3)))/b-1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))))))`

**Maxima** [C] Result contains complex when optimal does not.

time = 0.30, size = 136, normalized size = 0.58

$$\frac{3 \left( 135135 \sqrt{2} \sqrt{\pi} \left( (i+1) \cos(a) - (i-1) \sin(a) \right) \operatorname{erf}\left(\sqrt{i b} x^{\frac{1}{3}}\right) + (-i-1) \cos(a) + (i+1) \sin(a) \right) \operatorname{erf}\left(\sqrt{-i b} x^{\frac{1}{3}}\right) b^{\frac{15}{4}} + 208 \left( 16 b^7 x^{\frac{4}{3}} - 396 b^5 x^{\frac{2}{3}} + 3465 b^3 \sqrt{x} \right) \cos\left(b x^{\frac{1}{3}} + a \right) + 8 \left( 64 b^8 x^{\frac{4}{3}} - 2288 b^6 x^{\frac{2}{3}} + 36036 b^4 x^{\frac{1}{3}} - 135135 b^2 x^{\frac{1}{3}} \right) \sin\left(b x^{\frac{1}{3}} + a \right)}{512 b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*cos(a+b\*x^(1/3)),x, algorithm="maxima")

[Out]  $\frac{3}{512} \cdot (135135 \sqrt{2}) \sqrt{\pi} \cdot ((I + 1) \cos(a) - (I - 1) \sin(a)) \operatorname{erf}(\sqrt{I \cdot b} \cdot x^{1/6}) + (- (I - 1) \cos(a) + (I + 1) \sin(a)) \operatorname{erf}(\sqrt{-I \cdot b} \cdot x^{1/6}) \cdot b^{3/2} + 208 \cdot (16 \cdot b^7 \cdot x^{11/6} - 396 \cdot b^5 \cdot x^{7/6} + 3465 \cdot b^3 \sqrt{x}) \cdot \cos(b \cdot x^{1/3} + a) + 8 \cdot (64 \cdot b^8 \cdot x^{13/6} - 2288 \cdot b^6 \cdot x^{3/2} + 36036 \cdot b^4 \cdot x^{5/6} - 135135 \cdot b^2 \cdot x^{1/6}) \cdot \sin(b \cdot x^{1/3} + a) / b^9$

**Fricas** [A]

time = 0.36, size = 145, normalized size = 0.62

$$\frac{3 \left( 135135 \sqrt{2} \pi \sqrt{\frac{b}{\pi}} \cos(a) S \left( \sqrt{2} x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}} \right) + 135135 \sqrt{2} \pi \sqrt{\frac{b}{\pi}} C \left( \sqrt{2} x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}} \right) \sin(a) + 52 \left( 16 b^6 x^{\frac{11}{6}} - 396 b^4 x^{\frac{7}{6}} + 3465 b^2 \sqrt{x} \right) \cos(bx^{\frac{1}{3}} + a) - 2 \left( 2288 b^5 x^{\frac{3}{2}} - 36036 b^3 x^{\frac{5}{6}} - (64 b^7 x^2 - 135135 b) x^{\frac{1}{6}} \right) \sin(bx^{\frac{1}{3}} + a) \right)}{128 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*cos(a+b\*x^(1/3)),x, algorithm="fricas")

[Out]  $\frac{3}{128} \cdot (135135 \sqrt{2}) \cdot \pi \sqrt{b/\pi} \cdot \cos(a) \cdot \operatorname{fresnel\_sin}(\sqrt{2} \cdot x^{1/6} \sqrt{b/\pi}) + 135135 \sqrt{2} \cdot \pi \sqrt{b/\pi} \cdot \operatorname{fresnel\_cos}(\sqrt{2} \cdot x^{1/6} \sqrt{b/\pi}) \cdot \sin(a) + 52 \cdot (16 \cdot b^6 \cdot x^{11/6} - 396 \cdot b^4 \cdot x^{7/6} + 3465 \cdot b^2 \sqrt{x}) \cdot \cos(b \cdot x^{1/3} + a) - 2 \cdot (2288 \cdot b^5 \cdot x^{3/2} - 36036 \cdot b^3 \cdot x^{5/6} - (64 \cdot b^7 \cdot x^2 - 135135 \cdot b) \cdot x^{1/6}) \cdot \sin(b \cdot x^{1/3} + a) / b^8$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \cos(a + b\sqrt[3]{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)\*cos(a+b\*x\*\*(1/3)),x)

[Out] Integral(x\*\*(3/2)\*cos(a + b\*x\*\*(1/3)), x)

**Giac** [C] Result contains complex when optimal does not.

time = 0.44, size = 241, normalized size = 1.03

$$\frac{3 \left( 64 b^7 x^2 - 416 b^5 x^2 - 2288 b^3 x^2 + 10296 b^2 x^2 + 36036 b x^2 - 90090 \sqrt{x} - 135135 x^2 \right) e^{(i a + i x)} - 3 \left( -64 b^7 x^2 - 416 b^5 x^2 + 2288 b^3 x^2 + 10296 b^2 x^2 - 36036 b x^2 + 90090 \sqrt{x} + 135135 x^2 \right) e^{(-i a - i x)}}{128 b^7} + \frac{405405 \sqrt{2} \sqrt{\pi} \operatorname{erf} \left( -\frac{1}{2} \sqrt{2} x^{\frac{1}{6}} \left( -\frac{b}{\pi} + 1 \right) \sqrt{b} \right) e^{i a}}{256 b^7 \left( -\frac{b}{\pi} + 1 \right) \sqrt{b}} - \frac{405405 \sqrt{2} \sqrt{\pi} \operatorname{erf} \left( -\frac{1}{2} \sqrt{2} x^{\frac{1}{6}} \left( \frac{b}{\pi} + 1 \right) \sqrt{b} \right) e^{-i a}}{256 b^7 \left( \frac{b}{\pi} + 1 \right) \sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*cos(a+b\*x^(1/3)),x, algorithm="giac")

[Out]  $-3/128 \cdot (64 \cdot I \cdot b^6 \cdot x^{13/6} - 416 \cdot b^5 \cdot x^{11/6} - 2288 \cdot I \cdot b^4 \cdot x^{3/2} + 10296 \cdot b^3 \cdot x^{7/6} + 36036 \cdot I \cdot b^2 \cdot x^{5/6} - 90090 \cdot b \sqrt{x} - 135135 \cdot I \cdot x^{1/6}) \cdot e^{(I \cdot b \cdot x^{1/3} + I \cdot a) / b^7} - 3/128 \cdot (-64 \cdot I \cdot b^6 \cdot x^{13/6} - 416 \cdot b^5 \cdot x^{11/6} + 2288 \cdot I \cdot b^4 \cdot x^{3/2} + 10296 \cdot b^3 \cdot x^{7/6} - 36036 \cdot I \cdot b^2 \cdot x^{5/6} - 90090 \cdot b \sqrt{x} + 135135 \cdot I \cdot x^{1/6}) \cdot e^{(-I \cdot b \cdot x^{1/3} - I \cdot a) / b^7} + 405405 / 256 \cdot I \cdot \sqrt{2} \cdot \sqrt{\pi} \cdot \operatorname{erf} \left( -\frac{1}{2} \sqrt{2} x^{\frac{1}{6}} \left( -\frac{b}{\pi} + 1 \right) \sqrt{b} \right) e^{i a} + 405405 / 256 \cdot I \cdot \sqrt{2} \cdot \sqrt{\pi} \cdot \operatorname{erf} \left( -\frac{1}{2} \sqrt{2} x^{\frac{1}{6}} \left( \frac{b}{\pi} + 1 \right) \sqrt{b} \right) e^{-i a}$

```
pi)*erf(-1/2*sqrt(2)*x^(1/6)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b^7*(
-I*b/abs(b) + 1)*sqrt(abs(b))) - 405405/256*I*sqrt(2)*sqrt(pi)*erf(-1/2*sq
r(2)*x^(1/6)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b^7*(I*b/abs(b) + 1)*
sqrt(abs(b)))
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2} \cos(a + b x^{1/3}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*cos(a + b\*x^(1/3)),x)

[Out] int(x^(3/2)\*cos(a + b\*x^(1/3)), x)

### 3.50 $\int \sqrt{x} \cos(a + b\sqrt[3]{x}) dx$

**Optimal.** Leaf size=169

$$\frac{315\sqrt[6]{x} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{21x^{5/6} \cos(a + b\sqrt[3]{x})}{2b^2} + \frac{315\sqrt{\frac{\pi}{2}} \cos(a) \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right)}{8b^{9/2}} - \frac{315\sqrt{\frac{\pi}{2}} S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right)}{8b^{9/2}}$$

[Out]  $-315/8*x^{(1/6)}*\cos(a+b*x^{(1/3)})/b^4+21/2*x^{(5/6)}*\cos(a+b*x^{(1/3)})/b^2+3*x^{(7/6)}*\sin(a+b*x^{(1/3)})/b+315/16*\cos(a)*\operatorname{FresnelC}(x^{(1/6)}*b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)})*2^{(1/2)}*\pi^{(1/2)}/b^{(9/2)}-315/16*\operatorname{FresnelS}(x^{(1/6)}*b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)})*\sin(a)*2^{(1/2)}*\pi^{(1/2)}/b^{(9/2)}-105/4*\sin(a+b*x^{(1/3)})*x^{(1/2)}/b^3$

**Rubi [A]**

time = 0.15, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {3497, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{315\sqrt{\frac{\pi}{2}} \cos(a) \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt[6]{x}\right)}{8b^{9/2}} - \frac{315\sqrt{\frac{\pi}{2}} \sin(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right)}{8b^{9/2}} - \frac{315\sqrt[6]{x} \cos(a + b\sqrt[3]{x})}{8b^4} - \frac{105\sqrt{x} \sin(a + b\sqrt[3]{x})}{4b^3} + \frac{21x^{5/6} \cos(a + b\sqrt[3]{x})}{2b^2} + \frac{3x^{7/6} \sin(a + b\sqrt[3]{x})}{b}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[x]*Cos[a + b*x^(1/3)], x]`

[Out]  $(-315*x^{(1/6)}*\cos[a + b*x^{(1/3)}])/(8*b^4) + (21*x^{(5/6)}*\cos[a + b*x^{(1/3)}])/(2*b^2) + (315*\sqrt{\pi/2}*\cos[a]*\operatorname{FresnelC}[\sqrt{b}*\sqrt{2/\pi}*x^{(1/6)}])/(8*b^{(9/2)}) - (315*\sqrt{\pi/2}*\operatorname{FresnelS}[\sqrt{b}*\sqrt{2/\pi}*x^{(1/6)}]*\sin[a])/(8*b^{(9/2)}) - (105*\sqrt{x}*\sin[a + b*x^{(1/3)}])/(4*b^3) + (3*x^{(7/6)}*\sin[a + b*x^{(1/3)}])/b$

Rule 3377

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 3385

`Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

Rule 3386

`Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}`



, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3387

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

#### Rule 3432

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

#### Rule 3433

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

#### Rule 3497

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)<sup>n</sup>])\*(b\_.))<sup>p</sup>\*(x\_)<sup>m</sup>, x\_Symbol] := Module[{k = Denominator[n]}, Dist[k, Subst[Int[x<sup>(k\*(m + 1) - 1)</sup>\*(a + b\*Cos[c + d\*x<sup>(k\*n)</sup>]]<sup>p</sup>, x], x, x<sup>(1/k)</sup>], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p] && FractionQ[n]

#### Rubi steps

$$\begin{aligned}
\int \sqrt{x} \cos(a + b\sqrt[3]{x}) dx &= 3\text{Subst}\left(\int x^{7/2} \cos(a + bx) dx, x, \sqrt[3]{x}\right) \\
&= \frac{3x^{7/6} \sin(a + b\sqrt[3]{x})}{b} - \frac{21\text{Subst}(\int x^{5/2} \sin(a + bx) dx, x, \sqrt[3]{x})}{2b} \\
&= \frac{21x^{5/6} \cos(a + b\sqrt[3]{x})}{2b^2} + \frac{3x^{7/6} \sin(a + b\sqrt[3]{x})}{b} - \frac{105\text{Subst}(\int x^{3/2} \cos(a + bx) dx, x, \sqrt[3]{x})}{4b^2} \\
&= \frac{21x^{5/6} \cos(a + b\sqrt[3]{x})}{2b^2} - \frac{105\sqrt{x} \sin(a + b\sqrt[3]{x})}{4b^3} + \frac{3x^{7/6} \sin(a + b\sqrt[3]{x})}{b} + \frac{315\sqrt{x} \cos(a + b\sqrt[3]{x})}{8b^4} \\
&= -\frac{315\sqrt[6]{x} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{21x^{5/6} \cos(a + b\sqrt[3]{x})}{2b^2} - \frac{105\sqrt{x} \sin(a + b\sqrt[3]{x})}{4b^3} + \frac{315\sqrt{x} \cos(a + b\sqrt[3]{x})}{8b^4} \\
&= -\frac{315\sqrt[6]{x} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{21x^{5/6} \cos(a + b\sqrt[3]{x})}{2b^2} - \frac{105\sqrt{x} \sin(a + b\sqrt[3]{x})}{4b^3} + \frac{315\sqrt{x} \cos(a + b\sqrt[3]{x})}{8b^4} \\
&= -\frac{315\sqrt[6]{x} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{21x^{5/6} \cos(a + b\sqrt[3]{x})}{2b^2} - \frac{105\sqrt{x} \sin(a + b\sqrt[3]{x})}{4b^3} + \frac{315\sqrt{x} \cos(a + b\sqrt[3]{x})}{8b^4} \\
&= -\frac{315\sqrt[6]{x} \cos(a + b\sqrt[3]{x})}{8b^4} + \frac{21x^{5/6} \cos(a + b\sqrt[3]{x})}{2b^2} + \frac{315\sqrt{\frac{\pi}{2}} \cos(a) C\left(\sqrt{b} \sqrt[3]{x}\right)}{8b^{9/2}}
\end{aligned}$$

**Mathematica [A]**

time = 0.24, size = 141, normalized size = 0.83

$$\frac{315\sqrt{2\pi} \cos(a) \text{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[3]{x}\right) - 315\sqrt{2\pi} S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[3]{x}\right) \sin(a) + 6\sqrt{b} \sqrt[3]{x} (7(-15 + 4b^2x^{2/3}) \cos(a + b\sqrt[3]{x}) + 2b(-35 + 4b^2x^{2/3}) \sqrt[3]{x} \sin(a + b\sqrt[3]{x}))}{16b^{9/2}}$$

Antiderivative was successfully verified.

**[In]** Integrate[Sqrt[x]\*Cos[a + b\*x^(1/3)], x]

**[Out]** (315\*Sqrt[2\*Pi]\*Cos[a]\*FresnelC[Sqrt[b]\*Sqrt[2/Pi]\*x^(1/6)] - 315\*Sqrt[2\*Pi]\*FresnelS[Sqrt[b]\*Sqrt[2/Pi]\*x^(1/6)]\*Sin[a] + 6\*Sqrt[b]\*x^(1/6)\*(7\*(-15 + 4\*b^2\*x^(2/3))\*Cos[a + b\*x^(1/3)] + 2\*b\*(-35 + 4\*b^2\*x^(2/3))\*x^(1/3)\*Sin[a + b\*x^(1/3)]))/(16\*b^(9/2))

**Maple [A]**

time = 0.06, size = 131, normalized size = 0.78

method	result
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derivativedivides	$\frac{3x^{\frac{7}{6}} \sin\left(a+bx^{\frac{1}{3}}\right)}{b} - \frac{21 \left( -\frac{x^{\frac{5}{6}} \cos\left(a+bx^{\frac{1}{3}}\right)}{2b} + \frac{5\sqrt{x} \sin\left(a+bx^{\frac{1}{3}}\right)}{4b} - \frac{15 \left( -\frac{x^{\frac{1}{6}} \cos\left(a+bx^{\frac{1}{3}}\right)}{2b} + \frac{\sqrt{2} \sqrt{\pi} \left( \cos(a) \operatorname{FresnelC}\left(x^{\frac{1}{6}}\right) b^{\frac{1}{2}} \right)}{b} \right)}{b} \right)}{b}$
default	$\frac{3x^{\frac{7}{6}} \sin\left(a+bx^{\frac{1}{3}}\right)}{b} - \frac{21 \left( -\frac{x^{\frac{5}{6}} \cos\left(a+bx^{\frac{1}{3}}\right)}{2b} + \frac{5\sqrt{x} \sin\left(a+bx^{\frac{1}{3}}\right)}{4b} - \frac{15 \left( -\frac{x^{\frac{1}{6}} \cos\left(a+bx^{\frac{1}{3}}\right)}{2b} + \frac{\sqrt{2} \sqrt{\pi} \left( \cos(a) \operatorname{FresnelC}\left(x^{\frac{1}{6}}\right) b^{\frac{1}{2}} \right)}{b} \right)}{b} \right)}{b}$
meijerg	$\frac{24\sqrt{2} \cos(a) \sqrt{\pi} \left( -\frac{x^{\frac{1}{6}} \sqrt{2} (b^2)^{\frac{9}{4}} (-252x^{\frac{2}{3}} b^2 + 945) \cos\left(bx^{\frac{1}{3}}\right)}{1152 \sqrt{\pi} b^4} - \frac{\sqrt{x} \sqrt{2} (b^2)^{\frac{9}{4}} (-36x^{\frac{2}{3}} b^2 + 315) \sin\left(bx^{\frac{1}{3}}\right)}{576 \sqrt{\pi} b^3} \right)}{(b^2)^{\frac{9}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*cos(a+b*x^(1/3)),x,method=_RETURNVERBOSE)`

[Out]  $3x^{7/6} \sin(a+bx^{1/3})/b - 21/b * (-1/2/b * x^{5/6} * \cos(a+bx^{1/3}) + 5/2/b * (1/2/b * x^{1/2} * \sin(a+bx^{1/3}) - 3/2/b * (-1/2/b * x^{1/6} * \cos(a+bx^{1/3}) + 1/4/b^{3/2} * 2^{1/2} * \pi^{1/2} * (\cos(a) * \operatorname{FresnelC}(x^{1/6} * b^{1/2} * 2^{1/2} / \pi^{1/2}) - \sin(a) * \operatorname{FresnelS}(x^{1/6} * b^{1/2} * 2^{1/2} / \pi^{1/2})))$

**Maxima [C]** Result contains complex when optimal does not.

time = 0.30, size = 112, normalized size = 0.66

$$\frac{3 \left( 105 \sqrt{2} \sqrt{\pi} \left( (-i-1) \cos(a) - (i+1) \sin(a) \right) \operatorname{erf}\left(\sqrt{i b} x^{\frac{1}{3}}\right) + ((i+1) \cos(a) + (i-1) \sin(a)) \operatorname{erf}\left(\sqrt{-i b} x^{\frac{1}{3}}\right) \right) b^{\frac{3}{2}} + 56 \left( 4b^4 x^{\frac{5}{6}} - 15b^2 x^{\frac{1}{6}} \right) \cos\left(bx^{\frac{1}{3}} + a\right) + 16 \left( 4b^5 x^{\frac{7}{6}} - 35b^3 \sqrt{x} \right) \sin\left(bx^{\frac{1}{3}} + a\right)}{64 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*cos(a+b*x^(1/3)),x, algorithm="maxima")`

[Out]  $3/64 * (105 * \sqrt{2} * \sqrt{\pi} * ((-I-1) * \cos(a) - (I+1) * \sin(a)) * \operatorname{erf}(\sqrt{I*b} * x^{1/6}) + ((I+1) * \cos(a) + (I-1) * \sin(a)) * \operatorname{erf}(\sqrt{-I*b} * x^{1/6})) * b^{3/2} + 56 * (4 * b^4 * x^{5/6} - 15 * b^2 * x^{1/6}) * \cos(b * x^{1/3} + a) + 16 * (4 * b^5 * x^{7/6} - 35 * b^3 * \sqrt{x}) * \sin(b * x^{1/3} + a) / b^6$

**Fricas [A]**

time = 0.38, size = 118, normalized size = 0.70

$$\frac{3 \left( 105 \sqrt{2} \pi \sqrt{\frac{b}{\pi}} \cos(a) C \left( \sqrt{2} x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}} \right) - 105 \sqrt{2} \pi \sqrt{\frac{b}{\pi}} S \left( \sqrt{2} x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}} \right) \sin(a) + 14 \left( 4 b^3 x^{\frac{5}{6}} - 15 b x^{\frac{1}{6}} \right) \cos \left( b x^{\frac{1}{3}} + a \right) + 4 \left( 4 b^4 x^{\frac{7}{6}} - 35 b^2 \sqrt{x} \right) \sin \left( b x^{\frac{1}{3}} + a \right) \right)}{16 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(x^(1/2)\*cos(a+b\*x^(1/3)),x, algorithm="fricas")

**[Out]** 3/16\*(105\*sqrt(2)\*pi\*sqrt(b/pi)\*cos(a)\*fresnel\_cos(sqrt(2)\*x^(1/6)\*sqrt(b/pi)) - 105\*sqrt(2)\*pi\*sqrt(b/pi)\*fresnel\_sin(sqrt(2)\*x^(1/6)\*sqrt(b/pi))\*sin(a) + 14\*(4\*b^3\*x^(5/6) - 15\*b\*x^(1/6))\*cos(b\*x^(1/3) + a) + 4\*(4\*b^4\*x^(7/6) - 35\*b^2\*sqrt(x))\*sin(b\*x^(1/3) + a)/b^5

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \cos(a + b\sqrt[3]{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(x\*\*(1/2)\*cos(a+b\*x\*\*(1/3)),x)**[Out]** Integral(sqrt(x)\*cos(a + b\*x\*\*(1/3)), x)**Giac [C]** Result contains complex when optimal does not.

time = 0.44, size = 193, normalized size = 1.14

$$\frac{3 \left( 8i b^3 x^{\frac{5}{6}} - 28 b^2 x^{\frac{1}{6}} - 70i b \sqrt{x} + 105 x^{\frac{1}{6}} \right) e^{(i a x^{\frac{1}{3}} + i a)} - 3 \left( -8i b^3 x^{\frac{5}{6}} - 28 b^2 x^{\frac{1}{6}} + 70i b \sqrt{x} + 105 x^{\frac{1}{6}} \right) e^{(-i a x^{\frac{1}{3}} - i a)}}{16 b^4} - \frac{315 \sqrt{2} \sqrt{\pi} \operatorname{erf} \left( -\frac{1}{2} \sqrt{2} x^{\frac{1}{6}} \left( -\frac{10}{b} + 1 \right) \sqrt{|b|} \right) e^{i a}}{32 b^4 \left( -\frac{10}{b} + 1 \right) \sqrt{|b|}} - \frac{315 \sqrt{2} \sqrt{\pi} \operatorname{erf} \left( -\frac{1}{2} \sqrt{2} x^{\frac{1}{6}} \left( \frac{10}{b} + 1 \right) \sqrt{|b|} \right) e^{-i a}}{32 b^4 \left( \frac{10}{b} + 1 \right) \sqrt{|b|}}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(x^(1/2)\*cos(a+b\*x^(1/3)),x, algorithm="giac")

**[Out]** -3/16\*(8\*I\*b^3\*x^(7/6) - 28\*b^2\*x^(5/6) - 70\*I\*b\*sqrt(x) + 105\*x^(1/6))\*e^(I\*b\*x^(1/3) + I\*a)/b^4 - 3/16\*(-8\*I\*b^3\*x^(7/6) - 28\*b^2\*x^(5/6) + 70\*I\*b\*sqrt(x) + 105\*x^(1/6))\*e^(-I\*b\*x^(1/3) - I\*a)/b^4 - 315/32\*sqrt(2)\*sqrt(pi)\*erf(-1/2\*sqrt(2)\*x^(1/6)\*(-I\*b/abs(b) + 1)\*sqrt(abs(b)))\*e^(I\*a)/(b^4\*(-I\*b/abs(b) + 1)\*sqrt(abs(b))) - 315/32\*sqrt(2)\*sqrt(pi)\*erf(-1/2\*sqrt(2)\*x^(1/6)\*(I\*b/abs(b) + 1)\*sqrt(abs(b)))\*e^(-I\*a)/(b^4\*(I\*b/abs(b) + 1)\*sqrt(abs(b)))

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x} \cos(a + b x^{1/3}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(x^(1/2)\*cos(a + b\*x^(1/3)),x)**[Out]** int(x^(1/2)\*cos(a + b\*x^(1/3)), x)

$$3.51 \quad \int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} dx$$

**Optimal.** Leaf size=99

$$\frac{3\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right)}{b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right) \sin(a)}{b^{3/2}} + \frac{3\sqrt[6]{x} \sin(a+b\sqrt[3]{x})}{b}$$

[Out]  $3*x^{(1/6)}*\sin(a+b*x^{(1/3)})/b-3/2*\cos(a)*\operatorname{FresnelS}(x^{(1/6)}*b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)})*2^{(1/2)}*\pi^{(1/2)}/b^{(3/2)}-3/2*\operatorname{FresnelC}(x^{(1/6)}*b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)})*\sin(a)*2^{(1/2)}*\pi^{(1/2)}/b^{(3/2)}$

**Rubi [A]**

time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {3497, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{3\sqrt{\frac{\pi}{2}} \sin(a) \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt[6]{x}\right)}{b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right)}{b^{3/2}} + \frac{3\sqrt[6]{x} \sin(a+b\sqrt[3]{x})}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[a + b*x^{(1/3)}]/\operatorname{Sqrt}[x], x]$

[Out]  $(-3*\operatorname{Sqrt}[\pi/2]*\operatorname{Cos}[a]*\operatorname{FresnelS}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\pi]*x^{(1/6)}])/b^{(3/2)} - (3*\operatorname{Sqrt}[\pi/2]*\operatorname{FresnelC}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\pi]*x^{(1/6)}]*\operatorname{Sin}[a])/b^{(3/2)} + (3*x^{(1/6)}*\operatorname{Sin}[a + b*x^{(1/3)}])/b$

**Rule 3377**

$\operatorname{Int}[(c + d*x)^m * \sin(e + f*x), x\_Symbol] \rightarrow \operatorname{Simp}[-(c + d*x)^m * (\operatorname{Cos}[e + f*x]/f), x] + \operatorname{Dist}[d*(m/f), \operatorname{Int}[(c + d*x)^{m-1} * \operatorname{Cos}[e + f*x], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \operatorname{GtQ}[m, 0]$

**Rule 3385**

$\operatorname{Int}[\sin[\pi/2 + (e + f*x)]/\operatorname{Sqrt}[(c + d*x)], x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[\operatorname{Cos}[f*(x^2/d)], x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \operatorname{ComplexFreeQ}[f] \ \&\& \operatorname{EqQ}[d*e - c*f, 0]$

**Rule 3386**

$\operatorname{Int}[\sin[(e + f*x)]/\operatorname{Sqrt}[(c + d*x)], x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[\operatorname{Sin}[f*(x^2/d)], x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{c, d, e, f\}$

, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

### Rule 3387

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

### Rule 3432

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

### Rule 3433

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

### Rule 3497

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)<sup>n</sup>])\*(b\_.)<sup>p</sup>\*(x\_)<sup>m</sup>, x\_Symbol] := Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*Cos[c + d\*x^(k\*n)])<sup>p</sup>, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p] && FractionQ[n]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cos(a + b\sqrt[3]{x})}{\sqrt{x}} dx &= 3\text{Subst}\left(\int \sqrt{x} \cos(a + bx) dx, x, \sqrt[3]{x}\right) \\
 &= \frac{3\sqrt[6]{x} \sin(a + b\sqrt[3]{x})}{b} - \frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{x}} dx, x, \sqrt[3]{x}\right)}{2b} \\
 &= \frac{3\sqrt[6]{x} \sin(a + b\sqrt[3]{x})}{b} - \frac{(3 \cos(a))\text{Subst}\left(\int \frac{\sin(bx)}{\sqrt{x}} dx, x, \sqrt[3]{x}\right)}{2b} - \frac{(3 \sin(a))\text{Subst}\left(\int \frac{\cos(bx)}{\sqrt{x}} dx, x, \sqrt[3]{x}\right)}{2b} \\
 &= \frac{3\sqrt[6]{x} \sin(a + b\sqrt[3]{x})}{b} - \frac{(3 \cos(a))\text{Subst}\left(\int \sin(bx^2) dx, x, \sqrt[6]{x}\right)}{b} - \frac{(3 \sin(a))\text{Subst}\left(\int \cos(bx^2) dx, x, \sqrt[6]{x}\right)}{b} \\
 &= -\frac{3\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right)}{b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}} C\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right) \sin(a)}{b^{3/2}} + \frac{3\sqrt[6]{x} \sin(a)}{b}
 \end{aligned}$$

**Mathematica [A]**

time = 0.11, size = 94, normalized size = 0.95

$$\frac{3 \left( \sqrt{2\pi} \cos(a) S \left( \sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[3]{x} \right) + \sqrt{2\pi} \operatorname{FresnelC} \left( \sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[3]{x} \right) \sin(a) - 2\sqrt{b} \sqrt[3]{x} \sin(a + b\sqrt[3]{x}) \right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

**[In]** Integrate[Cos[a + b\*x^(1/3)]/Sqrt[x],x]

**[Out]**  $(-3*(\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Cos}[a]*\operatorname{FresnelS}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\operatorname{Pi}]*x^{1/6}] + \operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{FresnelC}[\operatorname{Sqrt}[b]*\operatorname{Sqrt}[2/\operatorname{Pi}]*x^{1/6}]*\operatorname{Sin}[a] - 2*\operatorname{Sqrt}[b]*x^{1/6}*\operatorname{Sin}[a + b*x^{1/3}])/(2*b^{3/2}))$

**Maple [A]**

time = 0.05, size = 64, normalized size = 0.65

method	result
derivativedivides	$\frac{3x^{1/6} \sin(a + bx^{1/3})}{b} - \frac{3\sqrt{2} \sqrt{\pi} \left( \cos(a) S \left( \frac{x^{1/6} \sqrt{b} \sqrt{2}}{\sqrt{\pi}} \right) + \sin(a) \operatorname{FresnelC} \left( \frac{x^{1/6} \sqrt{b} \sqrt{2}}{\sqrt{\pi}} \right) \right)}{2b^{3/2}}$
default	$\frac{3x^{1/6} \sin(a + bx^{1/3})}{b} - \frac{3\sqrt{2} \sqrt{\pi} \left( \cos(a) S \left( \frac{x^{1/6} \sqrt{b} \sqrt{2}}{\sqrt{\pi}} \right) + \sin(a) \operatorname{FresnelC} \left( \frac{x^{1/6} \sqrt{b} \sqrt{2}}{\sqrt{\pi}} \right) \right)}{2b^{3/2}}$
meijerg	$\frac{3 \cos(a) \sqrt{\pi} \sqrt{2} \left( \frac{x^{1/6} \sqrt{2} (b^2)^{3/4} \sin(bx^{1/3})}{2\sqrt{\pi} b} - \frac{(b^2)^{3/4} S \left( \frac{x^{1/6} \sqrt{b} \sqrt{2}}{\sqrt{\pi}} \right)}{2b^{3/2}} \right)}{(b^2)^{3/4}} - \frac{3 \sin(a) \sqrt{\pi} \sqrt{2} \left( -\frac{x^{1/6} \sqrt{2} \sqrt{\pi}}{2\sqrt{\pi} b} \right)}{(b^2)^{3/4}}$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(cos(a+b\*x^(1/3))/x^(1/2),x,method=\_RETURNVERBOSE)

**[Out]**  $3*x^{1/6}*\sin(a+b*x^{1/3})/b-3/2/b^{3/2}*2^{1/2}*Pi^{1/2}*(\cos(a)*\operatorname{FresnelS}(x^{1/6}*b^{1/2}*2^{1/2}/Pi^{1/2})+\sin(a)*\operatorname{FresnelC}(x^{1/6}*b^{1/2}*2^{1/2}/Pi^{1/2}))$

**Maxima [C]** Result contains complex when optimal does not.

time = 0.29, size = 73, normalized size = 0.74

$$\frac{3 \left( \sqrt{2} \sqrt{\pi} \left( (-i+1) \cos(a) + (i-1) \sin(a) \right) \operatorname{erf} \left( \sqrt{ib} x^{1/6} \right) + ((i-1) \cos(a) - (i+1) \sin(a)) \operatorname{erf} \left( \sqrt{-ib} x^{1/6} \right) \right) b^{3/2} + 8b^2 x^{1/6} \sin(bx^{1/3} + a)}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cos(a+b\*x^(1/3))/x^(1/2),x, algorithm="maxima")

[Out]  $3/8*(\sqrt{2}*\sqrt{\pi})*((-I + 1)*\cos(a) + (I - 1)*\sin(a))*\operatorname{erf}(\sqrt{I*b}*x^{1/6}) + ((I - 1)*\cos(a) - (I + 1)*\sin(a))*\operatorname{erf}(\sqrt{-I*b}*x^{1/6}))*b^{3/2} + 8*b^2*x^{1/6}*\sin(b*x^{1/3} + a))/b^3$

**Fricas** [A]

time = 0.38, size = 78, normalized size = 0.79

$$\frac{3 \left( \sqrt{2} \pi \sqrt{\frac{b}{\pi}} \cos(a) S \left( \sqrt{2} x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}} \right) + \sqrt{2} \pi \sqrt{\frac{b}{\pi}} C \left( \sqrt{2} x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}} \right) \sin(a) - 2 b x^{\frac{1}{6}} \sin \left( b x^{\frac{1}{3}} + a \right) \right)}{2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))/x^(1/2),x, algorithm="fricas")`

[Out]  $-3/2*(\sqrt{2}*\pi*\sqrt{b/\pi}*\cos(a)*\operatorname{fresnel\_sin}(\sqrt{2}*x^{1/6}*\sqrt{b/\pi}) + \sqrt{2}*\pi*\sqrt{b/\pi}*\operatorname{fresnel\_cos}(\sqrt{2}*x^{1/6}*\sqrt{b/\pi}))*\sin(a) - 2*b*x^{1/6}*\sin(b*x^{1/3} + a))/b^2$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + b\sqrt[3]{x})}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x**(1/3))/x**(1/2),x)`

[Out] `Integral(cos(a + b*x**(1/3))/sqrt(x), x)`

**Giac** [C] Result contains complex when optimal does not.

time = 0.49, size = 143, normalized size = 1.44

$$-\frac{3i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x^{\frac{1}{6}}\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{ia}}{4b\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}} + \frac{3i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x^{\frac{1}{6}}\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{-ia}}{4b\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}} - \frac{3ix^{\frac{1}{6}}e^{ibx^{\frac{1}{3}}+ia}}{2b} + \frac{3ix^{\frac{1}{6}}e^{-ibx^{\frac{1}{3}}-ia}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))/x^(1/2),x, algorithm="giac")`

[Out]  $-3/4*I*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}(-1/2*\sqrt{2}*x^{1/6}*(-I*b/\operatorname{abs}(b) + 1)*\sqrt{\operatorname{abs}(b)})*e^{I*a}/(b*(-I*b/\operatorname{abs}(b) + 1)*\sqrt{\operatorname{abs}(b)}) + 3/4*I*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}(-1/2*\sqrt{2}*x^{1/6}*(I*b/\operatorname{abs}(b) + 1)*\sqrt{\operatorname{abs}(b)})*e^{-I*a}/(b*(I*b/\operatorname{abs}(b) + 1)*\sqrt{\operatorname{abs}(b)}) - 3/2*I*x^{1/6}*e^{I*b*x^{1/3} + I*a}/b + 3/2*I*x^{1/6}*e^{-I*b*x^{1/3} - I*a}/b$

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + b x^{1/3})}{\sqrt{x}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^(1/3))/x^(1/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))/x^(1/2), x)
```

$$3.52 \quad \int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{3/2}} dx$$

**Optimal.** Leaf size=110

$$-\frac{2 \cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} - 4b^{3/2}\sqrt{2\pi} \cos(a) \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[3]{x}\right) + 4b^{3/2}\sqrt{2\pi} S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[3]{x}\right) \sin(a) + \frac{4b \sin(a)}{\sqrt{x}}$$

[Out]  $4*b*\sin(a+b*x^{(1/3)})/x^{(1/6)} - 4*b^{(3/2)}*\cos(a)*\operatorname{FresnelC}(x^{(1/6)}*b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)})*2^{(1/2)}*\pi^{(1/2)} + 4*b^{(3/2)}*\operatorname{FresnelS}(x^{(1/6)}*b^{(1/2)}*2^{(1/2)}/\pi^{(1/2)})*\sin(a)*2^{(1/2)}*\pi^{(1/2)} - 2*\cos(a+b*x^{(1/3)})/x^{(1/2)}$

**Rubi [A]**

time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {3497, 3378, 3387, 3386, 3432, 3385, 3433}

$$-4\sqrt{2\pi} b^{3/2} \cos(a) \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt[3]{x}\right) + 4\sqrt{2\pi} b^{3/2} \sin(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[3]{x}\right) + \frac{4b \sin(a+b\sqrt[3]{x})}{\sqrt[3]{x}} - \frac{2 \cos(a+b\sqrt[3]{x})}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^(1/3)]/x^(3/2), x]

[Out]  $(-2*\cos[a + b*x^{(1/3)}])/Sqrt[x] - 4*b^{(3/2)}*Sqrt[2*\pi]*\cos[a]*\operatorname{FresnelC}[Sqrt[b]*Sqrt[2/\pi]*x^{(1/6)}] + 4*b^{(3/2)}*Sqrt[2*\pi]*\operatorname{FresnelS}[Sqrt[b]*Sqrt[2/\pi]*x^{(1/6)}]*\sin[a] + (4*b*\sin[a + b*x^{(1/3)}])/x^{(1/6)}$

**Rule 3378**

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*(Sin[e + f\*x]/(d\*(m + 1))), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

**Rule 3385**

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

**Rule 3386**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)n])*(b_.)p*(x_)m, x_Symbol] := Module[{k = Denominator[n]}, Dist[k, Subst[Int[x(k*(m + 1) - 1)*(a +
b*Cos[c + d*x(k*n)]]p, x], x, x(1/k)], x] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a + b\sqrt[3]{x})}{x^{3/2}} dx &= 3 \text{Subst} \left( \int \frac{\cos(a + bx)}{x^{5/2}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{2 \cos(a + b\sqrt[3]{x})}{\sqrt{x}} - (2b) \text{Subst} \left( \int \frac{\sin(a + bx)}{x^{3/2}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{2 \cos(a + b\sqrt[3]{x})}{\sqrt{x}} + \frac{4b \sin(a + b\sqrt[3]{x})}{\sqrt[6]{x}} - (4b^2) \text{Subst} \left( \int \frac{\cos(a + bx)}{\sqrt{x}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{2 \cos(a + b\sqrt[3]{x})}{\sqrt{x}} + \frac{4b \sin(a + b\sqrt[3]{x})}{\sqrt[6]{x}} - (4b^2 \cos(a)) \text{Subst} \left( \int \frac{\cos(bx)}{\sqrt{x}} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{2 \cos(a + b\sqrt[3]{x})}{\sqrt{x}} + \frac{4b \sin(a + b\sqrt[3]{x})}{\sqrt[6]{x}} - (8b^2 \cos(a)) \text{Subst} \left( \int \cos(bx^2) dx, x, \sqrt[3]{x} \right) \\
&= -\frac{2 \cos(a + b\sqrt[3]{x})}{\sqrt{x}} - 4b^{3/2} \sqrt{2\pi} \cos(a) C \left( \sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x} \right) + 4b^{3/2} \sqrt{2\pi} S \left( \sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x} \right)
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 110, normalized size = 1.00

$$-\frac{2 \cos(a + b\sqrt[3]{x})}{\sqrt{x}} - 4b^{3/2}\sqrt{2\pi} \cos(a) \operatorname{FresnelC}\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right) + 4b^{3/2}\sqrt{2\pi} S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right) \sin(a) + \frac{4b \sin(a + b\sqrt[3]{x})}{\sqrt[6]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*x^(1/3)]/x^(3/2), x]

[Out] (-2\*Cos[a + b\*x^(1/3)]/Sqrt[x] - 4\*b^(3/2)\*Sqrt[2\*Pi]\*Cos[a]\*FresnelC[Sqrt[b]\*Sqrt[2/Pi]\*x^(1/6)] + 4\*b^(3/2)\*Sqrt[2\*Pi]\*FresnelS[Sqrt[b]\*Sqrt[2/Pi]\*x^(1/6)]\*Sin[a] + (4\*b\*Ssin[a + b\*x^(1/3)]/x^(1/6))

**Maple [A]**

time = 0.05, size = 78, normalized size = 0.71

method	result
derivativedivides	$-\frac{2 \cos(a + b x^{\frac{1}{3}})}{\sqrt{x}} - 4b \left( -\frac{\sin(a + b x^{\frac{1}{3}})}{x^{\frac{1}{6}}} + \sqrt{b} \sqrt{2} \sqrt{\pi} \left( \cos(a) \operatorname{FresnelC}\left(\frac{x^{\frac{1}{6}} \sqrt{b} \sqrt{2}}{\sqrt{\pi}}\right) - \sin(a) \operatorname{FresnelS}\left(\frac{x^{\frac{1}{6}} \sqrt{b} \sqrt{2}}{\sqrt{\pi}}\right) \right) \right)$
default	$-\frac{2 \cos(a + b x^{\frac{1}{3}})}{\sqrt{x}} - 4b \left( -\frac{\sin(a + b x^{\frac{1}{3}})}{x^{\frac{1}{6}}} + \sqrt{b} \sqrt{2} \sqrt{\pi} \left( \cos(a) \operatorname{FresnelC}\left(\frac{x^{\frac{1}{6}} \sqrt{b} \sqrt{2}}{\sqrt{\pi}}\right) - \sin(a) \operatorname{FresnelS}\left(\frac{x^{\frac{1}{6}} \sqrt{b} \sqrt{2}}{\sqrt{\pi}}\right) \right) \right)$
meijerg	$\frac{3 \cos(a) \sqrt{\pi} \sqrt{2} (b^2)^{\frac{3}{4}} \left( -\frac{8 \sqrt{2} \cos(b x^{\frac{1}{3}})}{3 \sqrt{\pi} \sqrt{x} (b^2)^{\frac{3}{4}}} + \frac{16 \sqrt{2} b \sin(b x^{\frac{1}{3}})}{3 \sqrt{\pi} x^{\frac{1}{6}} (b^2)^{\frac{3}{4}}} - \frac{32 b^{\frac{3}{2}} \operatorname{FresnelC}\left(\frac{x^{\frac{1}{6}} \sqrt{b} \sqrt{2}}{\sqrt{\pi}}\right)}{3 (b^2)^{\frac{3}{4}}} \right)}{8} - \frac{3 \sin(a) \sqrt{\pi} \sqrt{2} (b^2)^{\frac{3}{4}} \left( -\frac{8 \sqrt{2} \sin(b x^{\frac{1}{3}})}{3 \sqrt{\pi} \sqrt{x} (b^2)^{\frac{3}{4}}} + \frac{16 \sqrt{2} b \cos(b x^{\frac{1}{3}})}{3 \sqrt{\pi} x^{\frac{1}{6}} (b^2)^{\frac{3}{4}}} - \frac{32 b^{\frac{3}{2}} \operatorname{FresnelS}\left(\frac{x^{\frac{1}{6}} \sqrt{b} \sqrt{2}}{\sqrt{\pi}}\right)}{3 (b^2)^{\frac{3}{4}}} \right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*x^(1/3))/x^(3/2), x, method=\_RETURNVERBOSE)

[Out] -2\*cos(a+b\*x^(1/3))/x^(1/2)-4\*b\*(-1/x^(1/6)\*sin(a+b\*x^(1/3))+b^(1/2)\*2^(1/2)\*Pi^(1/2)\*(cos(a)\*FresnelC(x^(1/6)\*b^(1/2)\*2^(1/2)/Pi^(1/2))-sin(a)\*FresnelS(x^(1/6)\*b^(1/2)\*2^(1/2)/Pi^(1/2)))

**Maxima [C]** Result contains complex when optimal does not.

time = 0.58, size = 74, normalized size = 0.67

$$\frac{3 \left( (i-1) \sqrt{2} \Gamma\left(-\frac{3}{2}, i b x^{\frac{1}{3}}\right) - (i+1) \sqrt{2} \Gamma\left(-\frac{3}{2}, -i b x^{\frac{1}{3}}\right) \right) \cos(a) + \left( (i+1) \sqrt{2} \Gamma\left(-\frac{3}{2}, i b x^{\frac{1}{3}}\right) - (i-1) \sqrt{2} \Gamma\left(-\frac{3}{2}, -i b x^{\frac{1}{3}}\right) \right) \sin(a) \sqrt{b x^{\frac{1}{3}}}}{4 x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^(1/3))/x^(3/2), x, algorithm="maxima")

[Out] -3/4\*(((I - 1)\*sqrt(2)\*gamma(-3/2, I\*b\*x^(1/3)) - (I + 1)\*sqrt(2)\*gamma(-3/2, -I\*b\*x^(1/3)))\*cos(a) + ((I + 1)\*sqrt(2)\*gamma(-3/2, I\*b\*x^(1/3)) - (I - 1)\*sqrt(2)\*gamma(-3/2, -I\*b\*x^(1/3)))\*sin(a))\*sqrt(b\*x^(1/3))\*b/x^(1/6)

**Fricas [A]**

time = 0.36, size = 96, normalized size = 0.87

$$\frac{2 \left( 2 \sqrt{2} \pi b x \sqrt{\frac{b}{\pi}} \cos(a) C \left( \sqrt{2} x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}} \right) - 2 \sqrt{2} \pi b x \sqrt{\frac{b}{\pi}} S \left( \sqrt{2} x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}} \right) \sin(a) - 2 b x^{\frac{5}{6}} \sin \left( b x^{\frac{1}{3}} + a \right) + \sqrt{x} \cos \left( b x^{\frac{1}{3}} + a \right) \right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cos(a+b\*x^(1/3))/x^(3/2),x, algorithm="fricas")

**[Out]** -2\*(2\*sqrt(2)\*pi\*b\*x\*sqrt(b/pi)\*cos(a)\*fresnel\_cos(sqrt(2)\*x^(1/6)\*sqrt(b/pi)) - 2\*sqrt(2)\*pi\*b\*x\*sqrt(b/pi)\*fresnel\_sin(sqrt(2)\*x^(1/6)\*sqrt(b/pi))\*sin(a) - 2\*b\*x^(5/6)\*sin(b\*x^(1/3) + a) + sqrt(x)\*cos(b\*x^(1/3) + a))/x

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + b\sqrt[3]{x})}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cos(a+b\*x\*\*(1/3))/x\*\*(3/2),x)**[Out]** Integral(cos(a + b\*x\*\*(1/3))/x\*\*(3/2), x)**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cos(a+b\*x^(1/3))/x^(3/2),x, algorithm="giac")**[Out]** integrate(cos(b\*x^(1/3) + a)/x^(3/2), x)**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + b x^{1/3})}{x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(cos(a + b\*x^(1/3))/x^(3/2),x)**[Out]** int(cos(a + b\*x^(1/3))/x^(3/2), x)

$$3.53 \quad \int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{5/2}} dx$$

**Optimal.** Leaf size=184

$$-\frac{2 \cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2 \cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{32b^4 \cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} - \frac{32}{315} b^{9/2} \sqrt{2\pi} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right) - \frac{32}{315}$$

[Out]  $-2/3*\cos(a+b*x^{(1/3)})/x^{(3/2)}+8/105*b^2*\cos(a+b*x^{(1/3)})/x^{(5/6)}-32/315*b^4$   
 $*\cos(a+b*x^{(1/3)})/x^{(1/6)}+4/21*b*\sin(a+b*x^{(1/3)})/x^{(7/6)}-32/315*b^{(9/2)*c}$   
 $o$   
 $s(a)*\text{FresnelS}(x^{(1/6)*b^{(1/2)}*2^{(1/2)}/\text{Pi}^{(1/2)})*2^{(1/2)*\text{Pi}^{(1/2)}}-32/315*b^{($   
 $9/2)*\text{FresnelC}(x^{(1/6)*b^{(1/2)}*2^{(1/2)}/\text{Pi}^{(1/2)})*\sin(a)*2^{(1/2)*\text{Pi}^{(1/2)}}-16/$   
 $315*b^3*\sin(a+b*x^{(1/3)})/x^{(1/2)}$

**Rubi [A]**

time = 0.18, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {3497, 3378, 3387, 3386, 3432, 3385, 3433}

$$-\frac{32}{315} \sqrt{2\pi} b^{9/2} \sin(a) \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{b} \sqrt[6]{x}\right) - \frac{32}{315} \sqrt{2\pi} b^{9/2} \cos(a) S\left(\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt[6]{x}\right) - \frac{32b^4 \cos(a+b\sqrt[3]{x})}{315\sqrt[6]{x}} - \frac{16b^3 \sin(a+b\sqrt[3]{x})}{315\sqrt[6]{x}} + \frac{8b^2 \cos(a+b\sqrt[3]{x})}{105x^{5/6}} + \frac{4b \sin(a+b\sqrt[3]{x})}{21x^{7/6}} - \frac{2 \cos(a+b\sqrt[3]{x})}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^(1/3)]/x^(5/2), x]

[Out]  $(-2*\text{Cos}[a + b*x^{(1/3)}])/(3*x^{(3/2)}) + (8*b^2*\text{Cos}[a + b*x^{(1/3)}])/(105*x^{(5/6)}) - (32*b^4*\text{Cos}[a + b*x^{(1/3)}])/(315*x^{(1/6)}) - (32*b^{(9/2)*\text{Sqrt}[2*\text{Pi}]*\text{Cos}[a]*\text{FresnelS}[\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*x^{(1/6)}])/315 - (32*b^{(9/2)*\text{Sqrt}[2*\text{Pi}]*\text{FresnelC}[\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*x^{(1/6)}]*\text{Sin}[a])/315 + (4*b*\text{Sin}[a + b*x^{(1/3)}])/(21*x^{(7/6)}) - (16*b^3*\text{Sin}[a + b*x^{(1/3)}])/(315*\text{Sqrt}[x])$

Rule 3378

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*(Sin[e + f\*x]/(d\*(m + 1))), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3385

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

Rule 3386

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}

, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3387

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

#### Rule 3432

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

#### Rule 3433

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))<sup>2</sup>], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

#### Rule 3497

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)<sup>n</sup>])\*(b\_.))<sup>p</sup>\*(x\_)<sup>m</sup>, x\_Symbol] := Module[{k = Denominator[n]}, Dist[k, Subst[Int[x<sup>(k\*(m + 1) - 1)</sup>\*(a + b\*Cos[c + d\*x<sup>(k\*n)</sup>])<sup>p</sup>, x], x, x<sup>(1/k)</sup>], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p] && FractionQ[n]

#### Rubi steps

$$\begin{aligned}
\int \frac{\cos(a + b\sqrt[3]{x})}{x^{5/2}} dx &= 3\text{Subst}\left(\int \frac{\cos(a + bx)}{x^{11/2}} dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{3x^{3/2}} - \frac{1}{3}(2b)\text{Subst}\left(\int \frac{\sin(a + bx)}{x^{9/2}} dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{4b\sin(a + b\sqrt[3]{x})}{21x^{7/6}} - \frac{1}{21}(4b^2)\text{Subst}\left(\int \frac{\cos(a + bx)}{x^{7/2}} dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{105x^{5/6}} + \frac{4b\sin(a + b\sqrt[3]{x})}{21x^{7/6}} + \frac{1}{105}(8b^3)\text{Subst}\left(\int \frac{\sin(a + bx)}{x^{5/2}} dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{105x^{5/6}} + \frac{4b\sin(a + b\sqrt[3]{x})}{21x^{7/6}} - \frac{16b^3\sin(a + b\sqrt[3]{x})}{315\sqrt{x}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{105x^{5/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{315\sqrt[6]{x}} + \frac{4b\sin(a + b\sqrt[3]{x})}{21x^{7/6}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{105x^{5/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{315\sqrt[6]{x}} + \frac{4b\sin(a + b\sqrt[3]{x})}{21x^{7/6}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{105x^{5/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{315\sqrt[6]{x}} + \frac{4b\sin(a + b\sqrt[3]{x})}{21x^{7/6}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{105x^{5/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{315\sqrt[6]{x}} + \frac{4b\sin(a + b\sqrt[3]{x})}{21x^{7/6}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{105x^{5/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{315\sqrt[6]{x}} - \frac{32}{315}b^{9/2}\sqrt{2\pi}\cos(a)
\end{aligned}$$

**Mathematica [A]**

time = 0.16, size = 180, normalized size = 0.98

$$\frac{2\left(105\cos(a + b\sqrt[3]{x}) - 12b^2x^{2/3}\cos(a + b\sqrt[3]{x}) + 16b^4x^{4/3}\cos(a + b\sqrt[3]{x}) + 16b^{9/2}\sqrt{2\pi}x^{3/2}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{x}\right) + 16b^{9/2}\sqrt{2\pi}x^{3/2}\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{x}\right)\sin(a) - 30b\sqrt{x}\sin(a + b\sqrt[3]{x}) + 8b^3x\sin(a + b\sqrt[3]{x})\right)}{315x^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*x^(1/3)]/x^(5/2), x]`

```
[Out] (-*(105*Cos[a + b*x^(1/3)] - 12*b^2*x^(2/3)*Cos[a + b*x^(1/3)] + 16*b^4*x^(4/3)*Cos[a + b*x^(1/3)] + 16*b^(9/2)*Sqrt[2*Pi]*x^(3/2)*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)] + 16*b^(9/2)*Sqrt[2*Pi]*x^(3/2)*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a] - 30*b*x^(1/3)*Sin[a + b*x^(1/3)] + 8*b^3*x*Sin[a + b*x^(1/3)]))/(315*x^(3/2))
```

**Maple [A]**

time = 0.06, size = 129, normalized size = 0.70

method	result
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derivativedivides

$$\begin{aligned}
 & \frac{2 \cos\left(a + b x^{\frac{1}{3}}\right)}{3x^{\frac{3}{2}}} - \frac{4b \sin\left(a + b x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}} + \frac{2b \cos\left(a + b x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}} - \frac{2b \left( -\frac{\sin\left(a + b x^{\frac{1}{3}}\right)}{3\sqrt{x}} + \frac{\cos\left(a + b x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}} - \sqrt{b} \sqrt{2} \sqrt{\dots} \right)}{7}
 \end{aligned}$$

default

$$\begin{aligned}
 & \frac{2 \cos\left(a + b x^{\frac{1}{3}}\right)}{3x^{\frac{3}{2}}} - \frac{4b \sin\left(a + b x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}} + \frac{2b \cos\left(a + b x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}} - \frac{2b \left( -\frac{\sin\left(a + b x^{\frac{1}{3}}\right)}{3\sqrt{x}} + \frac{\cos\left(a + b x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}} - \sqrt{b} \sqrt{2} \sqrt{\dots} \right)}{7}
 \end{aligned}$$

meijerg	$3 \cos(a) \sqrt{\pi} \sqrt{2} (b^2)^{\frac{9}{4}} \left( -\frac{64 \sqrt{2} \left( \frac{16x^{\frac{4}{3}} b^4}{105} - \frac{4x^{\frac{2}{3}} b^2}{35} + 1 \right) \cos\left(bx^{\frac{1}{3}}\right)}{9 \sqrt{\pi} x^{\frac{3}{2}} (b^2)^{\frac{9}{4}}} + \frac{128 \sqrt{2} b \left( -4x^{\frac{2}{3}} b^2 + 15 \right) \sin\left(bx^{\frac{1}{3}}\right)}{945 \sqrt{\pi} x^{\frac{7}{6}} (b^2)^{\frac{9}{4}}} - \frac{2048 b^{\frac{9}{2}} S\left(\frac{x}{b^3}\right)}{945 \sqrt{\pi} x^{\frac{7}{6}} (b^2)^{\frac{9}{4}}} \right)$
	64

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*x^(1/3))/x^(5/2),x,method=_RETURNVERBOSE)`

[Out]  $-2/3 \cos(a+b*x^{1/3})/x^{3/2} - 4/3*b*(-1/7/x^{7/6}*\sin(a+b*x^{1/3})) + 2/7*b*(-1/5*\cos(a+b*x^{1/3})/x^{5/6} - 2/5*b*(-1/3/x^{1/2}*\sin(a+b*x^{1/3})) + 2/3*b*(-1/x^{1/6}*\cos(a+b*x^{1/3}) - b^{1/2}*2^{1/2}*Pi^{1/2}*(\cos(a)*\text{FresnelS}(x^{1/6}) * b^{1/2}*2^{1/2}/Pi^{1/2}) + \sin(a)*\text{FresnelC}(x^{1/6}) * b^{1/2}*2^{1/2}/Pi^{1/2}))$

**Maxima [C]** Result contains complex when optimal does not.

time = 0.59, size = 76, normalized size = 0.41

$$\frac{3 \left( (-i+1) \sqrt{2} \Gamma\left(-\frac{9}{2}, i b x^{\frac{1}{3}}\right) + (i-1) \sqrt{2} \Gamma\left(-\frac{9}{2}, -i b x^{\frac{1}{3}}\right) \right) \cos(a) + \left( (i-1) \sqrt{2} \Gamma\left(-\frac{9}{2}, i b x^{\frac{1}{3}}\right) - (i+1) \sqrt{2} \Gamma\left(-\frac{9}{2}, -i b x^{\frac{1}{3}}\right) \right) \sin(a) \sqrt{b x^{\frac{1}{3}} b^4}}{4 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))/x^(5/2),x, algorithm="maxima")`

[Out]  $3/4 * ((-I + 1) * \text{sqrt}(2) * \text{gamma}(-9/2, I * b * x^{1/3}) + (I - 1) * \text{sqrt}(2) * \text{gamma}(-9/2, -I * b * x^{1/3})) * \cos(a) + ((I - 1) * \text{sqrt}(2) * \text{gamma}(-9/2, I * b * x^{1/3}) - (I + 1) * \text{sqrt}(2) * \text{gamma}(-9/2, -I * b * x^{1/3})) * \sin(a) * \text{sqrt}(b * x^{1/3}) * b^4 / x^{1/6}$

**Fricas [A]**

time = 0.38, size = 134, normalized size = 0.73

$$\frac{2 \left( 16 \sqrt{2} \pi b^4 x^2 \sqrt{\frac{b}{\pi}} \cos(a) S\left(\sqrt{2} x^{\frac{1}{3}} \sqrt{\frac{b}{\pi}}\right) + 16 \sqrt{2} \pi b^4 x^2 \sqrt{\frac{b}{\pi}} C\left(\sqrt{2} x^{\frac{1}{3}} \sqrt{\frac{b}{\pi}}\right) \sin(a) + (16 b^4 x^{\frac{11}{6}} - 12 b^2 x^{\frac{7}{6}} + 105 \sqrt{x}) \cos(b x^{\frac{1}{3}} + a) + 2 (4 b^3 x^{\frac{11}{6}} - 15 b x^{\frac{7}{6}}) \sin(b x^{\frac{1}{3}} + a) \right)}{315 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))/x^(5/2),x, algorithm="fricas")`

[Out]  $-2/315 * (16 * \text{sqrt}(2) * \pi * b^4 * x^2 * \text{sqrt}(b/\pi) * \cos(a) * \text{fresnel\_sin}(\text{sqrt}(2) * x^{1/6} * \text{sqrt}(b/\pi)) + 16 * \text{sqrt}(2) * \pi * b^4 * x^2 * \text{sqrt}(b/\pi) * \text{fresnel\_cos}(\text{sqrt}(2) * x^{1/6} * \text{sqrt}(b/\pi)) * \sin(a) + (16 * b^4 * x^{11/6} - 12 * b^2 * x^{7/6} + 105 * \text{sqrt}(x)) * \cos(b * x^{1/3} + a) + 2 * (4 * b^3 * x^{11/6} - 15 * b * x^{5/6})) * \sin(b * x^{1/3} + a) / x^2$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + b\sqrt[3]{x})}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x**(1/3))/x**(5/2),x)`

[Out] `Integral(cos(a + b*x**(1/3))/x**(5/2), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))/x^(5/2),x, algorithm="giac")`

[Out] `integrate(cos(b*x^(1/3) + a)/x^(5/2), x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + b x^{1/3})}{x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x^(1/3))/x^(5/2),x)`

[Out] `int(cos(a + b*x^(1/3))/x^(5/2), x)`

$$3.54 \quad \int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}} dx$$

Optimal. Leaf size=250

$$-\frac{2 \cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2 \cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4 \cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{128b^6 \cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{256b^{15/2}\sqrt{2\pi} \cos(a)}{675675}$$

[Out]  $-2/5*\cos(a+b*x^{(1/3)})/x^{(5/2)}+8/715*b^2*\cos(a+b*x^{(1/3)})/x^{(11/6)}-32/45045*b^4*\cos(a+b*x^{(1/3)})/x^{(7/6)}+4/65*b*\sin(a+b*x^{(1/3)})/x^{(13/6)}-16/6435*b^3*\sin(a+b*x^{(1/3)})/x^{(3/2)}+64/225225*b^5*\sin(a+b*x^{(1/3)})/x^{(5/6)}-256/675675*b^7*\sin(a+b*x^{(1/3)})/x^{(1/6)}+256/675675*b^{(15/2)}*\cos(a)*\text{FresnelC}(x^{(1/6)})*b^{(1/2)}*2^{(1/2)}/\text{Pi}^{(1/2)})*2^{(1/2)}*\text{Pi}^{(1/2)}-256/675675*b^{(15/2)}*\text{FresnelS}(x^{(1/6)})*b^{(1/2)}*2^{(1/2)}/\text{Pi}^{(1/2)})*\sin(a)*2^{(1/2)}*\text{Pi}^{(1/2)}+128/675675*b^6*\cos(a+b*x^{(1/3)})/x^{(1/2)}$

Rubi [A]

time = 0.25, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {3497, 3378, 3387, 3386, 3432, 3385, 3433}

$$\frac{256\sqrt{2\pi}b^{15/2}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{x}\right)}{675675} - \frac{256\sqrt{2\pi}b^{15/2}\sin(a)\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{x}\right)}{675675} - \frac{256b^7\sin(a+b\sqrt[3]{x})}{675675\sqrt{x}} + \frac{128b^6\cos(a+b\sqrt[3]{x})}{675675\sqrt{x}} + \frac{64b^5\sin(a+b\sqrt[3]{x})}{225225x^{5/6}} - \frac{32b^4\cos(a+b\sqrt[3]{x})}{45045x^{7/6}} - \frac{16b^3\sin(a+b\sqrt[3]{x})}{6435x^{3/2}} + \frac{8b^2\cos(a+b\sqrt[3]{x})}{715x^{11/6}} + \frac{4b\sin(a+b\sqrt[3]{x})}{65x^{13/6}} - \frac{2\cos(a+b\sqrt[3]{x})}{5x^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^(1/3)]/x^(7/2), x]

[Out]  $(-2*\text{Cos}[a + b*x^{(1/3)}])/(5*x^{(5/2)}) + (8*b^2*\text{Cos}[a + b*x^{(1/3)}])/(715*x^{(11/6)}) - (32*b^4*\text{Cos}[a + b*x^{(1/3)}])/(45045*x^{(7/6)}) + (128*b^6*\text{Cos}[a + b*x^{(1/3)}])/(675675*\text{Sqrt}[x]) + (256*b^{(15/2)}*\text{Sqrt}[2*\text{Pi}]*\text{Cos}[a]*\text{FresnelC}[\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*x^{(1/6)}])/675675 - (256*b^{(15/2)}*\text{Sqrt}[2*\text{Pi}]*\text{FresnelS}[\text{Sqrt}[b]*\text{Sqrt}[2/\text{Pi}]*x^{(1/6)}]*\text{Sin}[a])/675675 + (4*b*\text{Sin}[a + b*x^{(1/3)}])/(65*x^{(13/6)}) - (16*b^3*\text{Sin}[a + b*x^{(1/3)}])/(6435*x^{(3/2)}) + (64*b^5*\text{Sin}[a + b*x^{(1/3)}])/(225225*x^{(5/6)}) - (256*b^7*\text{Sin}[a + b*x^{(1/3)}])/(675675*x^{(1/6)})$

Rule 3378

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*(Sin[e + f\*x]/(d\*(m + 1))), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3385

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d

, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3386

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[Sin[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3387

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

#### Rule 3432

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] :> Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

#### Rule 3433

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] :> Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

#### Rule 3497

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*(x\_)^(m\_.), x\_Symbol] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*Cos[c + d\*x^(k\*n)])^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p] && FractionQ[n]

#### Rubi steps

$$\begin{aligned}
\int \frac{\cos(a + b\sqrt[3]{x})}{x^{7/2}} dx &= 3\text{Subst}\left(\int \frac{\cos(a + bx)}{x^{17/2}} dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{5x^{5/2}} - \frac{1}{5}(2b)\text{Subst}\left(\int \frac{\sin(a + bx)}{x^{15/2}} dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{4b\sin(a + b\sqrt[3]{x})}{65x^{13/6}} - \frac{1}{65}(4b^2)\text{Subst}\left(\int \frac{\cos(a + bx)}{x^{13/2}} dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{715x^{11/6}} + \frac{4b\sin(a + b\sqrt[3]{x})}{65x^{13/6}} + \frac{1}{715}(8b^3)\text{Subst}\left(\int \frac{\sin(a + bx)}{x^{11/2}} dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{715x^{11/6}} + \frac{4b\sin(a + b\sqrt[3]{x})}{65x^{13/6}} - \frac{16b^3\sin(a + b\sqrt[3]{x})}{6435x^{3/2}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{715x^{11/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{45045x^{7/6}} + \frac{4b\sin(a + b\sqrt[3]{x})}{65x^{13/6}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{715x^{11/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{45045x^{7/6}} + \frac{4b\sin(a + b\sqrt[3]{x})}{65x^{13/6}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{715x^{11/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{45045x^{7/6}} + \frac{128b^6\cos(a + b\sqrt[3]{x})}{675675\sqrt{x}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{715x^{11/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{45045x^{7/6}} + \frac{128b^6\cos(a + b\sqrt[3]{x})}{675675\sqrt{x}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{715x^{11/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{45045x^{7/6}} + \frac{128b^6\cos(a + b\sqrt[3]{x})}{675675\sqrt{x}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{715x^{11/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{45045x^{7/6}} + \frac{128b^6\cos(a + b\sqrt[3]{x})}{675675\sqrt{x}} \\
&= -\frac{2\cos(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{8b^2\cos(a + b\sqrt[3]{x})}{715x^{11/6}} - \frac{32b^4\cos(a + b\sqrt[3]{x})}{45045x^{7/6}} + \frac{128b^6\cos(a + b\sqrt[3]{x})}{675675\sqrt{x}}
\end{aligned}$$

**Mathematica [A]**

time = 0.22, size = 238, normalized size = 0.95

$$\frac{2\left(-135135\cos(a + b\sqrt[3]{x}) + 3780b^2x^{2/3}\cos(a + b\sqrt[3]{x}) - 240b^4x^{4/3}\cos(a + b\sqrt[3]{x}) + 64b^6x^2\cos(a + b\sqrt[3]{x}) + 128b^{11/2}\sqrt{2x}x^{5/2}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{x}}\sqrt{x}\right) - 128b^{11/2}\sqrt{2x}x^{5/2}\sin\left(\sqrt{\frac{2}{x}}\sqrt{x}\right)\sin(a) + 20790b\sqrt{x}\sin(a + b\sqrt[3]{x}) - 840b^3x\sin(a + b\sqrt[3]{x}) + 96b^5x^{2/3}\sin(a + b\sqrt[3]{x}) - 128b^7x^{2/3}\sin(a + b\sqrt[3]{x})\right)}{675675x^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*x^(1/3)]/x^(7/2), x]`

```
[Out] (2*(-135135*Cos[a + b*x^(1/3)] + 3780*b^2*x^(2/3)*Cos[a + b*x^(1/3)] - 240*
b^4*x^(4/3)*Cos[a + b*x^(1/3)] + 64*b^6*x^2*Cos[a + b*x^(1/3)] + 128*b^(15/
2)*Sqrt[2*Pi]*x^(5/2)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)] - 128*b^(
15/2)*Sqrt[2*Pi]*x^(5/2)*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a] + 2079
0*b*x^(1/3)*Sin[a + b*x^(1/3)] - 840*b^3*x*Sin[a + b*x^(1/3)] + 96*b^5*x^(5
/3)*Sin[a + b*x^(1/3)] - 128*b^7*x^(7/3)*Sin[a + b*x^(1/3)]))/(675675*x^(5/
2))
```

**Maple [A]**

time = 0.06, size = 180, normalized size = 0.72

method	result
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$$2b - \frac{\sin\left(a + b x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}}$$

$$2b - \frac{\cos\left(a + b x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}$$

$$2b - \frac{\sin\left(a + b x^{\frac{1}{3}}\right)}{9x^{\frac{9}{2}}} +$$

$$2b - \frac{\cos\left(a + b x^{\frac{1}{3}}\right)}{11x^{\frac{11}{6}}}$$



$$2b - \frac{\sin\left(a + b x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}}$$

$$2b - \frac{\cos\left(a + b x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}$$

$$2b - \frac{\sin\left(a + b x^{\frac{1}{3}}\right)}{9x^{\frac{9}{2}}} +$$

$$2b - \frac{\cos\left(a + b x^{\frac{1}{3}}\right)}{11x^{\frac{11}{6}}}$$

meijerg	$3 \cos(a) \sqrt{\pi} \sqrt{2} (b^2)^{\frac{15}{4}} \left( \frac{512 \sqrt{2} \left( -\frac{64x^2 b^6}{135135} + \frac{16x^{\frac{4}{3}} b^4}{9009} - \frac{4x^{\frac{2}{3}} b^2}{143} + 1 \right) \cos\left(b x^{\frac{1}{3}}\right)}{15 \sqrt{\pi} x^{\frac{5}{2}} (b^2)^{\frac{15}{4}}} + \frac{1024 \sqrt{2} b \left( -64x^2 b^6 + 48x^{\frac{4}{3}} b^4 - 420x^{\frac{2}{3}} b^2 + 1 \right) \sin\left(b x^{\frac{1}{3}}\right)}{2027025 \sqrt{\pi} x^{\frac{13}{6}} (b^2)^{\frac{15}{4}}} \right)$	512
---------	--	-----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*x^(1/3))/x^(7/2),x,method=_RETURNVERBOSE)`

[Out]  $-2/5*\cos(a+b*x^{(1/3)})/x^{(5/2)}-4/5*b*(-1/13/x^{(13/6)}*\sin(a+b*x^{(1/3)})+2/13*b*(-1/11*\cos(a+b*x^{(1/3)})/x^{(11/6)}-2/11*b*(-1/9/x^{(3/2)}*\sin(a+b*x^{(1/3)}))+2/9*b*(-1/7/x^{(7/6)}*\cos(a+b*x^{(1/3)})-2/7*b*(-1/5/x^{(5/6)}*\sin(a+b*x^{(1/3)}))+2/5*b*(-1/3*\cos(a+b*x^{(1/3)})/x^{(1/2)}-2/3*b*(-1/x^{(1/6)}*\sin(a+b*x^{(1/3)}))+b^{(1/2)}*2^{(1/2)}*Pi^{(1/2)}*(\cos(a)*\text{FresnelC}(x^{(1/6)}*b^{(1/2)}*2^{(1/2)}/Pi^{(1/2)})-\sin(a)*\text{FresnelS}(x^{(1/6)}*b^{(1/2)}*2^{(1/2)}/Pi^{(1/2)}))))))$

**Maxima [C]** Result contains complex when optimal does not.

time = 0.57, size = 76, normalized size = 0.30

$$3 \left( \frac{\left( (i-1) \sqrt{2} \Gamma\left(-\frac{15}{2}, i b x^{\frac{1}{3}}\right) - (i+1) \sqrt{2} \Gamma\left(-\frac{15}{2}, -i b x^{\frac{1}{3}}\right) \right) \cos(a) + \left( (i+1) \sqrt{2} \Gamma\left(-\frac{15}{2}, i b x^{\frac{1}{3}}\right) - (i-1) \sqrt{2} \Gamma\left(-\frac{15}{2}, -i b x^{\frac{1}{3}}\right) \right) \sin(a)}{4 x^{\frac{1}{6}}} \sqrt{b x^{\frac{1}{3}}} b^7 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))/x^(7/2),x, algorithm="maxima")`

[Out]  $3/4*((I-1)*\sqrt{2}*\gamma(-15/2, I*b*x^{(1/3)}) - (I+1)*\sqrt{2}*\gamma(-15/2, -I*b*x^{(1/3)}))*\cos(a) + ((I+1)*\sqrt{2}*\gamma(-15/2, I*b*x^{(1/3)}) - (I-1)*\sqrt{2}*\gamma(-15/2, -I*b*x^{(1/3)}))*\sin(a))*\sqrt{b*x^{(1/3)}}*b^7/x^{(1/6)}$

**Fricas [A]**

time = 0.37, size = 164, normalized size = 0.66

$$\frac{2 \left( 128 \sqrt{2} \pi b^7 x^3 \sqrt{\frac{b}{\pi}} \cos(a) C\left(\sqrt{2} x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}}\right) - 128 \sqrt{2} \pi b^7 x^3 \sqrt{\frac{b}{\pi}} S\left(\sqrt{2} x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}}\right) \sin(a) - (240 b^4 x^{\frac{11}{6}} - 3780 b^2 x^{\frac{7}{6}} - (64 b^6 x^2 - 135135) \sqrt{x}) \cos(b x^{\frac{1}{3}} + a) + 2(48 b^5 x^{\frac{13}{6}} - 420 b^3 x^{\frac{9}{6}} - (64 b^7 x^2 - 10395 b) x^{\frac{5}{6}}) \sin(b x^{\frac{1}{3}} + a) \right)}{675675 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))/x^(7/2),x, algorithm="fricas")`

[Out]  $2/675675*(128*\sqrt{2}*\pi*b^7*x^3*\sqrt{b/\pi}*\cos(a)*\text{fresnel\_cos}(\sqrt{2}*x^{(1/6)}*\sqrt{b/\pi}) - 128*\sqrt{2}*\pi*b^7*x^3*\sqrt{b/\pi}*\text{fresnel\_sin}(\sqrt{2}*x^{(1/6)}*\sqrt{b/\pi}))*\sin(a) - (240*b^4*x^{(11/6)} - 3780*b^2*x^{(7/6)} - (64*b^6*x^2 - 135135)*\sqrt{x})*\cos(b*x^{(1/3)} + a) + 2*(48*b^5*x^{(13/6)} - 420*b^3*x^{(9/6)} - (64*b^7*x^2 - 10395*b)*x^{(5/6)})*\sin(b*x^{(1/3)} + a))/x^3$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + b\sqrt[3]{x})}{x^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x**(1/3))/x**(7/2),x)`

[Out] `Integral(cos(a + b*x**(1/3))/x**(7/2), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))/x^(7/2),x, algorithm="giac")`

[Out] `integrate(cos(b*x^(1/3) + a)/x^(7/2), x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a + b x^{1/3})}{x^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x^(1/3))/x^(7/2),x)`

[Out] `int(cos(a + b*x^(1/3))/x^(7/2), x)`

### 3.55 $\int x^{3/2} \cos^2(a + b\sqrt[3]{x}) dx$

Optimal. Leaf size=310

$$-\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x} \cos^2(a + b\sqrt[3]{x})}{2048b^6} - \frac{3861x^{7/6} \cos^2(a + b\sqrt[3]{x})}{128b^4} + \frac{39x^{11/6}}{16b^2}$$

[Out]  $3861/256*x^{(7/6)}/b^4 - 39/16*x^{(11/6)}/b^2 + 1/5*x^{(5/2)} - 3861/128*x^{(7/6)}*\cos(a + b*x^{(1/3)})^2/b^4 + 39/8*x^{(11/6)}*\cos(a + b*x^{(1/3)})^2/b^2 + 27027/512*x^{(5/6)}*\cos(a + b*x^{(1/3)})*\sin(a + b*x^{(1/3)})/b^5 - 429/32*x^{(3/2)}*\cos(a + b*x^{(1/3)})*\sin(a + b*x^{(1/3)})/b^3 + 3/2*x^{(13/6)}*\cos(a + b*x^{(1/3)})*\sin(a + b*x^{(1/3)})/b - 405405/16384*x^{(1/6)}*\sin(2*a + 2*b*x^{(1/3)})/b^7 + 405405/32768*\cos(2*a)*\text{FresnelS}(2*x^{(1/6)}*b^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/b^{(15/2)} + 405405/32768*\text{FresnelC}(2*x^{(1/6)}*b^{(1/2)}/\text{Pi}^{(1/2)})*\sin(2*a)*\text{Pi}^{(1/2)}/b^{(15/2)} - 135135/4096*x^{(1/2)}/b^6 + 135135/2048*\cos(a + b*x^{(1/3)})^2*x^{(1/2)}/b^6$

Rubi [A]

time = 0.25, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {3497, 3392, 30, 3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$\frac{405405\sqrt{x}\sin(2a)\text{FresnelC}\left(\frac{2\sqrt[3]{x}}{\sqrt{2b}}\right)}{32768b^{15/2}} + \frac{405405\sqrt{x}\cos(2a)\text{FresnelS}\left(\frac{2\sqrt[3]{x}}{\sqrt{2b}}\right)}{32768b^{15/2}} - \frac{405405\sqrt{x}\sin(2(a+b\sqrt[3]{x}))}{16384b^7} + \frac{135135\sqrt{x}\cos^2(a+b\sqrt[3]{x})}{2048b^6} + \frac{27027x^{5/6}\sin(a+b\sqrt[3]{x})\cos(a+b\sqrt[3]{x})}{512b^5} - \frac{3861x^{7/6}\cos^2(a+b\sqrt[3]{x})}{128b^4} - \frac{429x^{3/2}\sin(a+b\sqrt[3]{x})\cos(a+b\sqrt[3]{x})}{32b^3} + \frac{39x^{11/6}\sin(a+b\sqrt[3]{x})\cos(a+b\sqrt[3]{x})}{8b^2} + \frac{39x^{11/6}\sin(a+b\sqrt[3]{x})\cos(a+b\sqrt[3]{x})}{8b^2} - \frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)\*Cos[a + b\*x^(1/3)]^2,x]

[Out]  $(-135135*\text{Sqrt}[x])/(4096*b^6) + (3861*x^{(7/6)})/(256*b^4) - (39*x^{(11/6)})/(16*b^2) + x^{(5/2)}/5 + (135135*\text{Sqrt}[x]*\text{Cos}[a + b*x^{(1/3)}]^2)/(2048*b^6) - (3861*x^{(7/6)}*\text{Cos}[a + b*x^{(1/3)}]^2)/(128*b^4) + (39*x^{(11/6)}*\text{Cos}[a + b*x^{(1/3)}]^2)/(8*b^2) + (405405*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a]*\text{FresnelS}[(2*\text{Sqrt}[b]*x^{(1/6)})/\text{Sqrt}[\text{Pi}]])/(32768*b^{(15/2)}) + (405405*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[b]*x^{(1/6)})/\text{Sqrt}[\text{Pi}]]*\text{Sin}[2*a])/(32768*b^{(15/2)}) + (27027*x^{(5/6)}*\text{Cos}[a + b*x^{(1/3)}]*\text{Sin}[a + b*x^{(1/3)}])/(512*b^5) - (429*x^{(3/2)}*\text{Cos}[a + b*x^{(1/3)}]*\text{Sin}[a + b*x^{(1/3)}])/(32*b^3) + (3*x^{(13/6)}*\text{Cos}[a + b*x^{(1/3)}]*\text{Sin}[a + b*x^{(1/3)}])/(2*b) - (405405*x^{(1/6)}*\text{Sin}[2*(a + b*x^{(1/3)})])/(16384*b^7)$

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3377

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*sin[(e\_) + (f\_)\*(x\_)], x\_Symbol] := Simp[(-(c + d\*x)^m)\*(Cos[e + f\*x]/f), x] + Dist[d\*(m/f), Int[(c + d\*x)^(m - 1)\*Co

$s[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[m, 0]$

#### Rule 3385

$\text{Int}[\sin[\text{Pi}/2 + (e_{\cdot}) + (f_{\cdot})(x_{\cdot})]/\text{Sqrt}[(c_{\cdot}) + (d_{\cdot})(x_{\cdot})], x_{\text{Symbol}}] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Cos}[f*(x^2/d)], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3386

$\text{Int}[\sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]/\text{Sqrt}[(c_{\cdot}) + (d_{\cdot})(x_{\cdot})], x_{\text{Symbol}}] \rightarrow \text{Dist}[2/d, \text{Subst}[\text{Int}[\text{Sin}[f*(x^2/d)], x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

#### Rule 3387

$\text{Int}[\sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]/\text{Sqrt}[(c_{\cdot}) + (d_{\cdot})(x_{\cdot})], x_{\text{Symbol}}] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[c*(f/d) + f*x]/\text{Sqrt}[c + d*x], x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[c*(f/d) + f*x]/\text{Sqrt}[c + d*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{ComplexFreeQ}[f] \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

#### Rule 3392

$\text{Int}[(c_{\cdot}) + (d_{\cdot})(x_{\cdot})]^{(m_{\cdot})} * ((b_{\cdot}) * \sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})])^{(n_{\cdot})}, x_{\text{Symbol}}] \rightarrow \text{Simp}[d*m*(c + d*x)^{(m-1)} * ((b*\text{Sin}[e + f*x])^n / (f^{2*n^2})), x] + (\text{Dist}[b^{2*m} * ((n-1)/n), \text{Int}[(c + d*x)^m * (b*\text{Sin}[e + f*x])^{(n-2)}], x], x] - \text{Dist}[d^{2*m} * ((m-1)/(f^{2*n^2})), \text{Int}[(c + d*x)^{(m-2)} * (b*\text{Sin}[e + f*x])^n], x] - \text{Simp}[b*(c + d*x)^m * \text{Cos}[e + f*x] * ((b*\text{Sin}[e + f*x])^{(n-1)} / (f*n)), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 1]$

#### Rule 3393

$\text{Int}[(c_{\cdot}) + (d_{\cdot})(x_{\cdot})]^{(m_{\cdot})} * \sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]^{(n_{\cdot})}, x_{\text{Symbol}}] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x\} \ \&\& \ \text{IGtQ}[n, 1] \ \&\& \ (\text{!RationalQ}[m] \ || \ (\text{GeQ}[m, -1] \ \&\& \ \text{LtQ}[m, 1]))$

#### Rule 3432

$\text{Int}[\text{Sin}[(d_{\cdot}) * ((e_{\cdot}) + (f_{\cdot})(x_{\cdot}))^2], x_{\text{Symbol}}] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2] / (f * \text{Rt}[d, 2])) * \text{FresnelS}[\text{Sqrt}[2/\text{Pi}] * \text{Rt}[d, 2] * (e + f*x)], x] /; \text{FreeQ}\{d, e, f\}, x\}$

#### Rule 3433

$\text{Int}[\text{Cos}[(d_{\cdot}) * ((e_{\cdot}) + (f_{\cdot})(x_{\cdot}))^2], x_{\text{Symbol}}] \rightarrow \text{Simp}[(\text{Sqrt}[\text{Pi}/2] / (f * \text{Rt}[d, 2])) * \text{FresnelC}[\text{Sqrt}[2/\text{Pi}] * \text{Rt}[d, 2] * (e + f*x)], x] /; \text{FreeQ}\{d, e, f\}, x\}$

## Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol
] := Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)]]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

## Rubi steps

$$\begin{aligned}
\int x^{3/2} \cos^2(a + b\sqrt[3]{x}) dx &= 3 \text{Subst}\left(\int x^{13/2} \cos^2(a + bx) dx, x, \sqrt[3]{x}\right) \\
&= \frac{39x^{11/6} \cos^2(a + b\sqrt[3]{x})}{8b^2} + \frac{3x^{13/6} \cos(a + b\sqrt[3]{x}) \sin(a + b\sqrt[3]{x})}{2b} + \frac{3}{2} \text{Subst}\left(\int \right. \\
&= \frac{x^{5/2}}{5} - \frac{3861x^{7/6} \cos^2(a + b\sqrt[3]{x})}{128b^4} + \frac{39x^{11/6} \cos^2(a + b\sqrt[3]{x})}{8b^2} - \frac{429x^{3/2} \cos(a + b\sqrt[3]{x})}{128b^4} \\
&= -\frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x} \cos^2(a + b\sqrt[3]{x})}{2048b^6} - \frac{3861x^{7/6} \cos^2(a + b\sqrt[3]{x})}{128b^4} \\
&= \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x} \cos^2(a + b\sqrt[3]{x})}{2048b^6} - \frac{3861x^{7/6} \cos^2(a + b\sqrt[3]{x})}{128b^4} \\
&= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x} \cos^2(a + b\sqrt[3]{x})}{2048b^6} \\
&= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x} \cos^2(a + b\sqrt[3]{x})}{2048b^6} \\
&= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x} \cos^2(a + b\sqrt[3]{x})}{2048b^6} \\
&= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x} \cos^2(a + b\sqrt[3]{x})}{2048b^6}
\end{aligned}$$

**Mathematica [A]**

time = 0.43, size = 174, normalized size = 0.56

$$\frac{2027025\sqrt{\pi} \cos(2a) S\left(\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\pi}}\right) + 2027025\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\pi}}\right) \sin(2a) + 2\sqrt{b}\sqrt{x} (16384b^7x^{7/3} + 780(3465b\sqrt{x} - 1584b^2x + 256b^3x^{5/3}) \cos(2(a + b\sqrt{x})) + 15(-135135 + 144144b^2x^{2/3} - 36608b^4x^{4/3} + 4096b^6x^2) \sin(2(a + b\sqrt{x})))}{163840b^{5/2}}$$

Antiderivative was successfully verified.

**[In]** Integrate[x^(3/2)\*Cos[a + b\*x^(1/3)]^2,x]

[Out]  $(2027025*\sqrt{\pi}*\cos[2*a]*\text{FresnelS}[(2*\sqrt{b}*x^{(1/6)})/\sqrt{\pi}] + 2027025*\sqrt{\pi}*\text{FresnelC}[(2*\sqrt{b}*x^{(1/6)})/\sqrt{\pi}]*\sin[2*a] + 2*\sqrt{b}*x^{(1/6)}*(16384*b^7*x^{(7/3)} + 780*(3465*b*x^{(1/3)} - 1584*b^3*x + 256*b^5*x^{(5/3)})*\cos[2*(a + b*x^{(1/3)})] + 15*(-135135 + 144144*b^2*x^{(2/3)} - 36608*b^4*x^{(4/3)} + 4096*b^6*x^2)*\sin[2*(a + b*x^{(1/3)})]))/(163840*b^{(15/2)})$

Maple [A]

time = 0.08, size = 219, normalized size = 0.71

method	result
derivativedivides	$\frac{x^{\frac{5}{6}}}{5} + \frac{3x^{\frac{13}{6}} \sin\left(2a + 2bx^{\frac{1}{3}}\right)}{4b} - \left( \frac{x^{\frac{11}{6}} \cos\left(2a + 2bx^{\frac{1}{3}}\right)}{4b} + \frac{11x^{\frac{3}{2}} \sin\left(2a + 2bx^{\frac{1}{3}}\right)}{16b} - \left( \frac{x^{\frac{7}{6}} \cos\left(2a + 2bx^{\frac{1}{3}}\right)}{4b} + \frac{7x^{\frac{5}{6}} \sin\left(2a + 2bx^{\frac{1}{3}}\right)}{16b} \right) \right)$

<p>default</p>	$\frac{x^{\frac{5}{2}}}{5} + \frac{3x^{\frac{13}{6}} \sin(2a+2bx^{\frac{1}{3}})}{4b} -$	$39 - \frac{x^{\frac{11}{6}} \cos(2a+2bx^{\frac{1}{3}})}{4b} + \frac{11x^{\frac{3}{2}} \sin(2a+2bx^{\frac{1}{3}})}{16b} -$	$99 - \frac{x^{\frac{7}{6}} \cos(2a+2bx^{\frac{1}{3}})}{4b} + \frac{7x^{\frac{5}{6}} \sin(2a+2bx^{\frac{1}{3}})}{16b}$
----------------	---	--	--

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(a+b*x^(1/3))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^(5/2)+3/4/b*x^(13/6)*sin(2*a+2*b*x^(1/3))-39/4/b*(-1/4/b*x^(11/6)*cos(2*a+2*b*x^(1/3))+11/4/b*(1/4/b*x^(3/2)*sin(2*a+2*b*x^(1/3))-9/4/b*(-1/4/b*x^(7/6)*cos(2*a+2*b*x^(1/3))+7/4/b*(1/4/b*x^(5/6)*sin(2*a+2*b*x^(1/3))-5/4/b*(-1/4/b*x^(1/2)*cos(2*a+2*b*x^(1/3))+3/4/b*(1/4*x^(1/6)*sin(2*a+2*b*x^(1/3)))/b-1/8/b^(3/2)*Pi^(1/2)*(cos(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2))))))
```

**Maxima [C]** Result contains complex when optimal does not.  
time = 0.52, size = 161, normalized size = 0.52

262144\*b^9\*x^5/2 + 2027025\*4^(1/4)\*sqrt(2)\*sqrt(pi)\*(((I + 1)\*cos(2\*a) - (I - 1)\*sin(2\*a))\*erf(sqrt(2\*I\*b)\*x^(1/6)) + (-(I - 1)\*cos(2\*a) + (I + 1)\*sin(2\*a))\*erf(sqrt(-2\*I\*b)\*x^(1/6)))\*b^(3/2) + 12480\*(256\*b^7\*x^(13/6) - 1584\*b^5\*x^(11/6) + 3465\*b^3\*x^(9/6) - 4096\*b\*x^(7/6) + 144144\*b^5\*x^5/2 - 135135\*b^2\*x^3)sin(2\*b\*x^(1/3) + 2\*a)

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] 1/1310720*(262144*b^9*x^(5/2) + 2027025*4^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*cos(2*a) - (I - 1)*sin(2*a))*erf(sqrt(2*I*b)*x^(1/6)) + (-(I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf(sqrt(-2*I*b)*x^(1/6)))*b^(3/2) + 12480*(256*b^7*x^(13/6) - 1584*b^5*x^(11/6) + 3465*b^3*x^(9/6) - 4096*b*x^(7/6) + 144144*b^5*x^5/2 - 135135*b^2*x^3)sin(2*b*x^(1/3) + 2*a)
```



11/6) - 1584\*b^5\*x^(7/6) + 3465\*b^3\*sqrt(x))\*cos(2\*b\*x^(1/3) + 2\*a) + 240\*(4096\*b^8\*x^(13/6) - 36608\*b^6\*x^(3/2) + 144144\*b^4\*x^(5/6) - 135135\*b^2\*x^(1/6))\*sin(2\*b\*x^(1/3) + 2\*a))/b^9

**Fricas** [A]

time = 0.39, size = 184, normalized size = 0.59

$$\frac{399360 b^6 x^{\frac{11}{6}} - 2471040 b^4 x^{\frac{7}{6}} - 2027025 \pi \sqrt{\frac{b}{\pi}} \cos(2a) S\left(2x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}}\right) - 2027025 \pi \sqrt{\frac{b}{\pi}} C\left(2x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}}\right) \sin(2a) - 3120 (256 b^6 x^{\frac{11}{6}} - 1584 b^4 x^{\frac{7}{6}} + 3465 b^3 \sqrt{x}) \cos(bx^{\frac{1}{3}} + a)^2 + 60 (36608 b^5 x^{\frac{3}{2}} - 144144 b^4 x^{\frac{5}{6}} - (4096 b^7 x^2 - 135135 b) x^{\frac{1}{6}}) \cos(bx^{\frac{1}{3}} + a) \sin(bx^{\frac{1}{3}} + a) - 8 (4096 b^8 x^2 - 675675 b^2) \sqrt{x}}{163840 b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*cos(a+b\*x^(1/3))^2,x, algorithm="fricas")

[Out] -1/163840\*(399360\*b^6\*x^(11/6) - 2471040\*b^4\*x^(7/6) - 2027025\*pi\*sqrt(b/pi)\*cos(2\*a)\*fresnel\_sin(2\*x^(1/6)\*sqrt(b/pi)) - 2027025\*pi\*sqrt(b/pi)\*fresnel\_cos(2\*x^(1/6)\*sqrt(b/pi))\*sin(2\*a) - 3120\*(256\*b^6\*x^(11/6) - 1584\*b^4\*x^(7/6) + 3465\*b^2\*sqrt(x))\*cos(b\*x^(1/3) + a)^2 + 60\*(36608\*b^5\*x^(3/2) - 144144\*b^3\*x^(5/6) - (4096\*b^7\*x^2 - 135135\*b)\*x^(1/6))\*cos(b\*x^(1/3) + a)\*sin(b\*x^(1/3) + a) - 8\*(4096\*b^8\*x^2 - 675675\*b^2)\*sqrt(x))/b^8

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \cos^2(a + b\sqrt[3]{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)\*cos(a+b\*x\*\*(1/3))\*\*2,x)

[Out] Integral(x\*\*(3/2)\*cos(a + b\*x\*\*(1/3))\*\*2, x)

**Giac** [C] Result contains complex when optimal does not.

time = 0.42, size = 224, normalized size = 0.72

$$\frac{1}{5} x^{\frac{5}{2}} - \frac{3(4096 b^6 x^{\frac{11}{6}} - 13312 b^5 x^{\frac{7}{6}} + 36608 b^4 x^{\frac{3}{2}} + 82368 b^3 x^{\frac{1}{6}} - 180180 b \sqrt{x} - 135135 x^{\frac{1}{6}}) e^{(2 a + b x^{\frac{1}{3}})}}{32768 b^7} - \frac{3(-4096 b^6 x^{\frac{11}{6}} - 13312 b^5 x^{\frac{7}{6}} + 36608 b^4 x^{\frac{3}{2}} + 82368 b^3 x^{\frac{1}{6}} - 144144 b^2 x^{\frac{1}{6}} - 180180 b \sqrt{x} + 135135 x^{\frac{1}{6}}) e^{(-2 a + b x^{\frac{1}{3}})}}{32768 b^7} + \frac{405405 \sqrt{\pi} \operatorname{erf}\left(-\sqrt{b} x^{\frac{1}{6}} \left(\frac{b}{\pi} + 1\right)\right) e^{2 a}}{65536 b^{\frac{7}{2}} \left(\frac{b}{\pi} + 1\right)} - \frac{405405 \sqrt{\pi} \operatorname{erf}\left(-\sqrt{b} x^{\frac{1}{6}} \left(\frac{b}{\pi} + 1\right)\right) e^{-2 a}}{65536 b^{\frac{7}{2}} \left(\frac{b}{\pi} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*cos(a+b\*x^(1/3))^2,x, algorithm="giac")

[Out] 1/5\*x^(5/2) - 3/32768\*(4096\*I\*b^6\*x^(13/6) - 13312\*b^5\*x^(11/6) - 36608\*I\*b^4\*x^(3/2) + 82368\*b^3\*x^(7/6) + 144144\*I\*b^2\*x^(5/6) - 180180\*b\*sqrt(x) - 135135\*I\*x^(1/6))\*e^(2\*I\*b\*x^(1/3) + 2\*I\*a)/b^7 - 3/32768\*(-4096\*I\*b^6\*x^(13/6) - 13312\*b^5\*x^(11/6) + 36608\*I\*b^4\*x^(3/2) + 82368\*b^3\*x^(7/6) - 144144\*I\*b^2\*x^(5/6) - 180180\*b\*sqrt(x) + 135135\*I\*x^(1/6))\*e^(-2\*I\*b\*x^(1/3) - 2\*I\*a)/b^7 + 405405/65536\*I\*sqrt(pi)\*erf(-sqrt(b)\*x^(1/6)\*(-I\*b/abs(b) + 1))\*e^(2\*I\*a)/(b^(15/2)\*(-I\*b/abs(b) + 1)) - 405405/65536\*I\*sqrt(pi)\*erf(-sqrt(b)\*x^(1/6)\*(I\*b/abs(b) + 1))\*e^(-2\*I\*a)/(b^(15/2)\*(I\*b/abs(b) + 1))

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2} \cos(a + b x^{1/3})^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*cos(a + b*x^(1/3))^2,x)`

[Out] `int(x^(3/2)*cos(a + b*x^(1/3))^2, x)`

### 3.56 $\int \sqrt{x} \cos^2(a + b\sqrt[3]{x}) dx$

**Optimal.** Leaf size=218

$$\frac{315\sqrt[6]{x}}{256b^4} - \frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x} \cos^2(a + b\sqrt[3]{x})}{128b^4} + \frac{21x^{5/6} \cos^2(a + b\sqrt[3]{x})}{8b^2} + \frac{315\sqrt{\pi} \cos(2a) \text{FresnelC}\left(\frac{2\sqrt[6]{x}}{\sqrt{\pi}}\right)}{512b^{9/2}}$$

[Out]  $315/256*x^{(1/6)}/b^4 - 21/16*x^{(5/6)}/b^2 + 1/3*x^{(3/2)} - 315/128*x^{(1/6)}*\cos(a+b*x^{(1/3)})^2/b^4 + 21/8*x^{(5/6)}*\cos(a+b*x^{(1/3)})^2/b^2 + 3/2*x^{(7/6)}*\cos(a+b*x^{(1/3)})*\sin(a+b*x^{(1/3)})/b + 315/512*\cos(2*a)*\text{FresnelC}(2*x^{(1/6)}*b^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}/b^{(9/2)} - 315/512*\text{FresnelS}(2*x^{(1/6)}*b^{(1/2)}/\text{Pi}^{(1/2)})*\sin(2*a)*\text{Pi}^{(1/2)}/b^{(9/2)} - 105/32*\cos(a+b*x^{(1/3)})*\sin(a+b*x^{(1/3)})*x^{(1/2)}/b^3$

**Rubi [A]**

time = 0.17, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3497, 3392, 30, 3393, 3387, 3386, 3432, 3385, 3433}

$$\frac{315\sqrt{\pi} \cos(2a) \text{FresnelC}\left(\frac{2\sqrt[6]{x}}{\sqrt{\pi}}\right)}{512b^{9/2}} - \frac{315\sqrt{\pi} \sin(2a) \text{FresnelS}\left(\frac{2\sqrt[6]{x}}{\sqrt{\pi}}\right)}{512b^{9/2}} - \frac{315\sqrt[6]{x} \cos^2(a + b\sqrt[3]{x})}{128b^4} - \frac{105\sqrt{x} \sin(a + b\sqrt[3]{x}) \cos(a + b\sqrt[3]{x})}{32b^3} + \frac{21x^{5/6} \cos^2(a + b\sqrt[3]{x})}{8b^2} + \frac{3x^{7/6} \sin(a + b\sqrt[3]{x}) \cos(a + b\sqrt[3]{x})}{2b} + \frac{315\sqrt[6]{x}}{256b^4} - \frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]\*Cos[a + b\*x^(1/3)]^2,x]

[Out]  $(315*x^{(1/6)})/(256*b^4) - (21*x^{(5/6)})/(16*b^2) + x^{(3/2)}/3 - (315*x^{(1/6)}*\cos[a + b*x^{(1/3)}]^2)/(128*b^4) + (21*x^{(5/6)}*\cos[a + b*x^{(1/3)}]^2)/(8*b^2) + (315*\sqrt{\text{Pi}}*\cos[2*a]*\text{FresnelC}[(2*\sqrt{\text{Pi}}*b*x^{(1/6)})/\sqrt{\text{Pi}}])/(512*b^{(9/2)}) - (315*\sqrt{\text{Pi}}*\text{FresnelS}[(2*\sqrt{\text{Pi}}*b*x^{(1/6)})/\sqrt{\text{Pi}}]*\sin[2*a])/(512*b^{(9/2)}) - (105*\sqrt{x}*\cos[a + b*x^{(1/3)}]*\sin[a + b*x^{(1/3)}])/(32*b^3) + (3*x^{(7/6)}*\cos[a + b*x^{(1/3)}]*\sin[a + b*x^{(1/3)}])/(2*b)$

**Rule 30**

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

**Rule 3385**

Int[sin[Pi/2 + (e\_) + (f\_)\*(x\_)]/Sqrt[(c\_) + (d\_)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

**Rule 3386**

Int[sin[(e\_) + (f\_)\*(x\_)]/Sqrt[(c\_) + (d\_)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}

, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

### Rule 3387

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

### Rule 3392

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[d\*m\*(c + d\*x)^(m - 1)\*((b\*Sin[e + f\*x])^n/(f^2\*n^2)), x] + (Dist[b^2\*((n - 1)/n), Int[(c + d\*x)^m\*(b\*Sin[e + f\*x])^(n - 2), x], x] - Dist[d^2\*m\*((m - 1)/(f^2\*n^2)), Int[(c + d\*x)^(m - 2)\*(b\*Sin[e + f\*x])^n, x], x] - Simp[b\*(c + d\*x)^m\*Cos[e + f\*x]\*((b\*Sin[e + f\*x])^(n - 1)/(f\*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rule 3393

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rule 3432

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

### Rule 3433

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^(2)], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

### Rule 3497

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k\*(m + 1) - 1)\*(a + b\*Cos[c + d\*x^(k\*n)])^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p] && FractionQ[n]

### Rubi steps

$$\begin{aligned}
\int \sqrt{x} \cos^2(a + b\sqrt[3]{x}) dx &= 3\text{Subst}\left(\int x^{7/2} \cos^2(a + bx) dx, x, \sqrt[3]{x}\right) \\
&= \frac{21x^{5/6} \cos^2(a + b\sqrt[3]{x})}{8b^2} + \frac{3x^{7/6} \cos(a + b\sqrt[3]{x}) \sin(a + b\sqrt[3]{x})}{2b} + \frac{3}{2}\text{Subst}\left(\int \right) \\
&= \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x} \cos^2(a + b\sqrt[3]{x})}{128b^4} + \frac{21x^{5/6} \cos^2(a + b\sqrt[3]{x})}{8b^2} - \frac{105\sqrt{x} \cos(a - } \\
&= -\frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x} \cos^2(a + b\sqrt[3]{x})}{128b^4} + \frac{21x^{5/6} \cos^2(a + b\sqrt[3]{x})}{8b^2} - \frac{105\sqrt{x} \cos(a - } \\
&= \frac{315\sqrt[6]{x}}{256b^4} - \frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x} \cos^2(a + b\sqrt[3]{x})}{128b^4} + \frac{21x^{5/6} \cos^2(a + b\sqrt[3]{x})}{8b^2} \\
&= \frac{315\sqrt[6]{x}}{256b^4} - \frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x} \cos^2(a + b\sqrt[3]{x})}{128b^4} + \frac{21x^{5/6} \cos^2(a + b\sqrt[3]{x})}{8b^2} \\
&= \frac{315\sqrt[6]{x}}{256b^4} - \frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x} \cos^2(a + b\sqrt[3]{x})}{128b^4} + \frac{21x^{5/6} \cos^2(a + b\sqrt[3]{x})}{8b^2} \\
&= \frac{315\sqrt[6]{x}}{256b^4} - \frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x} \cos^2(a + b\sqrt[3]{x})}{128b^4} + \frac{21x^{5/6} \cos^2(a + b\sqrt[3]{x})}{8b^2}
\end{aligned}$$

**Mathematica [A]**

time = 0.29, size = 148, normalized size = 0.68

$$\frac{945\sqrt{\pi} \cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[3]{x}}{\sqrt{\pi}}\right) - 945\sqrt{\pi} S\left(\frac{2\sqrt{b}\sqrt[3]{x}}{\sqrt{\pi}}\right) \sin(2a) + 2\sqrt{b}\sqrt[3]{x} (63(-15 + 16b^2x^{2/3}) \cos(2(a + b\sqrt[3]{x})) + 4b\sqrt[3]{x} (64b^3x + 9(-35 + 16b^2x^{2/3}) \sin(2(a + b\sqrt[3]{x}))))}{1536b^{9/2}}$$

Antiderivative was successfully verified.

**[In]** Integrate[Sqrt[x]\*Cos[a + b\*x^(1/3)]^2,x]

**[Out]** (945\*Sqrt[Pi]\*Cos[2\*a]\*FresnelC[(2\*Sqrt[b]\*x^(1/6))/Sqrt[Pi]] - 945\*Sqrt[Pi]\*FresnelS[(2\*Sqrt[b]\*x^(1/6))/Sqrt[Pi]]\*Sin[2\*a] + 2\*Sqrt[b]\*x^(1/6)\*(63\*(-15 + 16\*b^2\*x^(2/3))\*Cos[2\*(a + b\*x^(1/3))] + 4\*b\*x^(1/3)\*(64\*b^3\*x + 9\*(-35 + 16\*b^2\*x^(2/3))\*Sin[2\*(a + b\*x^(1/3))])))/(1536\*b^(9/2))

**Maple [A]**

time = 0.07, size = 145, normalized size = 0.67

method	result
--------	--------

derivativedivides	$\frac{x^{\frac{3}{2}}}{3} + \frac{3x^{\frac{7}{6}} \sin(2a+2bx^{\frac{1}{3}})}{4b} - \frac{\left( \frac{x^{\frac{5}{6}} \cos(2a+2bx^{\frac{1}{3}})}{4b} + \frac{5\sqrt{x} \sin(2a+2bx^{\frac{1}{3}})}{16b} - \frac{15 \left( \frac{x^{\frac{1}{6}} \cos(2a+2bx^{\frac{1}{3}})}{4b} + \frac{\sqrt{\pi} \cos(2a)}{b} \right)}{b} \right)}{4b}$
default	$\frac{x^{\frac{3}{2}}}{3} + \frac{3x^{\frac{7}{6}} \sin(2a+2bx^{\frac{1}{3}})}{4b} - \frac{\left( \frac{x^{\frac{5}{6}} \cos(2a+2bx^{\frac{1}{3}})}{4b} + \frac{5\sqrt{x} \sin(2a+2bx^{\frac{1}{3}})}{16b} - \frac{15 \left( \frac{x^{\frac{1}{6}} \cos(2a+2bx^{\frac{1}{3}})}{4b} + \frac{\sqrt{\pi} \cos(2a)}{b} \right)}{b} \right)}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*cos(a+b*x^(1/3))^2,x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{3}x^{\frac{3}{2}} + \frac{3}{4}x^{\frac{7}{6}} \sin(2a+2bx^{\frac{1}{3}}) - \frac{21}{4}x^{\frac{5}{6}} \cos(2a+2bx^{\frac{1}{3}}) + \frac{5}{4}x^{\frac{1}{2}} \sin(2a+2bx^{\frac{1}{3}}) - \frac{3}{4}x^{\frac{1}{6}} \cos(2a+2bx^{\frac{1}{3}}) + \frac{1}{8}x^{\frac{1}{2}} \pi^{\frac{1}{2}} \cos(2a) \operatorname{FresnelC}(2x^{\frac{1}{6}}) - \frac{1}{8}x^{\frac{1}{2}} \pi^{\frac{1}{2}} \sin(2a) \operatorname{FresnelS}(2x^{\frac{1}{6}})$

**Maxima** [C] Result contains complex when optimal does not.

time = 0.51, size = 137, normalized size = 0.63

$$\frac{4096b^5x^{\frac{3}{2}} + 945 \cdot 4^{\frac{1}{4}} \sqrt{2} \sqrt{\pi} \left( (-i-1) \cos(2a) - (i+1) \sin(2a) \right) \operatorname{erf}(\sqrt{2i}b^{\frac{1}{2}}x^{\frac{1}{6}}) + ((i+1) \cos(2a) + (i-1) \sin(2a)) \operatorname{erf}(\sqrt{-2i}b^{\frac{1}{2}}x^{\frac{1}{6}})}{12288b^6} + 1008(16b^4x^{\frac{5}{6}} - 15b^2x^{\frac{1}{2}}) \cos(2bx^{\frac{1}{3}} + 2a) + 576(16b^5x^{\frac{7}{6}} - 35b^3\sqrt{x}) \sin(2bx^{\frac{1}{3}} + 2a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*cos(a+b*x^(1/3))^2,x, algorithm="maxima")`

[Out]  $\frac{1}{12288} (4096b^5x^{\frac{3}{2}} + 945 \cdot 4^{\frac{1}{4}} \sqrt{2} \sqrt{\pi} \left( (-i-1) \cos(2a) - (i+1) \sin(2a) \right) \operatorname{erf}(\sqrt{2i}b^{\frac{1}{2}}x^{\frac{1}{6}}) + ((i+1) \cos(2a) + (i-1) \sin(2a)) \operatorname{erf}(\sqrt{-2i}b^{\frac{1}{2}}x^{\frac{1}{6}})) b^{\frac{3}{2}} + 1008(16b^4x^{\frac{5}{6}} - 15b^2x^{\frac{1}{2}}) \cos(2bx^{\frac{1}{3}} + 2a) + 576(16b^5x^{\frac{7}{6}} - 35b^3\sqrt{x}) \sin(2bx^{\frac{1}{3}} + 2a) / b^6$

**Fricas** [A]

time = 0.38, size = 144, normalized size = 0.66

$$\frac{512b^5x^{\frac{3}{2}} - 2016b^3x^{\frac{5}{6}} + 945\pi\sqrt{\frac{b}{\pi}} \cos(2a) C\left(2x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right) - 945\pi\sqrt{\frac{b}{\pi}} S\left(2x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right) \sin(2a) + 252(16b^3x^{\frac{5}{6}} - 15bx^{\frac{1}{2}}) \cos(bx^{\frac{1}{3}} + a) + 144(16b^4x^{\frac{7}{6}} - 35b^2\sqrt{x}) \cos(bx^{\frac{1}{3}} + a) \sin(bx^{\frac{1}{3}} + a) + 1890bx^{\frac{1}{2}}}{1536b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)\*cos(a+b\*x^(1/3))^2,x, algorithm="fricas")

[Out] 1/1536\*(512\*b^5\*x^(3/2) - 2016\*b^3\*x^(5/6) + 945\*pi\*sqrt(b/pi)\*cos(2\*a)\*fresnel\_cos(2\*x^(1/6)\*sqrt(b/pi)) - 945\*pi\*sqrt(b/pi)\*fresnel\_sin(2\*x^(1/6)\*sqrt(b/pi))\*sin(2\*a) + 252\*(16\*b^3\*x^(5/6) - 15\*b\*x^(1/6))\*cos(b\*x^(1/3) + a)^2 + 144\*(16\*b^4\*x^(7/6) - 35\*b^2\*sqrt(x))\*cos(b\*x^(1/3) + a)\*sin(b\*x^(1/3) + a) + 1890\*b\*x^(1/6))/b^5

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \cos^2(a + b\sqrt[3]{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)\*cos(a+b\*x\*\*(1/3))\*\*2,x)

[Out] Integral(sqrt(x)\*cos(a + b\*x\*\*(1/3))\*\*2, x)

**Giac** [C] Result contains complex when optimal does not.

time = 0.46, size = 176, normalized size = 0.81

$$\frac{1}{3}x^{\frac{3}{2}} - \frac{3(64ib^2x^{\frac{5}{6}} - 112b^2x^{\frac{5}{6}} - 140ib\sqrt{x} + 105x^{\frac{1}{6}})e^{(2ibx^{\frac{1}{3}} + 2ia)}}{512b^4} - \frac{3(-64ib^2x^{\frac{5}{6}} - 112b^2x^{\frac{5}{6}} + 140ib\sqrt{x} + 105x^{\frac{1}{6}})e^{(-2ibx^{\frac{1}{3}} - 2ia)}}{512b^4} - \frac{315\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b}x^{\frac{1}{6}}\left(-\frac{ib}{|b|} + 1\right)\right)e^{(2ia)}}{1024b^{\frac{5}{2}}\left(-\frac{ib}{|b|} + 1\right)} - \frac{315\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b}x^{\frac{1}{6}}\left(\frac{ib}{|b|} + 1\right)\right)e^{(-2ia)}}{1024b^{\frac{5}{2}}\left(\frac{ib}{|b|} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)\*cos(a+b\*x^(1/3))^2,x, algorithm="giac")

[Out] 1/3\*x^(3/2) - 3/512\*(64\*I\*b^3\*x^(7/6) - 112\*b^2\*x^(5/6) - 140\*I\*b\*sqrt(x) + 105\*x^(1/6))\*e^(2\*I\*b\*x^(1/3) + 2\*I\*a)/b^4 - 3/512\*(-64\*I\*b^3\*x^(7/6) - 112\*b^2\*x^(5/6) + 140\*I\*b\*sqrt(x) + 105\*x^(1/6))\*e^(-2\*I\*b\*x^(1/3) - 2\*I\*a)/b^4 - 315/1024\*sqrt(pi)\*erf(-sqrt(b)\*x^(1/6)\*(-I\*b/abs(b) + 1))\*e^(2\*I\*a)/(b^(9/2)\*(-I\*b/abs(b) + 1)) - 315/1024\*sqrt(pi)\*erf(-sqrt(b)\*x^(1/6)\*(I\*b/abs(b) + 1))\*e^(-2\*I\*a)/(b^(9/2)\*(I\*b/abs(b) + 1))

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x} \cos(a + bx^{1/3})^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)\*cos(a + b\*x^(1/3))^2,x)

[Out] int(x^(1/2)\*cos(a + b\*x^(1/3))^2, x)

$$3.57 \quad \int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} dx$$

**Optimal.** Leaf size=102

$$\sqrt{x} - \frac{3\sqrt{\pi} \cos(2a)S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{3\sqrt{\pi} \operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) \sin(2a)}{8b^{3/2}} + \frac{3\sqrt[6]{x} \sin(2(a+b\sqrt[3]{x}))}{4b}$$

[Out]  $3/4*x^{(1/6)}*\sin(2*a+2*b*x^{(1/3)})/b-3/8*\cos(2*a)*\operatorname{FresnelS}(2*x^{(1/6)}*b^{(1/2)}/\operatorname{Pi}^{(1/2)})*\operatorname{Pi}^{(1/2)}/b^{(3/2)}-3/8*\operatorname{FresnelC}(2*x^{(1/6)}*b^{(1/2)}/\operatorname{Pi}^{(1/2)})*\sin(2*a)*\operatorname{Pi}^{(1/2)}/b^{(3/2)}+x^{(1/2)}$

**Rubi [A]**

time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {3497, 3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$-\frac{3\sqrt{\pi} \sin(2a)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{3\sqrt{\pi} \cos(2a)S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}} + \frac{3\sqrt[6]{x} \sin(2(a+b\sqrt[3]{x}))}{4b} + \sqrt{x}$$

Antiderivative was successfully verified.

[In] `Int[Cos[a + b*x^(1/3)]^2/Sqrt[x], x]`

[Out] `Sqrt[x] - (3*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]])/(8*b^(3/2)) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a])/(8*b^(3/2)) + (3*x^(1/6)*Sin[2*(a + b*x^(1/3))])/(4*b)`

Rule 3377

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 3385

`Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

Rule 3386

`Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`



Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol
] := Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)]]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a + b\sqrt[3]{x})}{\sqrt{x}} dx &= 3\text{Subst}\left(\int \sqrt{x} \cos^2(a + bx) dx, x, \sqrt[3]{x}\right) \\
&= 3\text{Subst}\left(\int \left(\frac{\sqrt{x}}{2} + \frac{1}{2}\sqrt{x} \cos(2a + 2bx)\right) dx, x, \sqrt[3]{x}\right) \\
&= \sqrt{x} + \frac{3}{2}\text{Subst}\left(\int \sqrt{x} \cos(2a + 2bx) dx, x, \sqrt[3]{x}\right) \\
&= \sqrt{x} + \frac{3\sqrt[6]{x} \sin(2(a + b\sqrt[3]{x}))}{4b} - \frac{3\text{Subst}\left(\int \frac{\sin(2a+2bx)}{\sqrt{x}} dx, x, \sqrt[3]{x}\right)}{8b} \\
&= \sqrt{x} + \frac{3\sqrt[6]{x} \sin(2(a + b\sqrt[3]{x}))}{4b} - \frac{(3 \cos(2a))\text{Subst}\left(\int \frac{\sin(2bx)}{\sqrt{x}} dx, x, \sqrt[3]{x}\right)}{8b} - \frac{(3 \sin(2a))\text{Subst}\left(\int \frac{\cos(2bx)}{\sqrt{x}} dx, x, \sqrt[3]{x}\right)}{8b} \\
&= \sqrt{x} + \frac{3\sqrt[6]{x} \sin(2(a + b\sqrt[3]{x}))}{4b} - \frac{(3 \cos(2a))\text{Subst}\left(\int \sin(2bx^2) dx, x, \sqrt[6]{x}\right)}{4b} - \frac{(3 \sin(2a))\text{Subst}\left(\int \cos(2bx^2) dx, x, \sqrt[6]{x}\right)}{4b} \\
&= \sqrt{x} - \frac{3\sqrt{\pi} \cos(2a)S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{3\sqrt{\pi} C\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) \sin(2a)}{8b^{3/2}} + \frac{3\sqrt[6]{x} \sin(2(a + b\sqrt[3]{x}))}{4b}
\end{aligned}$$

**Mathematica [A]**

time = 0.12, size = 103, normalized size = 1.01

$$\frac{-3\sqrt{\pi} \cos(2a)S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) - 3\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) \sin(2a) + 2\sqrt{b}\sqrt[6]{x} (4b\sqrt[3]{x} + 3 \sin(2(a + b\sqrt[3]{x})))}{8b^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*x^(1/3)]^2/Sqrt[x], x]`

```
[Out] (-3*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] - 3*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 2*Sqrt[b]*x^(1/6)*(4*b*x^(1/3) + 3*Sin[2*(a + b*x^(1/3))]))/(8*b^(3/2))
```

**Maple [A]**

time = 0.07, size = 67, normalized size = 0.66

method	result	size
derivativedivides	$ \sqrt{x} + \frac{3x^{\frac{1}{6}} \sin(2a + 2bx^{\frac{1}{3}})}{4b} - \frac{3\sqrt{\pi} \left( \cos(2a)S\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right) + \sin(2a) \text{FresnelC}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right) \right)}{8b^{\frac{3}{2}}} $	67

default	$\sqrt{x} + \frac{3x^{\frac{1}{6}} \sin(2a+2bx^{\frac{1}{3}})}{4b} - \frac{3\sqrt{\pi} \left( \cos(2a)S\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right) + \sin(2a) \operatorname{FresnelC}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right) \right)}{8b^{\frac{3}{2}}}$	67
---------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*x^(1/3))^2/x^(1/2),x,method=_RETURNVERBOSE)`

[Out] `x^(1/2)+3/4*x^(1/6)*sin(2*a+2*b*x^(1/3))/b-3/8/b^(3/2)*Pi^(1/2)*(cos(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2)))`

**Maxima** [C] Result contains complex when optimal does not.

time = 0.50, size = 96, normalized size = 0.94

$$\frac{3 \cdot 4^{\frac{1}{2}} \sqrt{\pi} \left( (i+1) \cos(2a) - (i-1) \sin(2a) \operatorname{erf}\left(\sqrt{2i} b^{\frac{1}{6}} x^{\frac{1}{6}}\right) + (-(i-1) \cos(2a) + (i+1) \sin(2a)) \operatorname{erf}\left(\sqrt{-2i} b^{\frac{1}{6}} x^{\frac{1}{6}}\right) \right) b^{\frac{3}{2}} - 64 b^3 \sqrt{x} - 48 b^2 x^{\frac{1}{6}} \sin\left(2 b x^{\frac{1}{3}} + 2a\right)}{64 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))^2/x^(1/2),x, algorithm="maxima")`

[Out] `-1/64*(3*4^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*cos(2*a) - (I - 1)*sin(2*a))*erf(sqrt(2*I*b)*x^(1/6)) + (-(I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf(sqrt(-2*I*b)*x^(1/6)))*b^(3/2) - 64*b^3*sqrt(x) - 48*b^2*x^(1/6)*sin(2*b*x^(1/3) + 2*a))/b^3`

**Fricas** [A]

time = 0.38, size = 90, normalized size = 0.88

$$\frac{3\pi \sqrt{\frac{b}{\pi}} \cos(2a) S\left(2x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}}\right) + 3\pi \sqrt{\frac{b}{\pi}} C\left(2x^{\frac{1}{6}} \sqrt{\frac{b}{\pi}}\right) \sin(2a) - 12bx^{\frac{1}{6}} \cos\left(bx^{\frac{1}{3}} + a\right) \sin\left(bx^{\frac{1}{3}} + a\right) - 8b^2 \sqrt{x}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))^2/x^(1/2),x, algorithm="fricas")`

[Out] `-1/8*(3*pi*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x^(1/6)*sqrt(b/pi)) + 3*pi*sqrt(b/pi)*fresnel_cos(2*x^(1/6)*sqrt(b/pi))*sin(2*a) - 12*b*x^(1/6)*cos(b*x^(1/3) + a)*sin(b*x^(1/3) + a) - 8*b^2*sqrt(x))/b^2`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a + b\sqrt[3]{x})}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x**(1/3))**2/x**(1/2),x)`

[Out] Integral(cos(a + b\*x\*\*(1/3))\*\*2/sqrt(x), x)

**Giac** [C] Result contains complex when optimal does not.  
time = 0.44, size = 124, normalized size = 1.22

$$\sqrt{x} - \frac{3i x^{\frac{1}{6}} e^{(2i b x^{\frac{1}{3}} + 2i a)}}{8b} + \frac{3i x^{\frac{1}{6}} e^{(-2i b x^{\frac{1}{3}} - 2i a)}}{8b} - \frac{3i \sqrt{\pi} \operatorname{erf}\left(-\sqrt{b} x^{\frac{1}{6}} \left(-\frac{i b}{|b|} + 1\right)\right) e^{(2i a)}}{16 b^{\frac{3}{2}} \left(-\frac{i b}{|b|} + 1\right)} + \frac{3i \sqrt{\pi} \operatorname{erf}\left(-\sqrt{b} x^{\frac{1}{6}} \left(\frac{i b}{|b|} + 1\right)\right) e^{(-2i a)}}{16 b^{\frac{3}{2}} \left(\frac{i b}{|b|} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^(1/3))^2/x^(1/2),x, algorithm="giac")

[Out] sqrt(x) - 3/8\*I\*x^(1/6)\*e^(2\*I\*b\*x^(1/3) + 2\*I\*a)/b + 3/8\*I\*x^(1/6)\*e^(-2\*I\*b\*x^(1/3) - 2\*I\*a)/b - 3/16\*I\*sqrt(pi)\*erf(-sqrt(b)\*x^(1/6)\*(-I\*b/abs(b) + 1))\*e^(2\*I\*a)/(b^(3/2)\*(-I\*b/abs(b) + 1)) + 3/16\*I\*sqrt(pi)\*erf(-sqrt(b)\*x^(1/6)\*(I\*b/abs(b) + 1))\*e^(-2\*I\*a)/(b^(3/2)\*(I\*b/abs(b) + 1))

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + b x^{1/3})^2}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^(1/3))^2/x^(1/2),x)

[Out] int(cos(a + b\*x^(1/3))^2/x^(1/2), x)

$$3.58 \quad \int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}} dx$$

**Optimal.** Leaf size=116

$$-\frac{2 \cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} - 8b^{3/2} \sqrt{\pi} \cos(2a) \operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) + 8b^{3/2} \sqrt{\pi} S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) \sin(2a) + \frac{8b \cos(a)}{\sqrt[6]{x}}$$

[Out] 8\*b\*cos(a+b\*x^(1/3))\*sin(a+b\*x^(1/3))/x^(1/6)-8\*b^(3/2)\*cos(2\*a)\*FresnelC(2\*x^(1/6)\*b^(1/2)/Pi^(1/2))\*Pi^(1/2)+8\*b^(3/2)\*FresnelS(2\*x^(1/6)\*b^(1/2)/Pi^(1/2))\*sin(2\*a)\*Pi^(1/2)-2\*cos(a+b\*x^(1/3))^2/x^(1/2)

**Rubi [A]**

time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3497, 3395, 30, 3393, 3387, 3386, 3432, 3385, 3433}

$$-8\sqrt{\pi} b^{3/2} \cos(2a) \operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) + 8\sqrt{\pi} b^{3/2} \sin(2a) S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) - \frac{2 \cos^2(a+b\sqrt[3]{x})}{\sqrt{x}} + \frac{8b \sin(a+b\sqrt[3]{x}) \cos(a+b\sqrt[3]{x})}{\sqrt[6]{x}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^(1/3)]^2/x^(3/2), x]

[Out] (-2\*Cos[a + b\*x^(1/3)]^2)/Sqrt[x] - 8\*b^(3/2)\*Sqrt[Pi]\*Cos[2\*a]\*FresnelC[(2\*Sqrt[b]\*x^(1/6))/Sqrt[Pi]] + 8\*b^(3/2)\*Sqrt[Pi]\*FresnelS[(2\*Sqrt[b]\*x^(1/6))/Sqrt[Pi]]\*Sin[2\*a] + (8\*b\*Cos[a + b\*x^(1/3)]\*Sin[a + b\*x^(1/3)])/x^(1/6)

**Rule 30**

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

**Rule 3385**

Int[sin[Pi/2 + (e\_) + (f\_)\*(x\_)]/Sqrt[(c\_) + (d\_)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

**Rule 3386**

Int[sin[(e\_) + (f\_)\*(x\_)]/Sqrt[(c\_) + (d\_)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

**Rule 3387**

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

### Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] := Simp[(c + d*x)^(m + 1)*((b*SIN[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*SIN[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*SIN[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*SIN[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

### Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

### Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

### Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)]^(n_)]*(b_.)^(p_.)*(x_)^m_, x_Symbol
] := Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*COS[c + d*x^(k*n)]]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a + b\sqrt[3]{x})}{x^{3/2}} dx &= 3\text{Subst}\left(\int \frac{\cos^2(a + bx)}{x^{5/2}} dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos^2(a + b\sqrt[3]{x})}{\sqrt{x}} + \frac{8b\cos(a + b\sqrt[3]{x})\sin(a + b\sqrt[3]{x})}{\sqrt[6]{x}} + (8b^2)\text{Subst}\left(\int \frac{1}{\sqrt{x}} a\right) \\
&= 16b^2\sqrt[6]{x} - \frac{2\cos^2(a + b\sqrt[3]{x})}{\sqrt{x}} + \frac{8b\cos(a + b\sqrt[3]{x})\sin(a + b\sqrt[3]{x})}{\sqrt[6]{x}} - (16b^2)\text{Subst}\left(\int \frac{1}{\sqrt{x}} a\right) \\
&= -\frac{2\cos^2(a + b\sqrt[3]{x})}{\sqrt{x}} + \frac{8b\cos(a + b\sqrt[3]{x})\sin(a + b\sqrt[3]{x})}{\sqrt[6]{x}} - (8b^2)\text{Subst}\left(\int \frac{\cos(2a)}{\sqrt{x}}\right) \\
&= -\frac{2\cos^2(a + b\sqrt[3]{x})}{\sqrt{x}} + \frac{8b\cos(a + b\sqrt[3]{x})\sin(a + b\sqrt[3]{x})}{\sqrt[6]{x}} - (8b^2\cos(2a))\text{Subst}\left(\int \frac{1}{\sqrt{x}}\right) \\
&= -\frac{2\cos^2(a + b\sqrt[3]{x})}{\sqrt{x}} + \frac{8b\cos(a + b\sqrt[3]{x})\sin(a + b\sqrt[3]{x})}{\sqrt[6]{x}} - (16b^2\cos(2a))\text{Subst}\left(\int \frac{1}{\sqrt{x}}\right) \\
&= -\frac{2\cos^2(a + b\sqrt[3]{x})}{\sqrt{x}} - 8b^{3/2}\sqrt{\pi}\cos(2a)C\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) + 8b^{3/2}\sqrt{\pi}S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)
\end{aligned}$$

**Mathematica [A]**

time = 0.17, size = 116, normalized size = 1.00

$$-8b^{3/2}\sqrt{\pi}\cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) + \frac{-1 - \cos(2(a + b\sqrt[3]{x})) + 8b^{3/2}\sqrt{\pi}\sqrt{x}S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a) + 4b\sqrt[3]{x}\sin(2(a + b\sqrt[3]{x}))}{\sqrt{x}}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*x^(1/3)]^2/x^(3/2), x]`

```
[Out] -8*b^(3/2)*Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] + (-1 - Cos[2*(a + b*x^(1/3))]) + 8*b^(3/2)*Sqrt[Pi]*Sqrt[x]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 4*b*x^(1/3)*Sin[2*(a + b*x^(1/3))]/Sqrt[x]
```

**Maple [A]**

time = 0.07, size = 87, normalized size = 0.75

method	result
derivativedivides	$-\frac{1}{\sqrt{x}} - \frac{\cos(2a+2bx^{\frac{1}{3}})}{\sqrt{x}} - 4b\left(-\frac{\sin(2a+2bx^{\frac{1}{3}})}{x^{\frac{1}{6}}} + 2\sqrt{b}\sqrt{\pi}\left(\cos(2a)\text{FresnelC}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)\right)\right)$
default	$-\frac{1}{\sqrt{x}} - \frac{\cos(2a+2bx^{\frac{1}{3}})}{\sqrt{x}} - 4b\left(-\frac{\sin(2a+2bx^{\frac{1}{3}})}{x^{\frac{1}{6}}} + 2\sqrt{b}\sqrt{\pi}\left(\cos(2a)\text{FresnelC}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*x^(1/3))^2/x^(3/2),x,method=_RETURNVERBOSE)`

[Out]  $-1/x^{1/2}-1/x^{1/2}*\cos(2*a+2*b*x^{1/3})-4*b*(-1/x^{1/6}*\sin(2*a+2*b*x^{1/3}))+2*b^{1/2}*Pi^{1/2}*(\cos(2*a)*\text{FresnelC}(2*x^{1/6}*b^{1/2}/Pi^{1/2})-\sin(2*a)*\text{FresnelS}(2*x^{1/6}*b^{1/2}/Pi^{1/2}))$

**Maxima** [C] Result contains complex when optimal does not.  
time = 0.57, size = 88, normalized size = 0.76

$$\frac{3\sqrt{2}\left(\left(-i-1\right)\sqrt{2}\Gamma\left(-\frac{3}{2},2i bx^{\frac{1}{3}}\right)+\left(i+1\right)\sqrt{2}\Gamma\left(-\frac{3}{2},-2i bx^{\frac{1}{3}}\right)\right)\cos(2a)+\left(-i+1\right)\sqrt{2}\Gamma\left(-\frac{3}{2},2i bx^{\frac{1}{3}}\right)+\left(i-1\right)\sqrt{2}\Gamma\left(-\frac{3}{2},-2i bx^{\frac{1}{3}}\right)\sin(2a)\sqrt{bx^{\frac{1}{3}}bx^{\frac{1}{3}}-4}}{4\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))^2/x^(3/2),x, algorithm="maxima")`

[Out]  $1/4*(3*\sqrt{2}*((-I-1)*\sqrt{2}*\gamma(-3/2,2*I*b*x^{1/3})+(I+1)*\sqrt{2}*\gamma(-3/2,-2*I*b*x^{1/3}))*\cos(2*a)+(-I+1)*\sqrt{2}*\gamma(-3/2,2*I*b*x^{1/3})+(I-1)*\sqrt{2}*\gamma(-3/2,-2*I*b*x^{1/3}))*\sin(2*a))*\sqrt{b*x^{1/3}}*b*x^{1/3}-4)/\sqrt{x}$

**Fricas** [A]

time = 0.38, size = 100, normalized size = 0.86

$$\frac{2\left(4\pi bx\sqrt{\frac{b}{\pi}}\cos(2a)C\left(2x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)-4\pi bx\sqrt{\frac{b}{\pi}}S\left(2x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin(2a)-4bx^{\frac{5}{6}}\cos\left(bx^{\frac{1}{3}}+a\right)\sin\left(bx^{\frac{1}{3}}+a\right)+\sqrt{x}\cos\left(bx^{\frac{1}{3}}+a\right)^2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))^2/x^(3/2),x, algorithm="fricas")`

[Out]  $-2*(4*pi*b*x*\sqrt{b/pi}*\cos(2*a)*\text{fresnel\_cos}(2*x^{1/6}*\sqrt{b/pi})-4*pi*b*x*\sqrt{b/pi}*\text{fresnel\_sin}(2*x^{1/6}*\sqrt{b/pi}))*\sin(2*a)-4*b*x^{5/6}*\cos(b*x^{1/3}+a)*\sin(b*x^{1/3}+a)+\sqrt{x}*\cos(b*x^{1/3}+a)^2/x$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+b\sqrt[3]{x})}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x**(1/3))**2/x**(3/2),x)`

[Out] `Integral(cos(a + b*x**(1/3))**2/x**(3/2), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)^2/x^(3/2), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + b x^{1/3})^2}{x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^(1/3))^2/x^(3/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))^2/x^(3/2), x)
```

$$3.59 \quad \int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}} dx$$

**Optimal.** Leaf size=228

$$-\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2\cos^2(a+b\sqrt[3]{x})}{3x^{3/2}} + \frac{32b^2\cos^2(a+b\sqrt[3]{x})}{105x^{5/6}} - \frac{512b^4\cos^2(a+b\sqrt[3]{x})}{315\sqrt[6]{x}} - \frac{512}{315}b^{9/2}\sqrt{\pi}\cos(2a)$$

[Out]  $-16/105*b^2/x^{(5/6)}+256/315*b^4/x^{(1/6)}-2/3*\cos(a+b*x^{(1/3)})^2/x^{(3/2)}+32/105*b^2*\cos(a+b*x^{(1/3)})^2/x^{(5/6)}-512/315*b^4*\cos(a+b*x^{(1/3)})^2/x^{(1/6)}+8/21*b*\cos(a+b*x^{(1/3)})*\sin(a+b*x^{(1/3)})/x^{(7/6)}-512/315*b^{(9/2)}*\cos(2*a)*\text{FresnelS}(2*x^{(1/6)}*b^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}-512/315*b^{(9/2)}*\text{FresnelC}(2*x^{(1/6)}*b^{(1/2)}/\text{Pi}^{(1/2)})*\sin(2*a)*\text{Pi}^{(1/2)}-128/315*b^3*\cos(a+b*x^{(1/3)})*\sin(a+b*x^{(1/3)})/x^{(1/2)}$

**Rubi [A]**

time = 0.17, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {3497, 3395, 30, 3394, 12, 3387, 3386, 3432, 3385, 3433}

$$-\frac{512}{315}\sqrt{\pi}b^{9/2}\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)-\frac{512}{315}\sqrt{\pi}b^{9/2}\cos(2a)\text{S}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)-\frac{512b^4\cos^2(a+b\sqrt[3]{x})}{315\sqrt[6]{x}}-\frac{128b^3\sin(a+b\sqrt[3]{x})\cos(a+b\sqrt[3]{x})}{315\sqrt[6]{x}}+\frac{32b^2\cos^2(a+b\sqrt[3]{x})}{105x^{5/6}}-\frac{2\cos^2(a+b\sqrt[3]{x})}{3x^{3/2}}+\frac{8b\sin(a+b\sqrt[3]{x})\cos(a+b\sqrt[3]{x})}{21x^{7/6}}+\frac{256b^4}{315\sqrt[6]{x}}-\frac{16b^2}{105x^{5/6}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^(1/3)]^2/x^(5/2), x]

[Out]  $(-16*b^2)/(105*x^{(5/6)}) + (256*b^4)/(315*x^{(1/6)}) - (2*\text{Cos}[a + b*x^{(1/3)}]^2)/(3*x^{(3/2)}) + (32*b^2*\text{Cos}[a + b*x^{(1/3)}]^2)/(105*x^{(5/6)}) - (512*b^4*\text{Cos}[a + b*x^{(1/3)}]^2)/(315*x^{(1/6)}) - (512*b^{(9/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a]*\text{FresnelS}[(2*\text{Sqrt}[b]*x^{(1/6)})/\text{Sqrt}[\text{Pi}]])/315 - (512*b^{(9/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelC}[(2*\text{Sqrt}[b]*x^{(1/6)})/\text{Sqrt}[\text{Pi}]]*\text{Sin}[2*a])/315 + (8*b*\text{Cos}[a + b*x^{(1/3)}]*\text{Sin}[a + b*x^{(1/3)}])/(21*x^{(7/6)}) - (128*b^3*\text{Cos}[a + b*x^{(1/3)}]*\text{Sin}[a + b*x^{(1/3)}])/(315*\text{Sqrt}[x])$

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3385

Int[sin[Pi/2 + (e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Cos[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d

, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3386

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[Sin[f\*(x^2/d)], x], x, Sqrt[c + d\*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d\*e - c\*f, 0]

#### Rule 3387

Int[sin[(e\_.) + (f\_.)\*(x\_)]/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[c\*(f/d) + f\*x]/Sqrt[c + d\*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d\*e - c\*f, 0]

#### Rule 3394

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*(Sin[e + f\*x]^n/(d\*(m + 1))), x] - Dist[f\*(n/(d\*(m + 1))), Int[ExpandTrigReduce[(c + d\*x)^(m + 1), Cos[e + f\*x]\*Sin[e + f\*x]^(n - 1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

#### Rule 3395

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*((b\*Sin[e + f\*x])^n/(d\*(m + 1))), x] + (Dist[b^2\*f^2\*n\*((n - 1)/(d^2\*(m + 1)\*(m + 2))), Int[(c + d\*x)^(m + 2)\*(b\*Sin[e + f\*x])^(n - 2), x], x] - Dist[f^2\*(n^2/(d^2\*(m + 1)\*(m + 2))), Int[(c + d\*x)^(m + 2)\*(b\*Sin[e + f\*x])^n, x], x] - Simp[b\*f\*n\*(c + d\*x)^(m + 2)\*Cos[e + f\*x]\*((b\*Sin[e + f\*x])^(n - 1)/(d^2\*(m + 1)\*(m + 2))), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

#### Rule 3432

Int[Sin[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelS[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

#### Rule 3433

Int[Cos[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^2], x\_Symbol] := Simp[(Sqrt[Pi/2]/(f\*Rt[d, 2]))\*FresnelC[Sqrt[2/Pi]\*Rt[d, 2]\*(e + f\*x)], x] /; FreeQ[{d, e, f}, x]

#### Rule 3497

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)]^(n\_)]\*(b\_.)^(p\_.)\*(x\_)^m, x\_Symbol] := Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k\*(m + 1) - 1)\*(a +

$b \cdot \text{Cos}[c + d \cdot x^{(k \cdot n)}]^{p, x], x, x^{(1/k)}, x]] /;$  FreeQ[{a, b, c, d, m}, x]  
&& IntegerQ[p] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a + b\sqrt[3]{x})}{x^{5/2}} dx &= 3 \text{Subst} \left( \int \frac{\cos^2(a + bx)}{x^{11/2}} dx, x, \sqrt[3]{x} \right) \\ &= -\frac{2 \cos^2(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{8b \cos(a + b\sqrt[3]{x}) \sin(a + b\sqrt[3]{x})}{21x^{7/6}} + \frac{1}{21} (8b^2) \text{Subst} \left( \int \frac{1}{x^{7/2}} \right) \\ &= -\frac{16b^2}{105x^{5/6}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{105x^{5/6}} + \frac{8b \cos(a + b\sqrt[3]{x}) \sin(a + b\sqrt[3]{x})}{21x^{7/6}} \\ &= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{105x^{5/6}} - \frac{512b^4 \cos^2(a + b\sqrt[3]{x})}{315\sqrt[6]{x}} \\ &= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{105x^{5/6}} - \frac{512b^4 \cos^2(a + b\sqrt[3]{x})}{315\sqrt[6]{x}} \\ &= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{105x^{5/6}} - \frac{512b^4 \cos^2(a + b\sqrt[3]{x})}{315\sqrt[6]{x}} \\ &= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{105x^{5/6}} - \frac{512b^4 \cos^2(a + b\sqrt[3]{x})}{315\sqrt[6]{x}} \\ &= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{3x^{3/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{105x^{5/6}} - \frac{512b^4 \cos^2(a + b\sqrt[3]{x})}{315\sqrt[6]{x}} \end{aligned}$$

**Mathematica [A]**

time = 0.18, size = 185, normalized size = 0.81

$$\frac{-105 - 105 \cos(2(a + b\sqrt[3]{x})) + 48b^2 x^{2/3} \cos(2(a + b\sqrt[3]{x})) - 256b^4 x^{4/3} \cos(2(a + b\sqrt[3]{x})) - 512b^{9/2} \sqrt{\pi} x^{3/2} \cos(2a) S\left(\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\pi}}\right) - 512b^{9/2} \sqrt{\pi} x^{3/2} \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\pi}}\right) \sin(2a) + 60b\sqrt[3]{x} \sin(2(a + b\sqrt[3]{x})) - 64b^3 x \sin(2(a + b\sqrt[3]{x}))}{315x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*x^(1/3)]^2/x^(5/2), x]

[Out] (-105 - 105\*Cos[2\*(a + b\*x^(1/3))] + 48\*b^2\*x^(2/3)\*Cos[2\*(a + b\*x^(1/3))] - 256\*b^4\*x^(4/3)\*Cos[2\*(a + b\*x^(1/3))] - 512\*b^(9/2)\*Sqrt[Pi]\*x^(3/2)\*Cos[2\*a]\*FresnelS[(2\*Sqrt[b]\*x^(1/6))/Sqrt[Pi]] - 512\*b^(9/2)\*Sqrt[Pi]\*x^(3/2)\*FresnelC[(2\*Sqrt[b]\*x^(1/6))/Sqrt[Pi]]\*Sin[2\*a] + 60\*b\*x^(1/3)\*Sin[2\*(a + b\*x^(1/3))] - 64\*b^3\*x\*Sin[2\*(a + b\*x^(1/3))])/(315\*x^(3/2))

**Maple [A]**

time = 0.07, size = 146, normalized size = 0.64

method	result
derivativedivides	$-\frac{1}{3x^{\frac{3}{2}}} - \frac{\cos\left(2a+2bx^{\frac{1}{3}}\right)}{3x^{\frac{3}{2}}} - \left( 4b \frac{\sin\left(2a+2bx^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}} + \left( 4b \frac{\cos\left(2a+2bx^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}} - \left( 4b \frac{\sin\left(2a+2bx^{\frac{1}{3}}\right)}{3\sqrt{x}} + \left( 4b \frac{\cos\left(2a+2bx^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}} \right) \right) \right)$
default	$-\frac{1}{3x^{\frac{3}{2}}} - \frac{\cos\left(2a+2bx^{\frac{1}{3}}\right)}{3x^{\frac{3}{2}}} - \left( 4b \frac{\sin\left(2a+2bx^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}} + \left( 4b \frac{\cos\left(2a+2bx^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}} - \left( 4b \frac{\sin\left(2a+2bx^{\frac{1}{3}}\right)}{3\sqrt{x}} + \left( 4b \frac{\cos\left(2a+2bx^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}} \right) \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*x^(1/3))^2/x^(5/2),x,method=_RETURNVERBOSE)`

[Out] 
$$-1/3/x^{3/2}-1/3/x^{3/2}*\cos(2*a+2*b*x^{1/3})-4/3*b*(-1/7/x^{7/6}*\sin(2*a+2*b*x^{1/3}))+4/7*b*(-1/5/x^{5/6}*\cos(2*a+2*b*x^{1/3}))-4/5*b*(-1/3/x^{1/2}*\sin(2*a+2*b*x^{1/3}))+4/3*b*(-1/x^{1/6}*\cos(2*a+2*b*x^{1/3}))-2*b^{1/2}*Pi^{1/2}*(\cos(2*a)*\text{FresnelS}(2*x^{1/6}*b^{1/2}/Pi^{1/2})+\sin(2*a)*\text{FresnelC}(2*x^{1/6}*b^{1/2}/Pi^{1/2}))$$

**Maxima** [C] Result contains complex when optimal does not.  
time = 0.58, size = 90, normalized size = 0.39

$$\frac{18\sqrt{2}\left(\left((i+1)\sqrt{2}\Gamma\left(-\frac{9}{2},2ibx^{\frac{1}{3}}\right)-(i-1)\sqrt{2}\Gamma\left(-\frac{9}{2},-2ibx^{\frac{1}{3}}\right)\right)\cos(2a)+\left(-i-1\right)\sqrt{2}\Gamma\left(-\frac{9}{2},2ibx^{\frac{1}{3}}\right)+(i+1)\sqrt{2}\Gamma\left(-\frac{9}{2},-2ibx^{\frac{1}{3}}\right)\right)\sin(2a)\sqrt{bx^{\frac{1}{3}}b^4x^{\frac{4}{3}}+1}}{3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))^2/x^(5/2),x, algorithm="maxima")`

[Out] 
$$-1/3*(18*\sqrt{2})*(((I+1)*\sqrt{2})*\gamma(-9/2,2*I*b*x^{1/3})-(I-1)*\sqrt{2})*\gamma(-9/2,-2*I*b*x^{1/3}))*\cos(2*a)+(-(I-1)*\sqrt{2})*\gamma(-9/2,2*I*b*x^{1/3})+(I+1)*\sqrt{2})*\gamma(-9/2,-2*I*b*x^{1/3}))*\sin(2*a)*\sqrt{2}*(b*x^{1/3})^4*x^{4/3}+1)/x^{3/2}$$

**Fricas** [A]

time = 0.37, size = 154, normalized size = 0.68

$$\frac{2\left(256\pi b^4x^2\sqrt{\frac{b}{\pi}}\cos(2a)\text{S}\left(2x^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)+256\pi b^4x^2\sqrt{\frac{b}{\pi}}\text{C}\left(2x^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)\sin(2a)-128b^4x^{\frac{11}{6}}+24b^2x^{\frac{7}{6}}+(256b^4x^{\frac{11}{6}}-48b^2x^{\frac{7}{6}}+105\sqrt{x})\cos(bx^{\frac{1}{3}}+a)^2+4(16b^3x^{\frac{3}{2}}-15bx^{\frac{5}{6}})\cos(bx^{\frac{1}{3}}+a)\sin(bx^{\frac{1}{3}}+a)\right)}{315x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))^2/x^(5/2),x, algorithm="fricas")`

[Out] 
$$-2/315*(256*\pi*b^4*x^2*\sqrt{b/\pi}*\cos(2*a)*\text{fresnel\_sin}(2*x^{1/6}*\sqrt{b/\pi})+256*\pi*b^4*x^2*\sqrt{b/\pi}*\text{fresnel\_cos}(2*x^{1/6}*\sqrt{b/\pi}))*\sin(2*a)-128*b^4*x^{11/6}+24*b^2*x^{7/6}+(256*b^4*x^{11/6}-48*b^2*x^{7/6}+105*\sqrt{x})*\cos(b*x^{1/3}+a)^2+4*(16*b^3*x^{3/2}-15*b*x^{5/6})*\cos(b*x^{1/3}+a)*\sin(b*x^{1/3}+a))/x^2$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+b\sqrt[3]{x})}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x**(1/3))**2/x**(5/2),x)`

[Out] Integral(cos(a + b\*x\*\*(1/3))\*\*2/x\*\*(5/2), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^(1/3))^2/x^(5/2),x, algorithm="giac")

[Out] integrate(cos(b\*x^(1/3) + a)^2/x^(5/2), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a + b x^{1/3})^2}{x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^(1/3))^2/x^(5/2),x)

[Out] int(cos(a + b\*x^(1/3))^2/x^(5/2), x)

$$3.60 \quad \int \frac{\cos^2(a + b\sqrt[3]{x})}{x^{7/2}} dx$$

Optimal. Leaf size=328

$$-\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2\cos^2(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{32b^2\cos^2(a + b\sqrt[3]{x})}{715x^{11/6}} - \frac{512b^4\cos^2(a + b\sqrt[3]{x})}{45045x^{7/6}} + \frac{8192b^6\cos^2(a + b\sqrt[3]{x})}{675675\sqrt{x}}$$

[Out]  $-16/715*b^2/x^{(11/6)}+256/45045*b^4/x^{(7/6)}-2/5*\cos(a+b*x^{(1/3)})^2/x^{(5/2)}+2/715*b^2*\cos(a+b*x^{(1/3)})^2/x^{(11/6)}-512/45045*b^4*\cos(a+b*x^{(1/3)})^2/x^{(7/6)}+8/65*b*\cos(a+b*x^{(1/3)})*\sin(a+b*x^{(1/3)})/x^{(13/6)}-128/6435*b^3*\cos(a+b*x^{(1/3)})*\sin(a+b*x^{(1/3)})/x^{(3/2)}+2048/225225*b^5*\cos(a+b*x^{(1/3)})*\sin(a+b*x^{(1/3)})/x^{(5/6)}-32768/675675*b^7*\cos(a+b*x^{(1/3)})*\sin(a+b*x^{(1/3)})/x^{(1/6)}+32768/675675*b^{(15/2)}*\cos(2*a)*\text{FresnelC}(2*x^{(1/6)}*b^{(1/2)}/\text{Pi}^{(1/2)})*\text{Pi}^{(1/2)}-32768/675675*b^{(15/2)}*\text{FresnelS}(2*x^{(1/6)}*b^{(1/2)}/\text{Pi}^{(1/2)})*\sin(2*a)*\text{Pi}^{(1/2)}-4096/675675*b^6/x^{(1/2)}+8192/675675*b^6*\cos(a+b*x^{(1/3)})^2/x^{(1/2)}$

Rubi [A]

time = 0.24, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3497, 3395, 30, 3393, 3387, 3386, 3432, 3385, 3433}

$$\frac{32768\sqrt{a}\sqrt{b}\sqrt{x}\cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\pi}}\right)}{675675} - \frac{32768\sqrt{a}\sqrt{b}\sqrt{x}\sin(2a)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\pi}}\right)}{675675} - \frac{32768b^7\sin(a+b\sqrt[3]{x})\cos(a+b\sqrt[3]{x})}{675675\sqrt{x}} - \frac{8192b^6\cos^2(a+b\sqrt[3]{x})}{675675\sqrt{x}} - \frac{2048b^5\sin(a+b\sqrt[3]{x})\cos(a+b\sqrt[3]{x})}{225225x^{5/6}} - \frac{512b^4\cos^2(a+b\sqrt[3]{x})}{45045x^{7/6}} - \frac{128b^3\sin(a+b\sqrt[3]{x})\cos(a+b\sqrt[3]{x})}{6435x^{3/2}} - \frac{2048b^2\cos^2(a+b\sqrt[3]{x})}{715x^{11/6}} - \frac{2048b\cos(a+b\sqrt[3]{x})\sin(a+b\sqrt[3]{x})}{6435x^{13/6}} - \frac{8192\sin(a+b\sqrt[3]{x})\cos(a+b\sqrt[3]{x})}{675675\sqrt{x}} - \frac{8192b^6\cos^2(a+b\sqrt[3]{x})}{675675\sqrt{x}} - \frac{16b^2}{715x^{11/6}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^(1/3)]^2/x^(7/2), x]

[Out]  $(-16*b^2)/(715*x^{(11/6)}) + (256*b^4)/(45045*x^{(7/6)}) - (4096*b^6)/(675675*\text{qrt}[x]) - (2*\text{Cos}[a + b*x^{(1/3)}]^2)/(5*x^{(5/2)}) + (32*b^2*\text{Cos}[a + b*x^{(1/3)}]^2)/(715*x^{(11/6)}) - (512*b^4*\text{Cos}[a + b*x^{(1/3)}]^2)/(45045*x^{(7/6)}) + (8192*b^6*\text{Cos}[a + b*x^{(1/3)}]^2)/(675675*\text{Sqrt}[x]) + (32768*b^{(15/2)}*\text{Sqrt}[\text{Pi}]*\text{Cos}[2*a]*\text{FresnelC}[(2*\text{Sqrt}[b]*x^{(1/6)})/\text{Sqrt}[\text{Pi}]])/675675 - (32768*b^{(15/2)}*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[b]*x^{(1/6)})/\text{Sqrt}[\text{Pi}])*Sin[2*a])/675675 + (8*b*\text{Cos}[a + b*x^{(1/3)}]*Sin[a + b*x^{(1/3)}])/(65*x^{(13/6)}) - (128*b^3*\text{Cos}[a + b*x^{(1/3)}]*Sin[a + b*x^{(1/3)}])/(6435*x^{(3/2)}) + (2048*b^5*\text{Cos}[a + b*x^{(1/3)}]*Sin[a + b*x^{(1/3)}])/(225225*x^{(5/6)}) - (32768*b^7*\text{Cos}[a + b*x^{(1/3)}]*Sin[a + b*x^{(1/3)}])/(675675*x^{(1/6)})$

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3385



```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

#### Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

#### Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

#### Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

#### Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

#### Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

#### Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

#### Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
```

b\*Cos[c + d\*x^(k\*n)]^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]  
 && IntegerQ[p] && FractionQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^2(a + b\sqrt[3]{x})}{x^{7/2}} dx &= 3 \text{Subst} \left( \int \frac{\cos^2(a + bx)}{x^{17/2}} dx, x, \sqrt[3]{x} \right) \\
 &= -\frac{2 \cos^2(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{8b \cos(a + b\sqrt[3]{x}) \sin(a + b\sqrt[3]{x})}{65x^{13/6}} + \frac{1}{65} (8b^2) \text{Subst} \left( \int \frac{1}{x^{13/2}} dx, x, \sqrt[3]{x} \right) \\
 &= -\frac{16b^2}{715x^{11/6}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{715x^{11/6}} + \frac{8b \cos(a + b\sqrt[3]{x}) \sin(a + b\sqrt[3]{x})}{65x^{13/6}} \\
 &= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{715x^{11/6}} - \frac{512b^4 \cos^2(a + b\sqrt[3]{x})}{45045x^{7/6}} \\
 &= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{715x^{11/6}} \\
 &= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{65536b^8 \sqrt[6]{x}}{675675} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{715x^{11/6}} \\
 &= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{715x^{11/6}} \\
 &= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{715x^{11/6}} \\
 &= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{715x^{11/6}} \\
 &= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2 \cos^2(a + b\sqrt[3]{x})}{5x^{5/2}} + \frac{32b^2 \cos^2(a + b\sqrt[3]{x})}{715x^{11/6}}
 \end{aligned}$$

**Mathematica [A]**

time = 0.25, size = 249, normalized size = 0.76

$-\frac{135135 - 135135 \cos(2(a + b\sqrt[3]{x})) + 151200x^{2/3} \cos(2(a + b\sqrt[3]{x})) - 38400x^{4/3} \cos(2(a + b\sqrt[3]{x})) + 40960x^2 \cos(2(a + b\sqrt[3]{x})) + 327680x^{5/3} \sqrt{x} \cos(2a) \text{FresnelC}\left(\frac{\sqrt{x}\sqrt{x}}{\sqrt{x}}\right) - 327680x^{5/3} \sqrt{x} \sin^2\left(\frac{\sqrt{x}\sqrt{x}}{\sqrt{x}}\right) \sin(2a) + 41580a\sqrt{x} \sin(2(a + b\sqrt[3]{x})) - 67200x \sin(2(a + b\sqrt[3]{x})) + 30720x^{4/3} \sin(2(a + b\sqrt[3]{x})) - 163840x^{7/3} \sin(2(a + b\sqrt[3]{x}))}{675675x^{11/6}}$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*x^(1/3)]^2/x^(7/2), x]

```
[Out] (-135135 - 135135*Cos[2*(a + b*x^(1/3))] + 15120*b^2*x^(2/3)*Cos[2*(a + b*x
^(1/3))] - 3840*b^4*x^(4/3)*Cos[2*(a + b*x^(1/3))] + 4096*b^6*x^2*Cos[2*(a
+ b*x^(1/3))] + 32768*b^(15/2)*Sqrt[Pi]*x^(5/2)*Cos[2*a]*FresnelC[(2*Sqrt[b
]*x^(1/6))/Sqrt[Pi]] - 32768*b^(15/2)*Sqrt[Pi]*x^(5/2)*FresnelS[(2*Sqrt[b]*
x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 41580*b*x^(1/3)*Sin[2*(a + b*x^(1/3))] - 6720
*b^3*x*Sin[2*(a + b*x^(1/3))] + 3072*b^5*x^(5/3)*Sin[2*(a + b*x^(1/3))] - 1
6384*b^7*x^(7/3)*Sin[2*(a + b*x^(1/3))])/(675675*x^(5/2))
```

**Maple [A]**

time = 0.07, size = 207, normalized size = 0.63

method	result
--------	--------

$$4b - \frac{\cos\left(2a + 2b x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}$$

$$4b - \frac{\sin\left(2a + 2b x^{\frac{1}{3}}\right)}{9x^{\frac{3}{2}}} +$$

$$4b - \frac{\cos\left(2a + 2b x^{\frac{1}{3}}\right)}{11x^{\frac{11}{6}}}$$

$$4b - \frac{\cos\left(2a + 2b x^{\frac{1}{3}}\right)}{7x^{\frac{1}{6}}}$$

$$4b - \frac{\sin\left(2a + 2b x^{\frac{1}{3}}\right)}{9x^{\frac{3}{2}}} +$$

$$4b - \frac{\cos\left(2a + 2b x^{\frac{1}{3}}\right)}{11x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*x^(1/3))^2/x^(7/2),x,method=_RETURNVERBOSE)`

[Out]  $-1/5/x^{5/2}-1/5/x^{5/2}*\cos(2*a+2*b*x^{1/3})-4/5*b*(-1/13/x^{13/6}*\sin(2*a+2*b*x^{1/3}))+4/13*b*(-1/11/x^{11/6}*\cos(2*a+2*b*x^{1/3}))-4/11*b*(-1/9/x^{9/6})*\sin(2*a+2*b*x^{1/3}))+4/9*b*(-1/7/x^{7/6}*\cos(2*a+2*b*x^{1/3}))-4/7*b*(-1/5/x^{5/6})*\sin(2*a+2*b*x^{1/3}))+4/5*b*(-1/3/x^{1/2}*\cos(2*a+2*b*x^{1/3}))-4/3*b*(-1/x^{1/6})*\sin(2*a+2*b*x^{1/3}))+2*b^{1/2}*Pi^{1/2}*(\cos(2*a)*\text{FresnelC}(2*x^{1/6}*b^{1/2}/Pi^{1/2})-\sin(2*a)*\text{FresnelS}(2*x^{1/6}*b^{1/2}/Pi^{1/2})))$

**Maxima [C]** Result contains complex when optimal does not.

time = 0.60, size = 90, normalized size = 0.27

$$\frac{240\sqrt{2}\left(\left(-i-1\right)\sqrt{2}\Gamma\left(-\frac{15}{2},2ibx^{\frac{1}{3}}\right)+\left(i+1\right)\sqrt{2}\Gamma\left(-\frac{15}{2},-2ibx^{\frac{1}{3}}\right)\right)\cos(2a)+\left(\left(i+1\right)\sqrt{2}\Gamma\left(-\frac{15}{2},2ibx^{\frac{1}{3}}\right)+\left(i-1\right)\sqrt{2}\Gamma\left(-\frac{15}{2},-2ibx^{\frac{1}{3}}\right)\right)\sin(2a)}{5x^{\frac{5}{2}}}\sqrt{bx^{\frac{1}{3}}b^7x^{\frac{7}{3}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))^2/x^(7/2),x, algorithm="maxima")`

[Out]  $-1/5*(240*\sqrt{2})*\left(\left(-I-1\right)*\sqrt{2}*\gamma\left(-15/2,2*I*b*x^{1/3}\right)+\left(I+1\right)*\sqrt{2}*\gamma\left(-15/2,-2*I*b*x^{1/3}\right)\right)*\cos(2*a)+\left(\left(-I+1\right)*\sqrt{2}*\gamma\left(-15/2,2*I*b*x^{1/3}\right)+\left(I-1\right)*\sqrt{2}*\gamma\left(-15/2,-2*I*b*x^{1/3}\right)\right)*\sin(2*a))*\sqrt{b*x^{1/3}}*b^7*x^{7/3}+1)/x^{5/2}$

**Fricas [A]**

time = 0.38, size = 192, normalized size = 0.59

$$\frac{2\left(16384\pi^7x^3\sqrt{\frac{b}{\pi}}\cos(2a)C\left(2x^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)-16384\pi^7x^3\sqrt{\frac{b}{\pi}}S\left(2x^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)\sin(2a)-2048b^6x^{5/2}+1920b^4x^{11/6}-7560b^2x^{7/6}-(3840b^4x^{11/6}-15120b^2x^{7/6}-(4096b^6x^2-135135)\sqrt{x})\cos(bx^{1/3}+a)^2+4(768b^5x^{13/6}-1680b^3x^{3/2}-(4096b^7x^2-10395b)*x^{5/6})\cos(bx^{1/3}+a)\sin(bx^{1/3}+a)\right)}{675675x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))^2/x^(7/2),x, algorithm="fricas")`

[Out]  $2/675675*(16384*\pi*b^7*x^3*\sqrt{b/\pi}*\cos(2*a)*\text{fresnel\_cos}(2*x^{1/6})*\sqrt{b/\pi})-16384*\pi*b^7*x^3*\sqrt{b/\pi}*\text{fresnel\_sin}(2*x^{1/6})*\sqrt{b/\pi})*\sin(2*a)-2048*b^6*x^{5/2}+1920*b^4*x^{11/6}-7560*b^2*x^{7/6}-(3840*b^4*x^{11/6}-15120*b^2*x^{7/6}-(4096*b^6*x^2-135135)*\sqrt{x})*\cos(b*x^{1/3}+a)^2+4*(768*b^5*x^{13/6}-1680*b^3*x^{3/2}-(4096*b^7*x^2-10395*b)*x^{5/6}))*\cos(b*x^{1/3}+a)*\sin(b*x^{1/3}+a))/x^3$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+b\sqrt[3]{x})}{x^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x**(1/3))**2/x**(7/2),x)`

[Out] `Integral(cos(a + b*x**(1/3))**2/x**(7/2), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*x^(1/3))^2/x^(7/2),x, algorithm="giac")`

[Out] `integrate(cos(b*x^(1/3) + a)^2/x^(7/2), x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a + bx^{1/3})^2}{x^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x^(1/3))^2/x^(7/2),x)`

[Out] `int(cos(a + b*x^(1/3))^2/x^(7/2), x)`

### 3.61 $\int \cos^3(\sqrt[3]{x}) dx$

**Optimal.** Leaf size=86

$$4\sqrt[3]{x} \cos(\sqrt[3]{x}) + \frac{2}{3}\sqrt[3]{x} \cos^3(\sqrt[3]{x}) - \frac{14}{3} \sin(\sqrt[3]{x}) + 2x^{2/3} \sin(\sqrt[3]{x}) + x^{2/3} \cos^2(\sqrt[3]{x}) \sin(\sqrt[3]{x}) + \frac{2}{9} \sin^3(\sqrt[3]{x})$$

[Out]  $4x^{(1/3)}*\cos(x^{(1/3)})+2/3*x^{(1/3)}*\cos(x^{(1/3)})^3-14/3*\sin(x^{(1/3)})+2*x^{(2/3)}*\sin(x^{(1/3)})+x^{(2/3)}*\cos(x^{(1/3)})^2*\sin(x^{(1/3)})+2/9*\sin(x^{(1/3)})^3$

**Rubi [A]**

time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {3443, 3392, 3377, 2717, 2713}

$$2x^{2/3} \sin(\sqrt[3]{x}) + x^{2/3} \sin(\sqrt[3]{x}) \cos^2(\sqrt[3]{x}) + \frac{2}{9} \sin^3(\sqrt[3]{x}) - \frac{14}{3} \sin(\sqrt[3]{x}) + \frac{2}{3} \sqrt[3]{x} \cos^3(\sqrt[3]{x}) + 4\sqrt[3]{x} \cos(\sqrt[3]{x})$$

Antiderivative was successfully verified.

[In] Int[Cos[x^(1/3)]^3,x]

[Out]  $4x^{(1/3)}*\text{Cos}[x^{(1/3)}] + (2*x^{(1/3)}*\text{Cos}[x^{(1/3)}]^3)/3 - (14*\text{Sin}[x^{(1/3)}])/3 + 2*x^{(2/3)}*\text{Sin}[x^{(1/3)}] + x^{(2/3)}*\text{Cos}[x^{(1/3)}]^2*\text{Sin}[x^{(1/3)}] + (2*\text{Sin}[x^{(1/3)}]^3)/9$

Rule 2713

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2717

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(-(c + d\*x)^m)\*(Cos[e + f\*x]/f), x] + Dist[d\*(m/f), Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3392

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[d\*m\*(c + d\*x)^(m - 1)\*((b\*Sine[e + f\*x])^n/(f^2\*n^2)), x] + (Dist[b^2\*((n - 1)/n), Int[(c + d\*x)^m\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Dist[d^2\*m\*((m - 1)/(f^2\*n^2)), Int[(c + d\*x)^(m - 2)\*(b\*Sine[e + f\*x])^n, x], x]



- Simp[b\*(c + d\*x)^m\*Cos[e + f\*x]\*((b\*Sin[e + f\*x])^(n - 1)/(f\*n)), x] /;  
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rule 3443

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/(n\*f), Subst[Int[x^(1/n - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, (e + f\*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[1/n]

### Rubi steps

$$\begin{aligned}
 \int \cos^3(\sqrt[3]{x}) dx &= 3 \text{Subst}\left(\int x^2 \cos^3(x) dx, x, \sqrt[3]{x}\right) \\
 &= \frac{2}{3} \sqrt[3]{x} \cos^3(\sqrt[3]{x}) + x^{2/3} \cos^2(\sqrt[3]{x}) \sin(\sqrt[3]{x}) - \frac{2}{3} \text{Subst}\left(\int \cos^3(x) dx, x, \sqrt[3]{x}\right) + 2 \text{Subst}\left(\int (1-x^2) \cos^3(x) dx, x, \sqrt[3]{x}\right) \\
 &= \frac{2}{3} \sqrt[3]{x} \cos^3(\sqrt[3]{x}) + 2x^{2/3} \sin(\sqrt[3]{x}) + x^{2/3} \cos^2(\sqrt[3]{x}) \sin(\sqrt[3]{x}) + \frac{2}{3} \text{Subst}\left(\int (1-x^2) \cos^3(x) dx, x, \sqrt[3]{x}\right) \\
 &= 4\sqrt[3]{x} \cos(\sqrt[3]{x}) + \frac{2}{3} \sqrt[3]{x} \cos^3(\sqrt[3]{x}) - \frac{2}{3} \sin(\sqrt[3]{x}) + 2x^{2/3} \sin(\sqrt[3]{x}) + x^{2/3} \cos^2(\sqrt[3]{x}) \sin(\sqrt[3]{x}) \\
 &= 4\sqrt[3]{x} \cos(\sqrt[3]{x}) + \frac{2}{3} \sqrt[3]{x} \cos^3(\sqrt[3]{x}) - \frac{14}{3} \sin(\sqrt[3]{x}) + 2x^{2/3} \sin(\sqrt[3]{x}) + x^{2/3} \cos^2(\sqrt[3]{x}) \sin(\sqrt[3]{x})
 \end{aligned}$$

### Mathematica [A]

time = 0.07, size = 66, normalized size = 0.77

$$\frac{1}{36} (162\sqrt[3]{x} \cos(\sqrt[3]{x}) + 6\sqrt[3]{x} \cos(3\sqrt[3]{x}) + 81(-2 + x^{2/3}) \sin(\sqrt[3]{x}) + (-2 + 9x^{2/3}) \sin(3\sqrt[3]{x}))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x^(1/3)]^3, x]

[Out] (162\*x^(1/3)\*Cos[x^(1/3)] + 6\*x^(1/3)\*Cos[3\*x^(1/3)] + 81\*(-2 + x^(2/3))\*Sin[x^(1/3)] + (-2 + 9\*x^(2/3))\*Sin[3\*x^(1/3)])/36

### Maple [A]

time = 0.04, size = 58, normalized size = 0.67

method	result
derivativedivides	$x^{\frac{2}{3}} \left(2 + \cos^2\left(x^{\frac{1}{3}}\right)\right) \sin\left(x^{\frac{1}{3}}\right) - 4 \sin\left(x^{\frac{1}{3}}\right) + 4x^{\frac{1}{3}} \cos\left(x^{\frac{1}{3}}\right) + \frac{2x^{\frac{1}{3}} \left(\cos^3\left(x^{\frac{1}{3}}\right)\right)}{3} - \frac{2\left(2 + \cos^2\left(x^{\frac{1}{3}}\right)\right)}{3}$

default	$x^{\frac{2}{3}} \left( 2 + \cos^2 \left( x^{\frac{1}{3}} \right) \right) \sin \left( x^{\frac{1}{3}} \right) - 4 \sin \left( x^{\frac{1}{3}} \right) + 4x^{\frac{1}{3}} \cos \left( x^{\frac{1}{3}} \right) + \frac{2x^{\frac{1}{3}} \left( \cos^3 \left( x^{\frac{1}{3}} \right) \right)}{3} - \frac{2 \left( 2 + \cos^2 \left( x^{\frac{1}{3}} \right) \right)}{9}$
---------	--

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x^(1/3))^3,x,method=_RETURNVERBOSE)`

[Out]  $x^{(2/3)}*(2+\cos(x^{(1/3)})^2)*\sin(x^{(1/3)})-4*\sin(x^{(1/3)})+4*x^{(1/3)}*\cos(x^{(1/3)})+2/3*x^{(1/3)}*\cos(x^{(1/3)})^3-2/9*(2+\cos(x^{(1/3)})^2)*\sin(x^{(1/3)})$

**Maxima** [A]

time = 0.30, size = 47, normalized size = 0.55

$$\frac{1}{36} \left( 9x^{\frac{2}{3}} - 2 \right) \sin \left( 3x^{\frac{1}{3}} \right) + \frac{9}{4} \left( x^{\frac{2}{3}} - 2 \right) \sin \left( x^{\frac{1}{3}} \right) + \frac{1}{6} x^{\frac{1}{3}} \cos \left( 3x^{\frac{1}{3}} \right) + \frac{9}{2} x^{\frac{1}{3}} \cos \left( x^{\frac{1}{3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x^(1/3))^3,x, algorithm="maxima")`

[Out]  $1/36*(9*x^{(2/3)} - 2)*\sin(3*x^{(1/3)}) + 9/4*(x^{(2/3)} - 2)*\sin(x^{(1/3)}) + 1/6*x^{(1/3)}*\cos(3*x^{(1/3)}) + 9/2*x^{(1/3)}*\cos(x^{(1/3)})$

**Fricas** [A]

time = 0.35, size = 48, normalized size = 0.56

$$\frac{2}{3} x^{\frac{1}{3}} \cos \left( x^{\frac{1}{3}} \right)^3 + \frac{1}{9} \left( \left( 9x^{\frac{2}{3}} - 2 \right) \cos \left( x^{\frac{1}{3}} \right)^2 + 18x^{\frac{2}{3}} - 40 \right) \sin \left( x^{\frac{1}{3}} \right) + 4x^{\frac{1}{3}} \cos \left( x^{\frac{1}{3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x^(1/3))^3,x, algorithm="fricas")`

[Out]  $2/3*x^{(1/3)}*\cos(x^{(1/3)})^3 + 1/9*((9*x^{(2/3)} - 2)*\cos(x^{(1/3)})^2 + 18*x^{(2/3)} - 40)*\sin(x^{(1/3)}) + 4*x^{(1/3)}*\cos(x^{(1/3)})$

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(85) = 170.

time = 0.97, size = 513, normalized size = 5.97

$$\frac{\frac{\cos^3(x^{1/3})}{x^{1/3}}}{\cos^2(x^{1/3})} + \frac{\frac{\cos^2(x^{1/3})}{x^{1/3}}}{\cos(x^{1/3})} + \frac{\frac{\cos(x^{1/3})}{x^{1/3}}}{\cos(x^{1/3})} + \frac{\frac{\cos(x^{1/3})}{x^{1/3}}}{\cos(x^{1/3})} + \frac{\frac{\cos(x^{1/3})}{x^{1/3}}}{\cos(x^{1/3})} + \frac{\frac{\cos(x^{1/3})}{x^{1/3}}}{\cos(x^{1/3})} + \frac{\frac{\cos(x^{1/3})}{x^{1/3}}}{\cos(x^{1/3})} + \frac{\frac{\cos(x^{1/3})}{x^{1/3}}}{\cos(x^{1/3})} + \frac{\frac{\cos(x^{1/3})}{x^{1/3}}}{\cos(x^{1/3})} + \frac{\frac{\cos(x^{1/3})}{x^{1/3}}}{\cos(x^{1/3})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x**(1/3))**3,x)`

[Out]  $54*x^{(2/3)}*\tan(x^{(1/3)}/2)**5/(9*\tan(x^{(1/3)}/2)**6 + 27*\tan(x^{(1/3)}/2)**4 + 27*\tan(x^{(1/3)}/2)**2 + 9) + 36*x^{(2/3)}*\tan(x^{(1/3)}/2)**3/(9*\tan(x^{(1/3)}/2)**6 + 27*\tan(x^{(1/3)}/2)**4 + 27*\tan(x^{(1/3)}/2)**2 + 9) + 54*x^{(2/3)}*\tan(x^{(1/3)}/2)/(9*\tan(x^{(1/3)}/2)**6 + 27*\tan(x^{(1/3)}/2)**4 + 27*\tan(x^{(1/3)}/2)**2 + 9)$

```

**(1/3)/2)**2 + 9) - 42*x**(1/3)*tan(x**(1/3)/2)**6/(9*tan(x**(1/3)/2)**6 +
  27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9) - 18*x**(1/3)*tan(x**(1
/3)/2)**4/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2
)**2 + 9) + 18*x**(1/3)*tan(x**(1/3)/2)**2/(9*tan(x**(1/3)/2)**6 + 27*tan(x
**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9) + 42*x**(1/3)/(9*tan(x**(1/3)/2)
**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9) - 84*tan(x**(1/3)/
2)**5/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2
+ 9) - 152*tan(x**(1/3)/2)**3/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**
4 + 27*tan(x**(1/3)/2)**2 + 9) - 84*tan(x**(1/3)/2)/(9*tan(x**(1/3)/2)**6 +
  27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9)

```

**Giac [A]**

time = 0.42, size = 47, normalized size = 0.55

$$\frac{1}{36} \left( 9x^{\frac{2}{3}} - 2 \right) \sin \left( 3x^{\frac{1}{3}} \right) + \frac{9}{4} \left( x^{\frac{2}{3}} - 2 \right) \sin \left( x^{\frac{1}{3}} \right) + \frac{1}{6} x^{\frac{1}{3}} \cos \left( 3x^{\frac{1}{3}} \right) + \frac{9}{2} x^{\frac{1}{3}} \cos \left( x^{\frac{1}{3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x^(1/3))^3,x, algorithm="giac")

[Out] 1/36\*(9\*x^(2/3) - 2)\*sin(3\*x^(1/3)) + 9/4\*(x^(2/3) - 2)\*sin(x^(1/3)) + 1/6\*x^(1/3)\*cos(3\*x^(1/3)) + 9/2\*x^(1/3)\*cos(x^(1/3))

**Mupad [B]**

time = 0.50, size = 62, normalized size = 0.72

$$4x^{1/3} \cos(x^{1/3}) - \frac{2 \cos(x^{1/3})^2 \sin(x^{1/3})}{9} - \frac{40 \sin(x^{1/3})}{9} + 2x^{2/3} \sin(x^{1/3}) + \frac{2x^{1/3} \cos(x^{1/3})^3}{3} + x^{2/3} \cos(x^{1/3})^2 \sin(x^{1/3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x^(1/3))^3,x)

[Out] 4\*x^(1/3)\*cos(x^(1/3)) - (2\*cos(x^(1/3))^2\*sin(x^(1/3)))/9 - (40\*sin(x^(1/3)))/9 + 2\*x^(2/3)\*sin(x^(1/3)) + (2\*x^(1/3)\*cos(x^(1/3))^3)/3 + x^(2/3)\*cos(x^(1/3))^2\*sin(x^(1/3))

$$3.62 \quad \int \frac{\cos(\sqrt[6]{x})}{x^{5/6}} dx$$

Optimal. Leaf size=8

$$6 \sin(\sqrt[6]{x})$$

[Out] 6\*sin(x^(1/6))

Rubi [A]

time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3461, 2717}

$$6 \sin(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] Int[Cos[x^(1/6)]/x^(5/6),x]

[Out] 6\*Sin[x^(1/6)]

Rule 2717

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /;  
FreeQ[{c, d}, x]

Rule 3461

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol]  
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p,  
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(  
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(  
m + 1)/n], 0]))

Rubi steps

$$\int \frac{\cos(\sqrt[6]{x})}{x^{5/6}} dx = 6 \text{Subst} \left( \int \cos(x) dx, x, \sqrt[6]{x} \right) \\ = 6 \sin(\sqrt[6]{x})$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$6 \sin(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x^(1/6)]/x^(5/6),x]

[Out] 6\*Sin[x^(1/6)]

**Maple** [A]

time = 0.02, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$6 \sin\left(x^{\frac{1}{6}}\right)$	7
default	$6 \sin\left(x^{\frac{1}{6}}\right)$	7
meijerg	$6 \sin\left(x^{\frac{1}{6}}\right)$	7

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x^(1/6))/x^(5/6),x,method=\_RETURNVERBOSE)

[Out] 6\*sin(x^(1/6))

**Maxima** [A]

time = 0.28, size = 6, normalized size = 0.75

$$6 \sin\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x^(1/6))/x^(5/6),x, algorithm="maxima")

[Out] 6\*sin(x^(1/6))

**Fricas** [A]

time = 0.38, size = 6, normalized size = 0.75

$$6 \sin\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x^(1/6))/x^(5/6),x, algorithm="fricas")

[Out] 6\*sin(x^(1/6))

**Sympy** [A]

time = 7.56, size = 7, normalized size = 0.88

$$6 \sin\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/6))/x**(5/6),x)
```

```
[Out] 6*sin(x**(1/6))
```

**Giac [A]**

time = 0.42, size = 6, normalized size = 0.75

$$6 \sin \left( x^{\frac{1}{6}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/6))/x^(5/6),x, algorithm="giac")
```

```
[Out] 6*sin(x^(1/6))
```

**Mupad [B]**

time = 0.43, size = 6, normalized size = 0.75

$$6 \sin \left( x^{1/6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/6))/x^(5/6),x)
```

```
[Out] 6*sin(x^(1/6))
```

### 3.63 $\int (ex)^m (b \cos(c + dx^n))^p dx$

Optimal. Leaf size=21

$$\text{Int}((ex)^m (b \cos(c + dx^n))^p, x)$$

[Out] Unintegrable((e\*x)^m\*(b\*cos(c+d\*x^n))^p,x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ ,

Rules used = {}

$$\int (ex)^m (b \cos(c + dx^n))^p dx$$

Verification is not applicable to the result.

[In] Int[(e\*x)^m\*(b\*Cos[c + d\*x^n])^p,x]

[Out] Defer[Int] [(e\*x)^m\*(b\*Cos[c + d\*x^n])^p, x]

Rubi steps

$$\int (ex)^m (b \cos(c + dx^n))^p dx = \int (ex)^m (b \cos(c + dx^n))^p dx$$

Mathematica [A]

time = 0.65, size = 0, normalized size = 0.00

$$\int (ex)^m (b \cos(c + dx^n))^p dx$$

Verification is not applicable to the result.

[In] Integrate[(e\*x)^m\*(b\*Cos[c + d\*x^n])^p,x]

[Out] Integrate[(e\*x)^m\*(b\*Cos[c + d\*x^n])^p, x]

Maple [A]

time = 0.07, size = 0, normalized size = 0.00

$$\int (ex)^m (b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*(b\*cos(c+d\*x^n))^p,x)

[Out] `int((e*x)^m*(b*cos(c+d*x^n))^p,x)`

**Maxima** [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*(b*cos(c+d*x^n))^p,x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x^n + c))^p*(x*e)^m, x)`

**Fricas** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*(b*cos(c+d*x^n))^p,x, algorithm="fricas")`

[Out] `integral((b*cos(d*x^n + c))^p*(x*e)^m, x)`

**Sympy** [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos(c + dx^n))^p (ex)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*(b*cos(c+d*x**n))**p,x)`

[Out] `Integral((b*cos(c + d*x**n))**p*(e*x)**m, x)`

**Giac** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*(b*cos(c+d*x^n))^p,x, algorithm="giac")`

[Out] `integrate((b*cos(d*x^n + c))^p*(x*e)^m, x)`

**Mupad** [A]

time = 0.00, size = -1, normalized size = -0.05

$$\int (ex)^m (b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*(b*cos(c + d*x^n))^p,x)`

[Out] `int((e*x)^m*(b*cos(c + d*x^n))^p, x)`



### 3.64 $\int (ex)^m (a + b \cos(c + dx^n))^p dx$

Optimal. Leaf size=23

$$\text{Int}((ex)^m (a + b \cos(c + dx^n))^p, x)$$

[Out] Unintegrable((e\*x)^m\*(a+b\*cos(c+d\*x^n))^p,x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (ex)^m (a + b \cos(c + dx^n))^p dx$$

Verification is not applicable to the result.

[In] Int[(e\*x)^m\*(a + b\*cos[c + d\*x^n])^p,x]

[Out] Defer[Int] [(e\*x)^m\*(a + b\*cos[c + d\*x^n])^p, x]

Rubi steps

$$\int (ex)^m (a + b \cos(c + dx^n))^p dx = \int (ex)^m (a + b \cos(c + dx^n))^p dx$$

Mathematica [A]

time = 0.88, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b \cos(c + dx^n))^p dx$$

Verification is not applicable to the result.

[In] Integrate[(e\*x)^m\*(a + b\*cos[c + d\*x^n])^p,x]

[Out] Integrate[(e\*x)^m\*(a + b\*cos[c + d\*x^n])^p, x]

Maple [A]

time = 0.07, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*(a+b*cos(c+d*x^n))^p,x)`

[Out] `int((e*x)^m*(a+b*cos(c+d*x^n))^p,x)`

**Maxima** [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*(a+b*cos(c+d*x^n))^p,x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x^n + c) + a)^p*(x*e)^m, x)`

**Fricas** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*(a+b*cos(c+d*x^n))^p,x, algorithm="fricas")`

[Out] `integral((b*cos(d*x^n + c) + a)^p*(x*e)^m, x)`

**Sympy** [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*(a+b*cos(c+d*x**n))**p,x)`

[Out] `Integral((e*x)**m*(a + b*cos(c + d*x**n))**p, x)`

**Giac** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*(a+b*cos(c+d*x^n))^p,x, algorithm="giac")`

[Out] `integrate((b*cos(d*x^n + c) + a)^p*(x*e)^m, x)`

**Mupad** [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int (ex)^m (a + b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*cos(c + d*x^n))^p,x)
```

```
[Out] int((e*x)^m*(a + b*cos(c + d*x^n))^p, x)
```

### 3.65 $\int (ex)^{-1+n} (b \cos(c + dx^n))^p dx$

**Optimal.** Leaf size=93

$$-\frac{x^{-n}(ex)^n (b \cos(c + dx^n))^{1+p} {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; \cos^2(c + dx^n)\right) \sin(c + dx^n)}{bden(1+p) \sqrt{\sin^2(c + dx^n)}}$$

[Out]  $-(e*x)^n*(b*\cos(c+d*x^n))^{(1+p)}*hypergeom([1/2, 1/2+1/2*p], [3/2+1/2*p], \cos(c+d*x^n)^2)*\sin(c+d*x^n)/b/d/e/n/(1+p)/(x^n)/(\sin(c+d*x^n)^2)^{(1/2)}$

**Rubi [A]**

time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {3463, 3461, 2722}

$$-\frac{x^{-n}(ex)^n \sin(c + dx^n) (b \cos(c + dx^n))^{p+1} {}_2F_1\left(\frac{1}{2}, \frac{p+1}{2}; \frac{p+3}{2}; \cos^2(dx^n + c)\right)}{bden(p+1) \sqrt{\sin^2(c + dx^n)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e*x)^{-1+n}*(b*\text{Cos}[c + d*x^n])^p, x]$

[Out]  $-\left(\left((e*x)^n*(b*\text{Cos}[c + d*x^n])^{(1+p)}*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(1+p)}{2}, \frac{(3+p)}{2}, \text{Cos}[c + d*x^n]^2*\text{Sin}[c + d*x^n]\right]/(b*d*e*n*(1+p)*x^n*\text{Sqrt}[\text{Sin}[c + d*x^n]^2]\right)\right)$

Rule 2722

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)^n], x\_Symbol] \rightarrow \text{Simp}[\text{Cos}[c + d*x]*((b*\text{Sin}[c + d*x])^{(n+1)}/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]))*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(n+1)}{2}, \frac{(n+3)}{2}, \text{Sin}[c + d*x]^2\right], x] /; \text{FreeQ}\{b, c, d, n\}, x \&\& \text{IntegerQ}[2*n]$

Rule 3461

$\text{Int}[(a_*) + \text{Cos}[(c_*) + (d_*)*(x_*)^n]*(b_*)^{(p_*)}*(x_*)^{(m_*)}, x\_Symbol] \rightarrow \text{Dist}\left[\frac{1}{n}, \text{Subst}\left[\text{Int}\left[x^{(\text{Simplify}[(m+1)/n] - 1)}*(a + b*\text{Cos}[c + d*x])^p, x\right], x, x^n\right], x\right] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \&\& (\text{EqQ}[p, 1] \parallel \text{EqQ}[m, n-1] \parallel (\text{IntegerQ}[p] \&\& \text{GtQ}[\text{Simplify}[(m+1)/n], 0]))$

Rule 3463

$\text{Int}[(a_*) + \text{Cos}[(c_*) + (d_*)*(x_*)^n]*(b_*)^{(p_*)}*((e_*)*(x_*)^{(m_*)}), x\_Symbol] \rightarrow \text{Dist}\left[e^{\text{IntPart}[m]}*(e*x)^{\text{FracPart}[m]}/x^{\text{FracPart}[m]}, \text{Int}\left[x^m*(a + b*\text{Cos}[c + d*x^n])^p, x\right], x\right] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x \&\& \text{Inte}$

gerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int (ex)^{-1+n} (b \cos(c + dx^n))^p dx &= \frac{(x^{-n}(ex)^n) \int x^{-1+n} (b \cos(c + dx^n))^p dx}{e} \\ &= \frac{(x^{-n}(ex)^n) \text{Subst}(\int (b \cos(c + dx))^p dx, x, x^n)}{en} \\ &= -\frac{x^{-n}(ex)^n (b \cos(c + dx^n))^{1+p} {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; \cos^2(c + dx^n)\right) \sin(c + dx^n)}{bden(1+p) \sqrt{\sin^2(c + dx^n)}} \end{aligned}$$

**Mathematica [A]**

time = 0.12, size = 89, normalized size = 0.96

$$\frac{x^{1-n}(ex)^{-1+n} (b \cos(c + dx^n))^p \cot(c + dx^n) {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; \cos^2(c + dx^n)\right) \sqrt{\sin^2(c + dx^n)}}{dn(1+p)}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1 + n)\*(b\*cos[c + d\*x^n])^p,x]

[Out] -((x^(1 - n)\*(e\*x)^(-1 + n)\*(b\*cos[c + d\*x^n])^p\*Cot[c + d\*x^n]\*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d\*x^n]^2]\*Sqrt[Sin[c + d\*x^n]^2])/(d\*n\*(1 + p))

**Maple [F]**

time = 0.06, size = 0, normalized size = 0.00

$$\int (ex)^{-1+n} (b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+n)\*(b\*cos(c+d\*x^n))^p,x)

[Out] int((e\*x)^(-1+n)\*(b\*cos(c+d\*x^n))^p,x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(b\*cos(c+d\*x^n))^p,x, algorithm="maxima")

[Out] integrate((b\*cos(d\*x^n + c))^p\*(x\*e)^(n - 1), x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(b\*cos(c+d\*x^n))^p,x, algorithm="fricas")

[Out] integral((b\*cos(d\*x^n + c))^p\*(x\*e)^(n - 1), x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos(c + dx^n))^p (ex)^{n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+n)\*(b\*cos(c+d\*x\*\*n))\*\*p,x)

[Out] Integral((b\*cos(c + d\*x\*\*n))\*\*p\*(e\*x)\*\*(n - 1), x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(b\*cos(c+d\*x^n))^p,x, algorithm="giac")

[Out] integrate((b\*cos(d\*x^n + c))^p\*(x\*e)^(n - 1), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (ex)^{n-1} (b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(n - 1)\*(b\*cos(c + d\*x^n))^p,x)

[Out] int((e\*x)^(n - 1)\*(b\*cos(c + d\*x^n))^p, x)

### 3.66 $\int (ex)^{-1+2n} (b \cos(c + dx^n))^p dx$

Optimal. Leaf size=39

$$\frac{x^{-2n}(ex)^{2n}\text{Int}(x^{-1+2n}(b \cos(c + dx^n))^p, x)}{e}$$

[Out]  $(e*x)^{(2*n)}*\text{Unintegrable}(x^{(-1+2*n)}*(b*\cos(c+d*x^n))^p, x)/e/(x^{(2*n)})$

Rubi [A]

time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (ex)^{-1+2n} (b \cos(c + dx^n))^p dx$$

Verification is not applicable to the result.

[In]  $\text{Int}[(e*x)^{(-1 + 2*n)}*(b*\text{Cos}[c + d*x^n])^p, x]$

[Out]  $((e*x)^{(2*n)}*\text{Defer}[\text{Int}[x^{(-1 + 2*n)}*(b*\text{Cos}[c + d*x^n])^p, x]])/(e*x^{(2*n)})$

Rubi steps

$$\int (ex)^{-1+2n} (b \cos(c + dx^n))^p dx = \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n} (b \cos(c + dx^n))^p dx}{e}$$

Mathematica [A]

time = 0.60, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (b \cos(c + dx^n))^p dx$$

Verification is not applicable to the result.

[In]  $\text{Integrate}[(e*x)^{(-1 + 2*n)}*(b*\text{Cos}[c + d*x^n])^p, x]$

[Out]  $\text{Integrate}[(e*x)^{(-1 + 2*n)}*(b*\text{Cos}[c + d*x^n])^p, x]$

Maple [A]

time = 0.06, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x)`

[Out] `int((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x)`

**Maxima** [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x^n + c))^p*(x*e)^(2*n - 1), x)`

**Fricas** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x, algorithm="fricas")`

[Out] `integral((b*cos(d*x^n + c))^p*(x*e)^(2*n - 1), x)`

**Sympy** [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos(c + dx^n))^p (ex)^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**(-1+2*n)*(b*cos(c+d*x**n))**p,x)`

[Out] `Integral((b*cos(c + d*x**n))**p*(e*x)**(2*n - 1), x)`

**Giac** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x, algorithm="giac")`

[Out] `integrate((b*cos(d*x^n + c))^p*(x*e)^(2*n - 1), x)`

**Mupad** [A]

time = 0.00, size = -1, normalized size = -0.03

$$\int (ex)^{2n-1} (b \cos(c + dx^n))^p dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(2*n - 1)*(b*cos(c + d*x^n))^p, x)
```

```
[Out] int((e*x)^(2*n - 1)*(b*cos(c + d*x^n))^p, x)
```

### 3.67 $\int (ex)^{-1+n} (a + b \cos(c + dx^n))^p dx$

**Optimal.** Leaf size=131

$$\frac{\sqrt{2} x^{-n} (ex)^n F_1\left(\frac{1}{2}; \frac{1}{2}, -p; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx^n)), \frac{b(1 - \cos(c + dx^n))}{a+b}\right) (a + b \cos(c + dx^n))^p \left(\frac{a+b \cos(c + dx^n)}{a+b}\right)^{-p} \sin}{den \sqrt{1 + \cos(c + dx^n)}}$$

[Out] (e\*x)^n\*AppellF1(1/2, -p, 1/2, 3/2, b\*(1-cos(c+d\*x^n))/(a+b), 1/2-1/2\*cos(c+d\*x^n))\*(a+b\*cos(c+d\*x^n))^p\*sin(c+d\*x^n)\*2^(1/2)/d/e/n/(x^n)/(((a+b\*cos(c+d\*x^n))/(a+b))^p)/(1+cos(c+d\*x^n))^(1/2)

**Rubi [A]**

time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {3463, 3461, 2744, 144, 143}

$$\frac{\sqrt{2} x^{-n} (ex)^n \sin(c + dx^n) (a + b \cos(c + dx^n))^p \left(\frac{a+b \cos(c + dx^n)}{a+b}\right)^{-p} F_1\left(\frac{1}{2}; \frac{1}{2}, -p; \frac{3}{2}; \frac{1}{2}(1 - \cos(dx^n + c)), \frac{b(1 - \cos(dx^n + c))}{a+b}\right)}{den \sqrt{\cos(c + dx^n) + 1}}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^(-1 + n)\*(a + b\*Cos[c + d\*x^n])^p,x]

[Out] (Sqrt[2]\*(e\*x)^n\*AppellF1[1/2, 1/2, -p, 3/2, (1 - Cos[c + d\*x^n])/2, (b\*(1 - Cos[c + d\*x^n]))/(a + b)]\*(a + b\*Cos[c + d\*x^n])^p\*Sin[c + d\*x^n])/(d\*e\*n\*x^n\*Sqrt[1 + Cos[c + d\*x^n]]\*((a + b\*Cos[c + d\*x^n])/(a + b))^p)

**Rule 143**

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)/(b\*(m + 1)\*(b/(b\*c - a\*d))^n\*(b/(b\*e - a\*f))^p)\*AppellF1[m + 1, -n, -p, m + 2, (-d)\*((a + b\*x)/(b\*c - a\*d)), (-f)\*((a + b\*x)/(b\*e - a\*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b\*c - a\*d), 0] && GtQ[b/(b\*e - a\*f), 0] && !(GtQ[d/(d\*a - c\*b), 0] && GtQ[d/(d\*e - c\*f), 0] && SimplifierQ[c + d\*x, a + b\*x]) && !(GtQ[f/(f\*a - e\*b), 0] && GtQ[f/(f\*c - e\*d), 0] && SimplifierQ[e + f\*x, a + b\*x])

**Rule 144**

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] :> Dist[(e + f\*x)^FracPart[p]/((b/(b\*e - a\*f))^IntPart[p]\*(b\*((e + f\*x)/(b\*e - a\*f)))^FracPart[p]), Int[(a + b\*x)^m\*(c + d\*x)^n\*(b/(b\*e - a\*f) + b\*f\*(x/(b\*e - a\*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b\*c - a\*d), 0] && !GtQ[b/(b\*e - a\*f), 0]

Rule 2744

Int[((a\_) + (b\_)\*sin[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[Cos[c + d\*x]/(d\*Sqrt[1 + Sin[c + d\*x]]\*Sqrt[1 - Sin[c + d\*x]]), Subst[Int[(a + b\*x)^n/(Sqrt[1 + x]\*Sqrt[1 - x]), x], x, Sin[c + d\*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2\*n]

Rule 3461

Int[((a\_) + Cos[(c\_) + (d\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 3463

Int[((a\_) + Cos[(c\_) + (d\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((e\_)\*(x\_)^(m\_)), x\_Symbol] := Dist[e^IntPart[m]\*(e\*x)^FracPart[m]/x^FracPart[m], Int[x^m\*(a + b\*Cos[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int (ex)^{-1+n} (a + b \cos(c + dx^n))^p dx &= \frac{(x^{-n}(ex)^n) \int x^{-1+n} (a + b \cos(c + dx^n))^p dx}{e} \\
 &= \frac{(x^{-n}(ex)^n) \text{Subst}(\int (a + b \cos(c + dx))^p dx, x, x^n)}{en} \\
 &= -\frac{(x^{-n}(ex)^n \sin(c + dx^n)) \text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx^n)\right)}{den \sqrt{1 - \cos(c + dx^n)} \sqrt{1 + \cos(c + dx^n)}} \\
 &= -\frac{\left(x^{-n}(ex)^n (a + b \cos(c + dx^n))^p \left(-\frac{a+b \cos(c+dx^n)}{-a-b}\right)^{-p} \sin(c + dx^n)\right)}{den \sqrt{1 - \cos(c + dx^n)} \sqrt{1 + \cos(c + dx^n)}} \\
 &= \frac{\sqrt{2} x^{-n}(ex)^n F_1\left(\frac{1}{2}; \frac{1}{2}, -p; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx^n)), \frac{b(1 - \cos(c + dx^n))}{a+b}\right) (a + b \cos(c + dx^n))^{1+p} \csc(c + dx^n)}{den \sqrt{1 + \cos(c + dx^n)}}
 \end{aligned}$$

**Mathematica [A]**

time = 0.30, size = 149, normalized size = 1.14

$$\frac{x^{-n}(ex)^n F_1\left(1 + p; \frac{1}{2}, \frac{1}{2}; 2 + p; \frac{a+b \cos(c+dx^n)}{a-b}, \frac{a+b \cos(c+dx^n)}{a+b}\right) \sqrt{-\frac{b(-1 + \cos(c + dx^n))}{a+b}} \sqrt{\frac{b(1 + \cos(c + dx^n))}{-a+b}} (a + b \cos(c + dx^n))^{1+p} \csc(c + dx^n)}{bden(1 + p)}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1 + n)\*(a + b\*cos[c + d\*x^n])^p,x]

[Out] -(((e\*x)^n\*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b\*cos[c + d\*x^n])/(a - b), (a + b\*cos[c + d\*x^n])/(a + b)]\*Sqrt[-((b\*(-1 + Cos[c + d\*x^n]))/(a + b))])\*Sqrt[(b\*(1 + Cos[c + d\*x^n]))/(-a + b)]\*(a + b\*cos[c + d\*x^n])^(1 + p)\*Csc[c + d\*x^n])/(b\*d\*e\*n\*(1 + p)\*x^n)

**Maple [F]**

time = 0.17, size = 0, normalized size = 0.00

$$\int (ex)^{-1+n} (a + b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+n)\*(a+b\*cos(c+d\*x^n))^p,x)

[Out] int((e\*x)^(-1+n)\*(a+b\*cos(c+d\*x^n))^p,x)

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*cos(c+d\*x^n))^p,x, algorithm="maxima")

[Out] integrate((b\*cos(d\*x^n + c) + a)^p\*(x\*e)^(n - 1), x)

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*cos(c+d\*x^n))^p,x, algorithm="fricas")

[Out] integral((b\*cos(d\*x^n + c) + a)^p\*(x\*e)^(n - 1), x)

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+n)\*(a+b\*cos(c+d\*x\*\*n))\*\*p,x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*cos(c+d\*x^n))^p,x, algorithm="giac")

[Out] integrate((b\*cos(d\*x^n + c) + a)^p\*(x\*e)^(n - 1), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int (ex)^{n-1} (a + b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(n - 1)\*(a + b\*cos(c + d\*x^n))^p,x)

[Out] int((e\*x)^(n - 1)\*(a + b\*cos(c + d\*x^n))^p, x)

### 3.68 $\int (ex)^{-1+2n} (a + b \cos(c + dx^n))^p dx$

Optimal. Leaf size=41

$$\frac{x^{-2n}(ex)^{2n}\text{Int}(x^{-1+2n}(a + b \cos(c + dx^n))^p, x)}{e}$$

[Out]  $(e*x)^{(2*n)}*\text{Unintegrable}(x^{(-1+2*n)}*(a+b*\cos(c+d*x^n))^p,x)/e/(x^{(2*n)})$

Rubi [A]

time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (ex)^{-1+2n} (a + b \cos(c + dx^n))^p dx$$

Verification is not applicable to the result.

[In]  $\text{Int}[(e*x)^{(-1 + 2*n)}*(a + b*\text{Cos}[c + d*x^n])^p,x]$

[Out]  $((e*x)^{(2*n)}*\text{Defer}[\text{Int}[x^{(-1 + 2*n)}*(a + b*\text{Cos}[c + d*x^n])^p, x]])/(e*x^{(2*n)})$

Rubi steps

$$\int (ex)^{-1+2n} (a + b \cos(c + dx^n))^p dx = \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n} (a + b \cos(c + dx^n))^p dx}{e}$$

Mathematica [A]

time = 1.25, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (a + b \cos(c + dx^n))^p dx$$

Verification is not applicable to the result.

[In]  $\text{Integrate}[(e*x)^{(-1 + 2*n)}*(a + b*\text{Cos}[c + d*x^n])^p,x]$

[Out]  $\text{Integrate}[(e*x)^{(-1 + 2*n)}*(a + b*\text{Cos}[c + d*x^n])^p, x]$

Maple [A]

time = 0.07, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (a + b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x)^{-1+2*n}*(a+b*\cos(c+d*x^n))^p, x)$

[Out]  $\text{int}((e*x)^{-1+2*n}*(a+b*\cos(c+d*x^n))^p, x)$

**Maxima** [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x)^{-1+2*n}*(a+b*\cos(c+d*x^n))^p, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}((b*\cos(d*x^n + c) + a)^p*(x*e)^{(2*n - 1)}, x)$

**Fricas** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x)^{-1+2*n}*(a+b*\cos(c+d*x^n))^p, x, \text{algorithm}="fricas")$

[Out]  $\text{integral}((b*\cos(d*x^n + c) + a)^p*(x*e)^{(2*n - 1)}, x)$

**Sympy** [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x)**(-1+2*n)*(a+b*\cos(c+d*x**n))**p, x)$

[Out] Timed out

**Giac** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x)^{-1+2*n}*(a+b*\cos(c+d*x^n))^p, x, \text{algorithm}="giac")$

[Out]  $\text{integrate}((b*\cos(d*x^n + c) + a)^p*(x*e)^{(2*n - 1)}, x)$

**Mupad [A]**

time = 0.00, size = -1, normalized size = -0.02

$$\int (ex)^{2n-1} (a + b \cos(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(2\*n - 1)\*(a + b\*cos(c + d\*x^n))^p, x)

[Out] int((e\*x)^(2\*n - 1)\*(a + b\*cos(c + d\*x^n))^p, x)



$$3.69 \quad \int \frac{\cos(a+bx^n)}{x} dx$$

Optimal. Leaf size=26

$$\frac{\cos(a)\text{CosIntegral}(bx^n)}{n} - \frac{\sin(a)\text{Si}(bx^n)}{n}$$

[Out] Ci(b\*x^n)\*cos(a)/n-Si(b\*x^n)\*sin(a)/n

**Rubi** [A]

time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3459, 3457, 3456}

$$\frac{\cos(a)\text{CosIntegral}(bx^n)}{n} - \frac{\sin(a)\text{Si}(bx^n)}{n}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^n]/x, x]

[Out] (Cos[a]\*CosIntegral[b\*x^n])/n - (Sin[a]\*SinIntegral[b\*x^n])/n

Rule 3456

Int[Sin[(d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] :> Simp[SinIntegral[d\*x^n]/n, x] / ; FreeQ[{d, n}, x]

Rule 3457

Int[Cos[(d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] :> Simp[CosIntegral[d\*x^n]/n, x] / ; FreeQ[{d, n}, x]

Rule 3459

Int[Cos[(c\_) + (d\_.)\*(x\_)^(n\_)]/(x\_), x\_Symbol] :> Dist[Cos[c], Int[Cos[d\*x^n]/x, x], x] - Dist[Sin[c], Int[Sin[d\*x^n]/x, x], x] / ; FreeQ[{c, d, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos(a+bx^n)}{x} dx &= \cos(a) \int \frac{\cos(bx^n)}{x} dx - \sin(a) \int \frac{\sin(bx^n)}{x} dx \\ &= \frac{\cos(a)\text{Ci}(bx^n)}{n} - \frac{\sin(a)\text{Si}(bx^n)}{n} \end{aligned}$$

**Mathematica [A]**

time = 0.06, size = 24, normalized size = 0.92

$$\frac{\cos(a)\text{CosIntegral}(bx^n) - \sin(a)\text{Si}(bx^n)}{n}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*x^n]/x,x]``[Out] (Cos[a]*CosIntegral[b*x^n] - Sin[a]*SinIntegral[b*x^n])/n`**Maple [A]**

time = 0.12, size = 25, normalized size = 0.96

method	result	si
derivativedivides	$\frac{-\sin\text{Integral}(bx^n)\sin(a) + \cos\text{Integral}(bx^n)\cos(a)}{n}$	25
default	$\frac{-\sin\text{Integral}(bx^n)\sin(a) + \cos\text{Integral}(bx^n)\cos(a)}{n}$	25
risch	$\frac{ie^{-ia}\pi\text{csgn}(bx^n)}{2n} - \frac{ie^{-ia}\sin\text{Integral}(bx^n)}{n} - \frac{e^{-ia}\exp\text{Integral}(1,-ibx^n)}{2n} - \frac{e^{ia}\exp\text{Integral}(1,-ibx^n)}{2n}$	75
meijerg	$\frac{\sqrt{\pi} \left( \frac{2\gamma + 2n \ln(x) + \ln(b^2)}{\sqrt{\pi}} - \frac{2\gamma}{\sqrt{\pi}} - \frac{2 \ln(2)}{\sqrt{\pi}} - \frac{2 \ln\left(\frac{bx^n}{2}\right)}{\sqrt{\pi}} + \frac{2 \cos\text{Integral}(bx^n)}{\sqrt{\pi}} \right) \cos(a)}{2n} - \frac{\sin\text{Integral}(bx^n)\sin(a)}{n}$	75

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(a+b*x^n)/x,x,method=_RETURNVERBOSE)``[Out] 1/n*(-Si(b*x^n)*sin(a)+Ci(b*x^n)*cos(a))`**Maxima [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.41, size = 90, normalized size = 3.46

$$\frac{\left( \text{Ei}(ibx^n) + \text{Ei}(-ibx^n) + \text{Ei}\left(ibe^{(n\log(x))}\right) + \text{Ei}\left(-ibe^{(n\log(x))}\right) \right) \cos(a) + \left( i\text{Ei}(ibx^n) - i\text{Ei}(-ibx^n) + i\text{Ei}\left(ibe^{(n\log(x))}\right) - i\text{Ei}\left(-ibe^{(n\log(x))}\right) \right) \sin(a)}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(a+b*x^n)/x,x, algorithm="maxima")`
`[Out] 1/4*((Ei(I*b*x^n) + Ei(-I*b*x^n) + Ei(I*b*e^(n*conjugate(log(x)))) + Ei(-I*b*e^(n*conjugate(log(x)))))*cos(a) + (I*Ei(I*b*x^n) - I*Ei(-I*b*x^n) + I*Ei(I*b*e^(n*conjugate(log(x)))) - I*Ei(-I*b*e^(n*conjugate(log(x)))))*sin(a))`  
`/n`
**Fricas [A]**

time = 0.35, size = 35, normalized size = 1.35

$$\frac{\cos(a)\text{Ci}(bx^n) + \cos(a)\text{Ci}(-bx^n) - 2\sin(a)\text{Si}(bx^n)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)/x,x, algorithm="fricas")

[Out] 1/2\*(cos(a)\*cos\_integral(b\*x^n) + cos(a)\*cos\_integral(-b\*x^n) - 2\*sin(a)\*sin\_integral(b\*x^n))/n

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + bx^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x\*\*n)/x,x)

[Out] Integral(cos(a + b\*x\*\*n)/x, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)/x,x, algorithm="giac")

[Out] integrate(cos(b\*x^n + a)/x, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a + bx^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^n)/x,x)

[Out] int(cos(a + b\*x^n)/x, x)

### 3.70 $\int \frac{\cos^2(a+bx^n)}{x} dx$

Optimal. Leaf size=43

$$\frac{\cos(2a)\text{CosIntegral}(2bx^n)}{2n} + \frac{\log(x)}{2} - \frac{\sin(2a)\text{Si}(2bx^n)}{2n}$$

[Out] 1/2\*Ci(2\*b\*x^n)\*cos(2\*a)/n+1/2\*ln(x)-1/2\*Si(2\*b\*x^n)\*sin(2\*a)/n

Rubi [A]

time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3507, 3459, 3457, 3456}

$$\frac{\cos(2a)\text{CosIntegral}(2bx^n)}{2n} - \frac{\sin(2a)\text{Si}(2bx^n)}{2n} + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^n]^2/x,x]

[Out] (Cos[2\*a]\*CosIntegral[2\*b\*x^n])/(2\*n) + Log[x]/2 - (Sin[2\*a]\*SinIntegral[2\*b\*x^n])/(2\*n)

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] := Dist[Cos[c], Int[Cos[d*x^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x]
```

Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a + bx^n)}{x} dx &= \int \left( \frac{1}{2x} + \frac{\cos(2a + 2bx^n)}{2x} \right) dx \\
&= \frac{\log(x)}{2} + \frac{1}{2} \int \frac{\cos(2a + 2bx^n)}{x} dx \\
&= \frac{\log(x)}{2} + \frac{1}{2} \cos(2a) \int \frac{\cos(2bx^n)}{x} dx - \frac{1}{2} \sin(2a) \int \frac{\sin(2bx^n)}{x} dx \\
&= \frac{\cos(2a)\text{Ci}(2bx^n)}{2n} + \frac{\log(x)}{2} - \frac{\sin(2a)\text{Si}(2bx^n)}{2n}
\end{aligned}$$

**Mathematica [A]**

time = 0.09, size = 37, normalized size = 0.86

$$\frac{\cos(2a)\text{CosIntegral}(2bx^n) + n \log(x) - \sin(2a)\text{Si}(2bx^n)}{2n}$$

Antiderivative was successfully verified.

**[In]** Integrate[Cos[a + b\*x^n]^2/x,x]**[Out]** (Cos[2\*a]\*CosIntegral[2\*b\*x^n] + n\*Log[x] - Sin[2\*a]\*SinIntegral[2\*b\*x^n])/ (2\*n)**Maple [A]**

time = 0.09, size = 40, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{\sin\text{Integral}(2b x^n) \sin(2a)}{2} + \frac{\cosine\text{Integral}(2b x^n) \cos(2a)}{2} + \frac{\ln(b x^n)}{2}}{n}$
default	$\frac{-\frac{\sin\text{Integral}(2b x^n) \sin(2a)}{2} + \frac{\cosine\text{Integral}(2b x^n) \cos(2a)}{2} + \frac{\ln(b x^n)}{2}}{n}$
risch	$\frac{\ln(x)}{2} + \frac{ie^{-2ia} \pi \text{csgn}(bx^n)}{4n} - \frac{ie^{-2ia} \sin\text{Integral}(2b x^n)}{2n} - \frac{e^{-2ia} \exp\text{Integral}(1, -2ib x^n)}{4n} - \frac{e^{2ia} \exp\text{Integral}(1, -2ib x^n)}{4n}$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(cos(a+b\*x^n)^2/x,x,method=\_RETURNVERBOSE)**[Out]** 1/n\*(-1/2\*Si(2\*b\*x^n)\*sin(2\*a)+1/2\*Ci(2\*b\*x^n)\*cos(2\*a)+1/2\*ln(b\*x^n))**Maxima [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.40, size = 99, normalized size = 2.30

$$\frac{\left( \text{Ei}(2i bx^n) + \text{Ei}(-2i bx^n) + \text{Ei}\left(2i be^{\frac{n \log(x)}{b}}\right) + \text{Ei}\left(-2i be^{\frac{n \log(x)}{b}}\right) \right) \cos(2a) + 4n \log(x) + \left( i \text{Ei}(2i bx^n) - i \text{Ei}(-2i bx^n) + i \text{Ei}\left(2i be^{\frac{n \log(x)}{b}}\right) - i \text{Ei}\left(-2i be^{\frac{n \log(x)}{b}}\right) \right) \sin(2a)}{8n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)^2/x,x, algorithm="maxima")

[Out]  $\frac{1}{8} * ((Ei(2*I*b*x^n) + Ei(-2*I*b*x^n) + Ei(2*I*b*e^{(n*conjugate(log(x))))) + Ei(-2*I*b*e^{(n*conjugate(log(x))))) * cos(2*a) + 4*n*log(x) + (I*Ei(2*I*b*x^n) - I*Ei(-2*I*b*x^n) + I*Ei(2*I*b*e^{(n*conjugate(log(x))))) - I*Ei(-2*I*b*e^{(n*conjugate(log(x))))) * sin(2*a)) / n$

**Fricas** [A]

time = 0.36, size = 48, normalized size = 1.12

$$\frac{\cos(2a) Ci(2bx^n) + \cos(2a) Ci(-2bx^n) + 2n \log(x) - 2 \sin(2a) Si(2bx^n)}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)^2/x,x, algorithm="fricas")

[Out]  $\frac{1}{4} * (\cos(2*a) * \cos\_integral(2*b*x^n) + \cos(2*a) * \cos\_integral(-2*b*x^n) + 2*n * \log(x) - 2 * \sin(2*a) * \sin\_integral(2*b*x^n)) / n$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a + bx^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x\*\*n)\*\*2/x,x)

[Out] Integral(cos(a + b\*x\*\*n)\*\*2/x, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)^2/x,x, algorithm="giac")

[Out] integrate(cos(b\*x^n + a)^2/x, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(a + bx^n)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^n)^2/x,x)

[Out] int(cos(a + b\*x^n)^2/x, x)

$$3.71 \quad \int \frac{\cos^3(a+bx^n)}{x} dx$$

**Optimal.** Leaf size=67

$$\frac{3 \cos(a) \operatorname{CosIntegral}(bx^n)}{4n} + \frac{\cos(3a) \operatorname{CosIntegral}(3bx^n)}{4n} - \frac{3 \sin(a) \operatorname{Si}(bx^n)}{4n} - \frac{\sin(3a) \operatorname{Si}(3bx^n)}{4n}$$

[Out]  $3/4 * \operatorname{Ci}(b * x^n) * \cos(a) / n + 1/4 * \operatorname{Ci}(3 * b * x^n) * \cos(3 * a) / n - 3/4 * \operatorname{Si}(b * x^n) * \sin(a) / n - 1/4 * \operatorname{Si}(3 * b * x^n) * \sin(3 * a) / n$

**Rubi [A]**

time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3507, 3459, 3457, 3456}

$$\frac{3 \cos(a) \operatorname{CosIntegral}(bx^n)}{4n} + \frac{\cos(3a) \operatorname{CosIntegral}(3bx^n)}{4n} - \frac{3 \sin(a) \operatorname{Si}(bx^n)}{4n} - \frac{\sin(3a) \operatorname{Si}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

[In] `Int[Cos[a + b*x^n]^3/x, x]`

[Out]  $(3 * \operatorname{Cos}[a] * \operatorname{CosIntegral}[b * x^n]) / (4 * n) + (\operatorname{Cos}[3 * a] * \operatorname{CosIntegral}[3 * b * x^n]) / (4 * n) - (3 * \operatorname{Sin}[a] * \operatorname{SinIntegral}[b * x^n]) / (4 * n) - (\operatorname{Sin}[3 * a] * \operatorname{SinIntegral}[3 * b * x^n]) / (4 * n)$

Rule 3456

`Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] / ; FreeQ[{d, n}, x]`

Rule 3457

`Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[CosIntegral[d*x^n]/n, x] / ; FreeQ[{d, n}, x]`

Rule 3459

`Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] := Dist[Cos[c], Int[Cos[d*x^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] / ; FreeQ[{c, d, n}, x]`

Rule 3507

`Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_)*((e_.)*(x_)^(m_.), x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b * Cos[c + d * x^n])^p, x], x] / ; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]`

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(a + bx^n)}{x} dx &= \int \left( \frac{3 \cos(a + bx^n)}{4x} + \frac{\cos(3a + 3bx^n)}{4x} \right) dx \\ &= \frac{1}{4} \int \frac{\cos(3a + 3bx^n)}{x} dx + \frac{3}{4} \int \frac{\cos(a + bx^n)}{x} dx \\ &= \frac{1}{4} (3 \cos(a)) \int \frac{\cos(bx^n)}{x} dx + \frac{1}{4} \cos(3a) \int \frac{\cos(3bx^n)}{x} dx - \frac{1}{4} (3 \sin(a)) \int \frac{\sin(bx^n)}{x} dx \\ &= \frac{3 \cos(a) \text{Ci}(bx^n)}{4n} + \frac{\cos(3a) \text{Ci}(3bx^n)}{4n} - \frac{3 \sin(a) \text{Si}(bx^n)}{4n} - \frac{\sin(3a) \text{Si}(3bx^n)}{4n} \end{aligned}$$

**Mathematica [A]**

time = 0.12, size = 53, normalized size = 0.79

$$\frac{3 \cos(a) \text{CosIntegral}(bx^n) + \cos(3a) \text{CosIntegral}(3bx^n) - 3 \sin(a) \text{Si}(bx^n) - \sin(3a) \text{Si}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*x^n]^3/x,x]

[Out] (3\*Cos[a]\*CosIntegral[b\*x^n] + Cos[3\*a]\*CosIntegral[3\*b\*x^n] - 3\*Sin[a]\*SinIntegral[b\*x^n] - Sin[3\*a]\*SinIntegral[3\*b\*x^n])/(4\*n)

**Maple [A]**

time = 0.10, size = 52, normalized size = 0.78

method	result
derivativedivides	$\frac{-\frac{\sin \text{Integral}(3b x^n) \sin(3a)}{4} + \frac{\cosine \text{Integral}(3b x^n) \cos(3a)}{4} - \frac{3 \sin \text{Integral}(b x^n) \sin(a)}{4} + \frac{3 \cosine \text{Integral}(b x^n) \cos(a)}{4}}{n}$
default	$\frac{-\frac{\sin \text{Integral}(3b x^n) \sin(3a)}{4} + \frac{\cosine \text{Integral}(3b x^n) \cos(3a)}{4} - \frac{3 \sin \text{Integral}(b x^n) \sin(a)}{4} + \frac{3 \cosine \text{Integral}(b x^n) \cos(a)}{4}}{n}$
risch	$\frac{ie^{-3ia} \pi \text{csgn}(b x^n)}{8n} - \frac{ie^{-3ia} \sin \text{Integral}(3b x^n)}{4n} - \frac{e^{-3ia} \exp \text{Integral}(1, -3ib x^n)}{8n} + \frac{3ie^{-ia} \pi \text{csgn}(b x^n)}{8n} - \frac{3ie^{-ia} \sin \text{Integral}(b x^n)}{4n}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*x^n)^3/x,x,method=\_RETURNVERBOSE)

[Out] 1/n\*(-1/4\*Si(3\*b\*x^n)\*sin(3\*a)+1/4\*Ci(3\*b\*x^n)\*cos(3\*a)-3/4\*Si(b\*x^n)\*sin(a)+3/4\*Ci(b\*x^n)\*cos(a))

**Maxima [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.51, size = 180, normalized size = 2.69

$$\frac{\left( \text{Ei}(3i b x^n) + \text{Ei}(-3i b x^n) + \text{Ei}\left(\frac{3i b x^n}{\sqrt{3}}\right) + \text{Ei}\left(-\frac{3i b x^n}{\sqrt{3}}\right) \right) \cos(3a) + 3 \left( \text{Ei}(i b x^n) + \text{Ei}(-i b x^n) + \text{Ei}\left(\frac{i b x^n}{\sqrt{3}}\right) + \text{Ei}\left(-\frac{i b x^n}{\sqrt{3}}\right) \right) \cos(a) + \left( i \text{Ei}(3i b x^n) - i \text{Ei}(-3i b x^n) + i \text{Ei}\left(\frac{3i b x^n}{\sqrt{3}}\right) - i \text{Ei}\left(-\frac{3i b x^n}{\sqrt{3}}\right) \right) \sin(3a) - 3 \left( i \text{Ei}(i b x^n) + i \text{Ei}(-i b x^n) - i \text{Ei}\left(\frac{i b x^n}{\sqrt{3}}\right) + i \text{Ei}\left(-\frac{i b x^n}{\sqrt{3}}\right) \right) \sin(a)}{16n}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)^3/x,x, algorithm="maxima")

[Out]  $\frac{1}{16} * ((Ei(3*I*b*x^n) + Ei(-3*I*b*x^n) + Ei(3*I*b*e^{(n*\text{conjugate}(\log(x)))}) + Ei(-3*I*b*e^{(n*\text{conjugate}(\log(x)))})) * \cos(3*a) + 3 * (Ei(I*b*x^n) + Ei(-I*b*x^n) + Ei(I*b*e^{(n*\text{conjugate}(\log(x)))}) + Ei(-I*b*e^{(n*\text{conjugate}(\log(x)))})) * \cos(a) + (I*Ei(3*I*b*x^n) - I*Ei(-3*I*b*x^n) + I*Ei(3*I*b*e^{(n*\text{conjugate}(\log(x)))}) - I*Ei(-3*I*b*e^{(n*\text{conjugate}(\log(x)))})) * \sin(3*a) - 3 * (-I*Ei(I*b*x^n) + I*Ei(-I*b*x^n) - I*Ei(I*b*e^{(n*\text{conjugate}(\log(x)))}) + I*Ei(-I*b*e^{(n*\text{conjugate}(\log(x)))})) * \sin(a)) / n$

**Fricas** [A]

time = 0.37, size = 74, normalized size = 1.10

$$\frac{\cos(3a) \text{Ci}(3bx^n) + 3 \cos(a) \text{Ci}(bx^n) + 3 \cos(a) \text{Ci}(-bx^n) + \cos(3a) \text{Ci}(-3bx^n) - 2 \sin(3a) \text{Si}(3bx^n) - 6 \sin(a) \text{Si}(bx^n)}{8n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)^3/x,x, algorithm="fricas")

[Out]  $\frac{1}{8} * (\cos(3*a) * \cos\_integral(3*b*x^n) + 3 * \cos(a) * \cos\_integral(b*x^n) + 3 * \cos(a) * \cos\_integral(-b*x^n) + \cos(3*a) * \cos\_integral(-3*b*x^n) - 2 * \sin(3*a) * \sin\_integral(3*b*x^n) - 6 * \sin(a) * \sin\_integral(b*x^n)) / n$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(a + bx^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x\*\*n)\*\*3/x,x)

[Out] Integral(cos(a + b\*x\*\*n)\*\*3/x, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)^3/x,x, algorithm="giac")

[Out] integrate(cos(b\*x^n + a)^3/x, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + bx^n)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)^3/x,x)
```

```
[Out] int(cos(a + b*x^n)^3/x, x)
```

## 3.72 $\int \frac{\cos^4(a+bx^n)}{x} dx$

**Optimal.** Leaf size=79

$$\frac{\cos(2a)\text{CosIntegral}(2bx^n)}{2n} + \frac{\cos(4a)\text{CosIntegral}(4bx^n)}{8n} + \frac{3\log(x)}{8} - \frac{\sin(2a)\text{Si}(2bx^n)}{2n} - \frac{\sin(4a)\text{Si}(4bx^n)}{8n}$$

[Out]  $1/2*\text{Ci}(2*b*x^n)*\cos(2*a)/n+1/8*\text{Ci}(4*b*x^n)*\cos(4*a)/n+3/8*\ln(x)-1/2*\text{Si}(2*b*x^n)*\sin(2*a)/n-1/8*\text{Si}(4*b*x^n)*\sin(4*a)/n$

**Rubi [A]**

time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3507, 3459, 3457, 3456}

$$\frac{\cos(2a)\text{CosIntegral}(2bx^n)}{2n} + \frac{\cos(4a)\text{CosIntegral}(4bx^n)}{8n} - \frac{\sin(2a)\text{Si}(2bx^n)}{2n} - \frac{\sin(4a)\text{Si}(4bx^n)}{8n} + \frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

[In] `Int[Cos[a + b*x^n]^4/x, x]`

[Out]  $(\text{Cos}[2*a]*\text{CosIntegral}[2*b*x^n])/(2*n) + (\text{Cos}[4*a]*\text{CosIntegral}[4*b*x^n])/(8*n) + (3*\text{Log}[x])/8 - (\text{Sin}[2*a]*\text{SinIntegral}[2*b*x^n])/(2*n) - (\text{Sin}[4*a]*\text{SinIntegral}[4*b*x^n])/(8*n)$

Rule 3456

`Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] / ; FreeQ[{d, n}, x]`

Rule 3457

`Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[CosIntegral[d*x^n]/n, x] / ; FreeQ[{d, n}, x]`

Rule 3459

`Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] := Dist[Cos[c], Int[Cos[d*x^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] / ; FreeQ[{c, d, n}, x]`

Rule 3507

`Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_)*((e_.)*(x_)^(m_.), x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x] / ; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]`

Rubi steps

$$\begin{aligned} \int \frac{\cos^4(a + bx^n)}{x} dx &= \int \left( \frac{3}{8x} + \frac{\cos(2a + 2bx^n)}{2x} + \frac{\cos(4a + 4bx^n)}{8x} \right) dx \\ &= \frac{3 \log(x)}{8} + \frac{1}{8} \int \frac{\cos(4a + 4bx^n)}{x} dx + \frac{1}{2} \int \frac{\cos(2a + 2bx^n)}{x} dx \\ &= \frac{3 \log(x)}{8} + \frac{1}{2} \cos(2a) \int \frac{\cos(2bx^n)}{x} dx + \frac{1}{8} \cos(4a) \int \frac{\cos(4bx^n)}{x} dx - \frac{1}{2} \sin(2a) \int \frac{\sin(2bx^n)}{x} dx \\ &= \frac{\cos(2a) \text{Ci}(2bx^n)}{2n} + \frac{\cos(4a) \text{Ci}(4bx^n)}{8n} + \frac{3 \log(x)}{8} - \frac{\sin(2a) \text{Si}(2bx^n)}{2n} - \frac{\sin(4a) \text{Si}(4bx^n)}{8n} \end{aligned}$$

**Mathematica [A]**

time = 0.13, size = 66, normalized size = 0.84

$$\frac{3 \log(x)}{8} + \frac{4 \cos(2a) \text{CosIntegral}(2bx^n) + \cos(4a) \text{CosIntegral}(4bx^n) - 4 \sin(2a) \text{Si}(2bx^n) - \sin(4a) \text{Si}(4bx^n)}{8n}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*x^n]^4/x,x]

[Out] (3\*Log[x])/8 + (4\*Cos[2\*a]\*CosIntegral[2\*b\*x^n] + Cos[4\*a]\*CosIntegral[4\*b\*x^n] - 4\*Sin[2\*a]\*SinIntegral[2\*b\*x^n] - Sin[4\*a]\*SinIntegral[4\*b\*x^n])/(8\*n)

**Maple [A]**

time = 0.10, size = 66, normalized size = 0.84

method	result
derivativedivides	$\frac{-\frac{\sin \text{Integral}(4b x^n) \sin(4a)}{8} + \frac{\cos \text{Integral}(4b x^n) \cos(4a)}{8} - \frac{\sin \text{Integral}(2b x^n) \sin(2a)}{2} + \frac{\cos \text{Integral}(2b x^n) \cos(2a)}{2} + \frac{3 \ln(b x^n)}{8}}{n}$
default	$\frac{-\frac{\sin \text{Integral}(4b x^n) \sin(4a)}{8} + \frac{\cos \text{Integral}(4b x^n) \cos(4a)}{8} - \frac{\sin \text{Integral}(2b x^n) \sin(2a)}{2} + \frac{\cos \text{Integral}(2b x^n) \cos(2a)}{2} + \frac{3 \ln(b x^n)}{8}}{n}$
risch	$\frac{3 \ln(x)}{8} + \frac{ie^{-4ia} \pi \text{csgn}(b x^n)}{16n} - \frac{ie^{-4ia} \sin \text{Integral}(4b x^n)}{8n} - \frac{e^{-4ia} \exp \text{Integral}(1, -4ib x^n)}{16n} + \frac{ie^{-2ia} \pi \text{csgn}(b x^n)}{4n}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*x^n)^4/x,x,method=\_RETURNVERBOSE)

[Out] 1/n\*(-1/8\*Si(4\*b\*x^n)\*sin(4\*a)+1/8\*Ci(4\*b\*x^n)\*cos(4\*a)-1/2\*Si(2\*b\*x^n)\*sin(2\*a)+1/2\*Ci(2\*b\*x^n)\*cos(2\*a)+3/8\*ln(b\*x^n))

**Maxima [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.52, size = 189, normalized size = 2.39

$$\frac{\left( \text{Ei}(4i a b^n) + \text{Ei}(-4i a b^n) + \text{Ei}\left(4i b^n \sqrt{\frac{a^2 + b^2 x^{2n}}{b^2}}\right) + \text{Ei}\left(-4i b^n \sqrt{\frac{a^2 + b^2 x^{2n}}{b^2}}\right) \right) \cos(4a) + 4 \left( \text{Ei}(2i a b^n) + \text{Ei}(-2i a b^n) + \text{Ei}\left(2i b^n \sqrt{\frac{a^2 + b^2 x^{2n}}{b^2}}\right) + \text{Ei}\left(-2i b^n \sqrt{\frac{a^2 + b^2 x^{2n}}{b^2}}\right) \right) \cos(2a) + 12n \log(x) + \left( \text{Ei}(4i b^n) - \text{Ei}(-4i b^n) + \text{Ei}\left(4i b^n \sqrt{\frac{a^2 + b^2 x^{2n}}{b^2}}\right) - \text{Ei}\left(-4i b^n \sqrt{\frac{a^2 + b^2 x^{2n}}{b^2}}\right) \right) \sin(4a) - 4 \left( \text{Ei}(2i b^n) + \text{Ei}(-2i b^n) + \text{Ei}\left(2i b^n \sqrt{\frac{a^2 + b^2 x^{2n}}{b^2}}\right) + \text{Ei}\left(-2i b^n \sqrt{\frac{a^2 + b^2 x^{2n}}{b^2}}\right) \right) \sin(2a)}{32n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)^4/x,x, algorithm="maxima")

[Out]  $\frac{1}{32} * ((Ei(4*I*b*x^n) + Ei(-4*I*b*x^n) + Ei(4*I*b*e^{(n*\text{conjugate}(\log(x)))}) + Ei(-4*I*b*e^{(n*\text{conjugate}(\log(x)))})) * \cos(4*a) + 4 * (Ei(2*I*b*x^n) + Ei(-2*I*b*x^n) + Ei(2*I*b*e^{(n*\text{conjugate}(\log(x)))}) + Ei(-2*I*b*e^{(n*\text{conjugate}(\log(x)))})) * \cos(2*a) + 12*n*\log(x) + (I*Ei(4*I*b*x^n) - I*Ei(-4*I*b*x^n) + I*Ei(4*I*b*e^{(n*\text{conjugate}(\log(x)))}) - I*Ei(-4*I*b*e^{(n*\text{conjugate}(\log(x)))})) * \sin(4*a) - 4 * (-I*Ei(2*I*b*x^n) + I*Ei(-2*I*b*x^n) - I*Ei(2*I*b*e^{(n*\text{conjugate}(\log(x)))}) + I*Ei(-2*I*b*e^{(n*\text{conjugate}(\log(x)))})) * \sin(2*a)) / n$

**Fricas** [A]

time = 0.38, size = 87, normalized size = 1.10

$$\frac{\cos(4a) \text{Ci}(4bx^n) + 4 \cos(2a) \text{Ci}(2bx^n) + 4 \cos(2a) \text{Ci}(-2bx^n) + \cos(4a) \text{Ci}(-4bx^n) + 6n \log(x) - 2 \sin(4a) \text{Si}(4bx^n) - 8 \sin(2a) \text{Si}(2bx^n)}{16n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)^4/x,x, algorithm="fricas")

[Out]  $\frac{1}{16} * (\cos(4*a) * \cos\_integral(4*b*x^n) + 4 * \cos(2*a) * \cos\_integral(2*b*x^n) + 4 * \cos(2*a) * \cos\_integral(-2*b*x^n) + \cos(4*a) * \cos\_integral(-4*b*x^n) + 6*n*\log(x) - 2*\sin(4*a) * \sin\_integral(4*b*x^n) - 8*\sin(2*a) * \sin\_integral(2*b*x^n)) / n$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(a + bx^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x\*\*n)\*\*4/x,x)

[Out] Integral(cos(a + b\*x\*\*n)\*\*4/x, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)^4/x,x, algorithm="giac")

[Out] integrate(cos(b\*x^n + a)^4/x, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + b x^n)^4}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^n)^4/x,x)

[Out] int(cos(a + b\*x^n)^4/x, x)

### 3.73 $\int \cos(a + bx^n) dx$

Optimal. Leaf size=83

$$-\frac{e^{ia}x(-ibx^n)^{-1/n}\Gamma(\frac{1}{n}, -ibx^n)}{2n} - \frac{e^{-ia}x(ibx^n)^{-1/n}\Gamma(\frac{1}{n}, ibx^n)}{2n}$$

[Out]  $-1/2*\exp(I*a)*x*\text{GAMMA}(1/n, -I*b*x^n)/n/((-I*b*x^n)^{(1/n)}) - 1/2*x*\text{GAMMA}(1/n, I*b*x^n)/\exp(I*a)/n/((I*b*x^n)^{(1/n)})$

Rubi [A]

time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3447, 2239}

$$-\frac{e^{ia}x(-ibx^n)^{-1/n}\Gamma(\frac{1}{n}, -ibx^n)}{2n} - \frac{e^{-ia}x(ibx^n)^{-1/n}\Gamma(\frac{1}{n}, ibx^n)}{2n}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^n], x]

[Out]  $-1/2*(E^{(I*a)*x*\text{Gamma}[n^{(-1)}, (-I)*b*x^n]}/(n*((-I)*b*x^n)^{n^{(-1)}}) - (x*\text{Gamma}[n^{(-1)}, I*b*x^n])/(2*E^{(I*a)*n*(I*b*x^n)^{n^{(-1)}}})$

Rule 2239

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_)), x\_Symbol] := Simp[(-F^a)\*(c + d\*x)\*(Gamma[1/n, (-b)\*(c + d\*x)^n\*Log[F]]/(d\*n\*((-b)\*(c + d\*x)^n\*Log[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

Rule 3447

Int[Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_))^(n\_)], x\_Symbol] := Dist[1/2, Int[E^((-c)\*I - d\*I\*(e + f\*x)^n), x], x] + Dist[1/2, Int[E^(c\*I + d\*I\*(e + f\*x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \cos(a + bx^n) dx &= \frac{1}{2} \int e^{-ia-ibx^n} dx + \frac{1}{2} \int e^{ia+ibx^n} dx \\ &= -\frac{e^{ia}x(-ibx^n)^{-1/n}\Gamma(\frac{1}{n}, -ibx^n)}{2n} - \frac{e^{-ia}x(ibx^n)^{-1/n}\Gamma(\frac{1}{n}, ibx^n)}{2n} \end{aligned}$$

**Mathematica [A]**

time = 0.09, size = 92, normalized size = 1.11

$$\frac{x(b^2x^{2n})^{-1/n} \left( (-ibx^n)^{\frac{1}{n}} \Gamma\left(\frac{1}{n}, ibx^n\right) (\cos(a) - i \sin(a)) + (ibx^n)^{\frac{1}{n}} \Gamma\left(\frac{1}{n}, -ibx^n\right) (\cos(a) + i \sin(a)) \right)}{2n}$$

Antiderivative was successfully verified.

**[In]** Integrate[Cos[a + b\*x^n], x]

**[Out]**  $-1/2*(x*((-I)*b*x^n)^n \Gamma[n^(-1), I*b*x^n]*(\text{Cos}[a] - I*\text{Sin}[a]) + (I*b*x^n)^n \Gamma[n^(-1), (-I)*b*x^n]*(\text{Cos}[a] + I*\text{Sin}[a]))/(n*(b^2*x^(2*n))^n \Gamma[n^(-1)])$

**Maple [C]** Result contains higher order function than in optimal. Order 5 vs. order 4.

time = 0.08, size = 75, normalized size = 0.90

method	result	size
meijerg	$x \text{ hypergeom} \left( \left[ \frac{1}{2n} \right], \left[ \frac{1}{2}, 1 + \frac{1}{2n} \right], -\frac{x^{2n} b^2}{4} \right) \cos(a) - \frac{b x^{1+n} \text{ hypergeom} \left( \left[ \frac{1}{2} + \frac{1}{2n} \right], \left[ \frac{3}{2}, \frac{3}{2} + \frac{1}{2n} \right], -\frac{x^{2n} b^2}{4} \right) \sin(a)}{1+n}$	75

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(cos(a+b\*x^n), x, method=\_RETURNVERBOSE)

**[Out]**  $x*\text{hypergeom}([1/2/n], [1/2, 1+1/2/n], -1/4*x^(2*n)*b^2)*\cos(a) - b/(1+n)*x^(1+n)*\text{hypergeom}([1/2+1/2/n], [3/2, 3/2+1/2/n], -1/4*x^(2*n)*b^2)*\sin(a)$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cos(a+b\*x^n), x, algorithm="maxima")**[Out]** integrate(cos(b\*x^n + a), x)**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(cos(a+b\*x^n), x, algorithm="fricas")**[Out]** integral(cos(b\*x^n + a), x)



**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x\*\*n),x)

[Out] Integral(cos(a + b\*x\*\*n), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n),x, algorithm="giac")

[Out] integrate(cos(b\*x^n + a), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^n),x)

[Out] int(cos(a + b\*x^n), x)

### 3.74 $\int \cos^2(a + bx^n) dx$

**Optimal.** Leaf size=102

$$\frac{x}{2} - \frac{2^{-2-\frac{1}{n}} e^{2ia} x (-ibx^n)^{-1/n} \Gamma(\frac{1}{n}, -2ibx^n)}{n} - \frac{2^{-2-\frac{1}{n}} e^{-2ia} x (ibx^n)^{-1/n} \Gamma(\frac{1}{n}, 2ibx^n)}{n}$$

[Out]  $\frac{1}{2} x - 2^{(-2-1/n)} \exp(2I*a) x \text{GAMMA}(1/n, -2I*b*x^n) / n / ((-I*b*x^n)^{(1/n)}) - 2^{(-2-1/n)} x \text{GAMMA}(1/n, 2I*b*x^n) / \exp(2I*a) / n / (I*b*x^n)^{(1/n)}$

**Rubi [A]**

time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {3449, 3447, 2239}

$$-\frac{e^{2ia} 2^{-\frac{1}{n}-2} x (-ibx^n)^{-1/n} \Gamma(\frac{1}{n}, -2ibx^n)}{n} - \frac{e^{-2ia} 2^{-\frac{1}{n}-2} x (ibx^n)^{-1/n} \Gamma(\frac{1}{n}, 2ibx^n)}{n} + \frac{x}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*x^n]^2, x]

[Out]  $\frac{x}{2} - (2^{(-2-n^{(-1)})} E^{((2I)*a)} x \text{Gamma}[n^{(-1)}, (-2I)*b*x^n]) / (n * ((-I)*b*x^n)^{n^{(-1)}}) - (2^{(-2-n^{(-1)})} x \text{Gamma}[n^{(-1)}, (2I)*b*x^n]) / (E^{((2I)*a)} * n * (I*b*x^n)^{n^{(-1)}})$

**Rule 2239**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^n)), x\_Symbol] :> Simp[(-F^a)\*(c + d\*x)\*(Gamma[1/n, (-b)\*(c + d\*x)^n\*Log[F]]/(d\*n\*((-b)\*(c + d\*x)^n\*Log[F]))^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

**Rule 3447**

Int[Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_)^n)], x\_Symbol] :> Dist[1/2, Int[E^((-c)\*I - d\*I\*(e + f\*x)^n), x], x] + Dist[1/2, Int[E^(c\*I + d\*I\*(e + f\*x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]

**Rule 3449**

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_)^n)]\*(b\_.))^p, x\_Symbol] :> Int[ExpandTrigReduce[(a + b\*Cos[c + d\*(e + f\*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]

Rubi steps

$$\begin{aligned}
\int \cos^2(a + bx^n) dx &= \int \left( \frac{1}{2} + \frac{1}{2} \cos(2a + 2bx^n) \right) dx \\
&= \frac{x}{2} + \frac{1}{2} \int \cos(2a + 2bx^n) dx \\
&= \frac{x}{2} + \frac{1}{4} \int e^{-2ia - 2ibx^n} dx + \frac{1}{4} \int e^{2ia + 2ibx^n} dx \\
&= \frac{x}{2} - \frac{2^{-2-\frac{1}{n}} e^{2ia} x (-ibx^n)^{-1/n} \Gamma\left(\frac{1}{n}, -2ibx^n\right)}{n} - \frac{2^{-2-\frac{1}{n}} e^{-2ia} x (ibx^n)^{-1/n} \Gamma\left(\frac{1}{n}, 2ibx^n\right)}{n}
\end{aligned}$$

**Mathematica [A]**

time = 0.27, size = 94, normalized size = 0.92

$$\frac{x \left( -2n + 2^{-1/n} e^{2ia} (-ibx^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, -2ibx^n\right) + 2^{-1/n} e^{-2ia} (ibx^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, 2ibx^n\right) \right)}{4n}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*x^n]^2, x]`

```
[Out] -1/4*(x*(-2*n + (E^((2*I)*a))*Gamma[n^(-1), (-2*I)*b*x^n])/(2^n^(-1)*((-I)*b*x^n)^n^(-1)) + Gamma[n^(-1), (2*I)*b*x^n]/(2^n^(-1)*E^((2*I)*a)*(I*b*x^n)^n^(-1)))/n
```

**Maple [F]**

time = 0.07, size = 0, normalized size = 0.00

$$\int \cos^2(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(a+b*x^n)^2, x)``[Out] int(cos(a+b*x^n)^2, x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(a+b*x^n)^2, x, algorithm="maxima")``[Out] 1/2*x + 1/2*integrate(cos(2*b*x^n + 2*a), x)`

**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(a+b*x^n)^2,x, algorithm="fricas")``[Out] integral(cos(b*x^n + a)^2, x)`**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^2(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(a+b*x**n)**2,x)``[Out] Integral(cos(a + b*x**n)**2, x)`**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(a+b*x^n)^2,x, algorithm="giac")``[Out] integrate(cos(b*x^n + a)^2, x)`**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a + bx^n)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(a + b*x^n)^2,x)``[Out] int(cos(a + b*x^n)^2, x)`

### 3.75 $\int \cos^3(a + bx^n) dx$

**Optimal.** Leaf size=179

$$\frac{3e^{ia}x(-ibx^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, -ibx^n\right)}{8n} - \frac{3e^{-ia}x(ibx^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, ibx^n\right)}{8n} - \frac{3^{-1/n}e^{3ia}x(-ibx^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, 3ibx^n\right)}{8n}$$

[Out]  $-3/8*\exp(I*a)*x*\text{GAMMA}(1/n, -I*b*x^n)/n/((-I*b*x^n)^(1/n)) - 3/8*x*\text{GAMMA}(1/n, I*b*x^n)/\exp(I*a)/n/((I*b*x^n)^(1/n)) - 1/8*\exp(3*I*a)*x*\text{GAMMA}(1/n, -3*I*b*x^n)/(3^(1/n))/n/((-I*b*x^n)^(1/n)) - 1/8*x*\text{GAMMA}(1/n, 3*I*b*x^n)/(3^(1/n))/\exp(3*I*a)/n/((I*b*x^n)^(1/n))$

**Rubi [A]**

time = 0.06, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ ,

Rules used = {3449, 3447, 2239}

$$\frac{3e^{ia}x(-ibx^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, -ibx^n\right)}{8n} - \frac{e^{3ia}3^{-1/n}x(-ibx^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, -3ibx^n\right)}{8n} - \frac{3e^{-ia}x(ibx^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, ibx^n\right)}{8n} - \frac{e^{-3ia}3^{-1/n}x(ibx^n)^{-1/n} \text{Gamma}\left(\frac{1}{n}, 3ibx^n\right)}{8n}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[a + b*x^n]^3, x]$

[Out]  $(-3*E^{(I*a)*x}*Gamma[n^(-1), (-I)*b*x^n])/(8*n*((-I)*b*x^n)^n^(-1)) - (3*x*Gamma[n^(-1), I*b*x^n])/(8*E^{(I*a)*x}*((I*b*x^n)^n^(-1))) - (E^{((3*I)*a)*x}*Gamma[n^(-1), (-3*I)*b*x^n])/(8*3^n^(-1)*n*((-I)*b*x^n)^n^(-1)) - (x*Gamma[n^(-1), (3*I)*b*x^n])/(8*3^n^(-1)*E^{((3*I)*a)*x}*((I*b*x^n)^n^(-1)))$

**Rule 2239**

$\text{Int}[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))^(n_)), x\_Symbol] \rightarrow \text{Simp}[(-F^a)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)]^n*\text{Log}[F])/(d*n*((-b)*(c + d*x)]^n*\text{Log}[F])^(1/n)), x] /;$   $\text{FreeQ}\{F, a, b, c, d, n\}, x \&\& \text{IntegerQ}[2/n]$

**Rule 3447**

$\text{Int}[\text{Cos}[(c_) + (d_)*((e_) + (f_)*(x_))^(n_)], x\_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[E^{((-c)*I - d*I*(e + f*x)^n)}, x], x] + \text{Dist}[1/2, \text{Int}[E^{(c*I + d*I*(e + f*x)^n)}, x], x] /;$   $\text{FreeQ}\{c, d, e, f, n\}, x$

**Rule 3449**

$\text{Int}[(a_) + \text{Cos}[(c_) + (d_)*((e_) + (f_)*(x_))^(n_)])*(b_)^(p_), x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(a + b*\text{Cos}[c + d*(e + f*x)^n]]^p, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, n\}, x \&\& \text{IGtQ}[p, 1]$

Rubi steps

$$\begin{aligned}
\int \cos^3(a + bx^n) dx &= \int \left( \frac{3}{4} \cos(a + bx^n) + \frac{1}{4} \cos(3a + 3bx^n) \right) dx \\
&= \frac{1}{4} \int \cos(3a + 3bx^n) dx + \frac{3}{4} \int \cos(a + bx^n) dx \\
&= \frac{1}{8} \int e^{-3ia-3ibx^n} dx + \frac{1}{8} \int e^{3ia+3ibx^n} dx + \frac{3}{8} \int e^{-ia-ibx^n} dx + \frac{3}{8} \int e^{ia+ibx^n} dx \\
&= -\frac{3e^{ia}x(-ibx^n)^{-1/n} \Gamma(\frac{1}{n}, -ibx^n)}{8n} - \frac{3e^{-ia}x(ibx^n)^{-1/n} \Gamma(\frac{1}{n}, ibx^n)}{8n} - \frac{3^{-1/n}e^{3ia}x(-ibx^n)^{-1/n} \Gamma(\frac{1}{n}, -3ibx^n)}{8n} - \frac{3^{-1/n}e^{-3ia}x(ibx^n)^{-1/n} \Gamma(\frac{1}{n}, 3ibx^n)}{8n}
\end{aligned}$$

**Mathematica [A]**

time = 0.27, size = 173, normalized size = 0.97

$$\frac{3^{-1/n}e^{-3ia}x(b^2x^{2n})^{-1/n} \left( 3^{1+\frac{1}{n}}e^{4ia}(ibx^n)^{\frac{1}{n}} \Gamma(\frac{1}{n}, -ibx^n) + 3^{1+\frac{1}{n}}e^{2ia}(-ibx^n)^{\frac{1}{n}} \Gamma(\frac{1}{n}, ibx^n) + e^{6ia}(ibx^n)^{\frac{1}{n}} \Gamma(\frac{1}{n}, -3ibx^n) + (-ibx^n)^{\frac{1}{n}} \Gamma(\frac{1}{n}, 3ibx^n) \right)}{8n}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*x^n]^3, x]`

```
[Out] -1/8*(x*(3^(1 + n^(-1))*E^((4*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x^n] + 3^(1 + n^(-1))*E^((2*I)*a)*((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n] + E^((6*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-3*I)*b*x^n] + ((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), (3*I)*b*x^n]))/(3^n^(-1)*E^((3*I)*a)*n*(b^2*x^(2*n))^n^(-1))
```

**Maple [F]**

time = 0.08, size = 0, normalized size = 0.00

$$\int \cos^3(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(a+b*x^n)^3, x)``[Out] int(cos(a+b*x^n)^3, x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(cos(a+b*x^n)^3, x, algorithm="maxima")`

[Out] integrate(cos(b\*x^n + a)^3, x)

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)^3,x, algorithm="fricas")

[Out] integral(cos(b\*x^n + a)^3, x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^3(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x\*\*n)\*\*3,x)

[Out] Integral(cos(a + b\*x\*\*n)\*\*3, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*x^n)^3,x, algorithm="giac")

[Out] integrate(cos(b\*x^n + a)^3, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a + bx^n)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^n)^3,x)

[Out] int(cos(a + b\*x^n)^3, x)

### 3.76 $\int x^m \cos(a + bx^n) dx$

**Optimal.** Leaf size=105

$$-\frac{e^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -ibx^n\right)}{2n} - \frac{e^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, ibx^n\right)}{2n}$$

[Out]  $-1/2*\exp(I*a)*x^{(1+m)*\text{GAMMA}((1+m)/n, -I*b*x^n)/n/((-I*b*x^n)^{((1+m)/n)})-1/2*x^{(1+m)*\text{GAMMA}((1+m)/n, I*b*x^n)/\exp(I*a)/n/((I*b*x^n)^{((1+m)/n)})$

**Rubi [A]**

time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3505, 2250}

$$-\frac{e^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, -ibx^n\right)}{2n} - \frac{e^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, ibx^n\right)}{2n}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^m*\text{Cos}[a + b*x^n], x]$

[Out]  $-1/2*(E^{(I*a)*x^{(1+m)*\text{Gamma}[(1+m)/n, (-I)*b*x^n]})/(n*((-I)*b*x^n)^{((1+m)/n)}) - (x^{(1+m)*\text{Gamma}[(1+m)/n, I*b*x^n]})/(2*E^{(I*a)*n*(I*b*x^n)^{((1+m)/n)})$

Rule 2250

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((e_.) + (f_.)*(x_))^{(m_.)}], x\_Symbol] := \text{Simp}[(-F^a)*((e + f*x)^{(m+1)})/(f*n*((-b)*(c + d*x)^n*\text{Log}[F])^{((m+1)/n)})*\text{Gamma}[(m+1)/n, (-b)*(c + d*x)^n*\text{Log}[F]], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, m, n\}, x] \&\& \text{EqQ}[d*e - c*f, 0]$

Rule 3505

$\text{Int}[\text{Cos}[(c_.) + (d_.)*(x_)^{(n_.)}]*((e_.)*(x_))^{(m_.)}], x\_Symbol] := \text{Dist}[1/2, \text{Int}[(e*x)^m*\text{E}^{((-c)*I - d*I*x^n)}, x], x] + \text{Dist}[1/2, \text{Int}[(e*x)^m*\text{E}^{(c*I + d*I*x^n)}, x], x] /; \text{FreeQ}\{c, d, e, m, n\}, x]$

Rubi steps

$$\begin{aligned} \int x^m \cos(a + bx^n) dx &= \frac{1}{2} \int e^{-ia-ibx^n} x^m dx + \frac{1}{2} \int e^{ia+ibx^n} x^m dx \\ &= -\frac{e^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -ibx^n\right)}{2n} - \frac{e^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, ibx^n\right)}{2n} \end{aligned}$$



**Mathematica [A]**

time = 0.22, size = 115, normalized size = 1.10

$$\frac{x^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}} \left( (-ibx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, ibx^n\right) (\cos(a) - i \sin(a)) + (ibx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -ibx^n\right) (\cos(a) + i \sin(a)) \right)}{2n}$$

Antiderivative was successfully verified.

**[In]** Integrate[x^m\*cos[a + b\*x^n], x]

**[Out]**  $-1/2*(x^{(1+m)}*(((-I)*b*x^n)^{((1+m)/n)}*\Gamma[(1+m)/n, I*b*x^n]*(\cos[a] - I*\sin[a]) + (I*b*x^n)^{((1+m)/n)}*\Gamma[(1+m)/n, (-I)*b*x^n]*(\cos[a] + I*\sin[a]))/(n*(b^2*x^{(2*n)})^{((1+m)/n)})$

**Maple [C]** Result contains higher order function than in optimal. Order 5 vs. order 4.

time = 0.14, size = 111, normalized size = 1.06

method	result
meijerg	$\frac{x^{1+m} \operatorname{hypergeom}\left(\left[\frac{m}{2n} + \frac{1}{2n}\right], \left[\frac{1}{2}, 1 + \frac{m}{2n} + \frac{1}{2n}\right], -\frac{x^{2n}b^2}{4}\right) \cos(a)}{1+m} - \frac{bx^{n+m+1} \operatorname{hypergeom}\left(\left[\frac{1}{2} + \frac{m}{2n} + \frac{1}{2n}\right], \left[\frac{3}{2}, \frac{3}{2} + \frac{m}{2n} + \frac{1}{2n}\right], -\frac{x^{2n}b^2}{4}\right) \sin(a)}{n+m+1}$

Verification of antiderivative is not currently implemented for this CAS.

**[In]** int(x^m\*cos(a+b\*x^n), x, method=\_RETURNVERBOSE)

**[Out]**  $1/(1+m)*x^{(1+m)}*\operatorname{hypergeom}\left(\left[1/2/n*m+1/2/n\right], \left[1/2, 1+1/2/n*m+1/2/n\right], -1/4*x^{(2*n)}*b^2\right)*\cos(a) - b/(n+m+1)*x^{(n+m+1)}*\operatorname{hypergeom}\left(\left[1/2+1/2/n*m+1/2/n\right], \left[3/2, 3/2+1/2/n*m+1/2/n\right], -1/4*x^{(2*n)}*b^2\right)*\sin(a)$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(x^m\*cos(a+b\*x^n), x, algorithm="maxima")**[Out]** integrate(x^m\*cos(b\*x^n + a), x)**Fricas [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

**[In]** integrate(x^m\*cos(a+b\*x^n), x, algorithm="fricas")**[Out]** integral(x^m\*cos(b\*x^n + a), x)

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cos(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cos(a+b\*x\*\*n),x)

[Out] Integral(x\*\*m\*cos(a + b\*x\*\*n), x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+b\*x^n),x, algorithm="giac")

[Out] integrate(x^m\*cos(b\*x^n + a), x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cos(a + b x^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cos(a + b\*x^n),x)

[Out] int(x^m\*cos(a + b\*x^n), x)

### 3.77 $\int x^m \cos^2(a + bx^n) dx$

**Optimal.** Leaf size=141

$$\frac{x^{1+m}}{2(1+m)} - \frac{2^{-\frac{1+m+2n}{n}} e^{2ia} x^{1+m} (-ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2ibx^n\right)}{n} - \frac{2^{-\frac{1+m+2n}{n}} e^{-2ia} x^{1+m} (ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 2ibx^n\right)}{n}$$

[Out]  $1/2*x^{(1+m)/(1+m)-\exp(2*I*a)*x^{(1+m)*\Gamma((1+m)/n,-2*I*b*x^n)/(2^{((1+m+2*n)/n)}/n)/((-I*b*x^n)^{((1+m)/n)}-x^{(1+m)*\Gamma((1+m)/n,2*I*b*x^n)/(2^{((1+m+2*n)/n)}/n))/\exp(2*I*a)/n/((I*b*x^n)^{((1+m)/n)})$

**Rubi [A]**

time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3507, 3505, 2250}

$$\frac{e^{2ia} 2^{-\frac{m+2n+1}{n}} x^{m+1} (-ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, -2ibx^n\right)}{n} - \frac{e^{-2ia} 2^{-\frac{m+2n+1}{n}} x^{m+1} (ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, 2ibx^n\right)}{n} + \frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^m \text{Cos}[a + b*x^n]^2, x]$

[Out]  $x^{(1+m)/(2*(1+m))} - (E^{((2*I)*a)}*x^{(1+m)*\Gamma[(1+m)/n, (-2*I)*b*x^n])/(2^{((1+m+2*n)/n)*n*((-I)*b*x^n)^{((1+m)/n)}} - (x^{(1+m)*\Gamma[(1+m)/n, (2*I)*b*x^n]})/(2^{((1+m+2*n)/n)*E^{((2*I)*a)*n*(I*b*x^n)^{((1+m)/n)}})$

**Rule 2250**

$\text{Int}[(F_)^{((a_) + (b_)*(c_) + (d_)*(x_))^{(n_)}}*((e_) + (f_)*(x_))^{(m_)}], x\_Symbol] := \text{Simp}[(-F^a)*((e + f*x)^{(m+1)})/(f*n*((-b)*(c + d*x))^n*\text{Log}[F])^{((m+1)/n)})*\Gamma[(m+1)/n, (-b)*(c + d*x)^n*\text{Log}[F]], x] /; \text{FreeQ}[\{F, a, b, c, d, e, f, m, n\}, x] \&\& \text{EqQ}[d*e - c*f, 0]$

**Rule 3505**

$\text{Int}[\text{Cos}[(c_) + (d_)*(x_)]^{(n_)}*((e_)*(x_))^{(m_)}], x\_Symbol] := \text{Dist}[1/2, \text{Int}[(e*x)^m * E^{((-c)*I - d*I*x^n)}, x], x] + \text{Dist}[1/2, \text{Int}[(e*x)^m * E^{(c*I + d*I*x^n)}, x], x] /; \text{FreeQ}[\{c, d, e, m, n\}, x]$

**Rule 3507**

$\text{Int}[(a_) + \text{Cos}[(c_) + (d_)*(x_)]^{(n_)}]^{(p_)}*((e_)*(x_))^{(m_)}], x\_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(e*x)^m, (a + b*\text{Cos}[c + d*x^n])^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, n\}, x] \&\& \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned}
\int x^m \cos^2(a + bx^n) dx &= \int \left( \frac{x^m}{2} + \frac{1}{2} x^m \cos(2a + 2bx^n) \right) dx \\
&= \frac{x^{1+m}}{2(1+m)} + \frac{1}{2} \int x^m \cos(2a + 2bx^n) dx \\
&= \frac{x^{1+m}}{2(1+m)} + \frac{1}{4} \int e^{-2ia - 2ibx^n} x^m dx + \frac{1}{4} \int e^{2ia + 2ibx^n} x^m dx \\
&= \frac{x^{1+m}}{2(1+m)} - \frac{2^{-\frac{1+m+2n}{n}} e^{2ia} x^{1+m} (-ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2ibx^n\right)}{n} - \frac{2^{-\frac{1+m+2n}{n}} e^{-2ia} x^{1+m}}{n}
\end{aligned}$$

**Mathematica [A]**

time = 0.58, size = 129, normalized size = 0.91

$$\frac{x^{1+m} \left( -2n + 2^{-\frac{1+m}{n}} e^{2ia} (1+m) (-ibx^n)^{-\frac{1+m}{n}} \text{Gamma}\left(\frac{1+m}{n}, -2ibx^n\right) + 2^{-\frac{1+m}{n}} e^{-2ia} (1+m) (ibx^n)^{-\frac{1+m}{n}} \text{Gamma}\left(\frac{1+m}{n}, 2ibx^n\right) \right)}{4(1+m)n}$$

Antiderivative was successfully verified.

`[In] Integrate[x^m * Cos[a + b * x^n]^2, x]`

```
[Out] -1/4*(x^(1+m)*(-2*n + (E^((2*I)*a))*(1+m)*Gamma[(1+m)/n, (-2*I)*b*x^n])
)/(2^((1+m)/n)*((-I)*b*x^n)^((1+m)/n)) + ((1+m)*Gamma[(1+m)/n, (2*I
)*b*x^n])/(2^((1+m)/n)*E^((2*I)*a)*(I*b*x^n)^((1+m)/n)))/((1+m)*n)
```

**Maple [F]**

time = 0.06, size = 0, normalized size = 0.00

$$\int x^m (\cos^2(a + bx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^m*cos(a+b*x^n)^2,x)``[Out] int(x^m*cos(a+b*x^n)^2,x)`**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^m*cos(a+b*x^n)^2,x, algorithm="maxima")`

[Out]  $1/2*(x*x^m + (m + 1)*\text{integrate}(x^m*\cos(2*b*x^n + 2*a), x))/(m + 1)$

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*cos(a+b*x^n)^2,x, algorithm="fricas")`

[Out] `integral(x^m*cos(b*x^n + a)^2, x)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cos^2(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*cos(a+b*x**n)**2,x)`

[Out] `Integral(x**m*cos(a + b*x**n)**2, x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*cos(a+b*x^n)^2,x, algorithm="giac")`

[Out] `integrate(x^m*cos(b*x^n + a)^2, x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cos(a + bx^n)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*cos(a + b*x^n)^2,x)`

[Out] `int(x^m*cos(a + b*x^n)^2, x)`

### 3.78 $\int x^m \cos^3(a + bx^n) dx$

**Optimal.** Leaf size=229

$$\frac{3e^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -ibx^n\right)}{8n} - \frac{3e^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, ibx^n\right)}{8n} - \frac{3^{-\frac{1+m}{n}} e^{3ia} x^{1+m}}{8n}$$

[Out]  $-3/8*\exp(I*a)*x^{(1+m)*\text{GAMMA}((1+m)/n, -I*b*x^n)/n}/((-I*b*x^n)^{((1+m)/n)}) - 3/8*x^{(1+m)*\text{GAMMA}((1+m)/n, I*b*x^n)/\exp(I*a)/n}/((I*b*x^n)^{((1+m)/n)}) - 1/8*\exp(3*I*a)*x^{(1+m)*\text{GAMMA}((1+m)/n, -3*I*b*x^n)/(3^{((1+m)/n)})/n}/((-I*b*x^n)^{((1+m)/n)}) - 1/8*x^{(1+m)*\text{GAMMA}((1+m)/n, 3*I*b*x^n)/(3^{((1+m)/n)})/\exp(3*I*a)/n}/((I*b*x^n)^{((1+m)/n)})$

**Rubi [A]**

time = 0.14, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3507, 3505, 2250}

$$\frac{3e^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, -ibx^n\right)}{8n} - \frac{3e^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, ibx^n\right)}{8n} - \frac{e^{3ia}3^{-\frac{m+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, -3ibx^n\right)}{8n} - \frac{e^{-3ia}3^{-\frac{m+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, 3ibx^n\right)}{8n}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^m \text{Cos}[a + b*x^n]^3, x]$

[Out]  $(-3*E^{(I*a)*x^{(1+m)*\text{Gamma}[(1+m)/n, (-I)*b*x^n]}}/(8*n*((-I)*b*x^n)^{((1+m)/n)}) - (3*x^{(1+m)*\text{Gamma}[(1+m)/n, I*b*x^n]}/(8*E^{(I*a)*n*(I*b*x^n)^{((1+m)/n)}}) - (E^{((3*I)*a)*x^{(1+m)*\text{Gamma}[(1+m)/n, (-3*I)*b*x^n]}}/(8*3^{((1+m)/n)*n*((-I)*b*x^n)^{((1+m)/n)}}) - (x^{(1+m)*\text{Gamma}[(1+m)/n, (3*I)*b*x^n]}/(8*3^{((1+m)/n)*E^{((3*I)*a)*n*(I*b*x^n)^{((1+m)/n)}}))$

Rule 2250

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})*((e_.) + (f_.)*(x_))^{(m_.)}], x\_Symbol] := \text{Simp}[(-F^a)*((e + f*x)^{(m + 1)})/(f*n*((-b)*(c + d*x)^n*\text{Log}[F])^{((m + 1)/n)})*\text{Gamma}[m + 1, (-b)*(c + d*x)^n*\text{Log}[F]], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, m, n\}, x] \&\& \text{EqQ}[d*e - c*f, 0]$

Rule 3505

$\text{Int}[\text{Cos}[(c_.) + (d_.)*(x_)]^{(n_.)}*((e_.)*(x_))^{(m_.)}], x\_Symbol] := \text{Dist}[1/2, \text{Int}[(e*x)^m * E^{((-c)*I - d*I*x^n)}, x], x] + \text{Dist}[1/2, \text{Int}[(e*x)^m * E^{(c*I + d*I*x^n)}, x], x] /; \text{FreeQ}\{c, d, e, m, n\}, x]$

Rule 3507

$\text{Int}[(a_.) + \text{Cos}[(c_.) + (d_.)*(x_)]^{(n_.)}]^{(p_.)}*((e_.)*(x_))^{(m_.)}, x\_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(e*x)^m, (a + b*\text{Cos}[c + d*x^n])^p, x], x]$

```
;/ FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int x^m \cos^3(a + bx^n) dx &= \int \left( \frac{3}{4} x^m \cos(a + bx^n) + \frac{1}{4} x^m \cos(3a + 3bx^n) \right) dx \\ &= \frac{1}{4} \int x^m \cos(3a + 3bx^n) dx + \frac{3}{4} \int x^m \cos(a + bx^n) dx \\ &= \frac{1}{8} \int e^{-3ia-3ibx^n} x^m dx + \frac{1}{8} \int e^{3ia+3ibx^n} x^m dx + \frac{3}{8} \int e^{-ia-ibx^n} x^m dx + \frac{3}{8} \int e^{ia+ibx^n} x^m dx \\ &= -\frac{3e^{ia} x^{1+m} (-ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -ibx^n\right)}{8n} - \frac{3e^{-ia} x^{1+m} (ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, ibx^n\right)}{8n} \end{aligned}$$

Mathematica [A]

time = 0.64, size = 221, normalized size = 0.97

$$\frac{3^{-\frac{1+m}{n}} e^{-3ia} x^{1+m} (b^2 x^{2n})^{-\frac{1+m}{n}} \left( 3^{\frac{1+m+n}{n}} e^{4ia} (ibx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -ibx^n\right) + 3^{\frac{1+m+n}{n}} e^{2ia} (-ibx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, ibx^n\right) + e^{6ia} (ibx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -3ibx^n\right) + (-ibx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 3ibx^n\right) \right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m * Cos[a + b*x^n]^3, x]
```

```
[Out] -1/8*(x^(1 + m)*(3^((1 + m + n)/n)*E^((4*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-I)*b*x^n] + 3^((1 + m + n)/n)*E^((2*I)*a)*((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, I*b*x^n] + E^((6*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-3*I)*b*x^n] + ((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (3*I)*b*x^n]))/(3^((1 + m)/n)*E^((3*I)*a)*n*(b^2*x^(2*n))^((1 + m)/n))
```

Maple [F]

time = 0.11, size = 0, normalized size = 0.00

$$\int x^m (\cos^3(a + bx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+b*x^n)^3,x)
```

```
[Out] int(x^m*cos(a+b*x^n)^3,x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*cos(a+b*x^n)^3,x, algorithm="maxima")`

[Out] `integrate(x^m*cos(b*x^n + a)^3, x)`

**Fricas** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*cos(a+b*x^n)^3,x, algorithm="fricas")`

[Out] `integral(x^m*cos(b*x^n + a)^3, x)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cos^3(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*cos(a+b*x**n)**3,x)`

[Out] `Integral(x**m*cos(a + b*x**n)**3, x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*cos(a+b*x^n)^3,x, algorithm="giac")`

[Out] `integrate(x^m*cos(b*x^n + a)^3, x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^m \cos(a + bx^n)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*cos(a + b*x^n)^3,x)`

[Out] `int(x^m*cos(a + b*x^n)^3, x)`



### 3.79 $\int x^{-1-n} \cos(a + bx^n) dx$

Optimal. Leaf size=47

$$-\frac{x^{-n} \cos(a + bx^n)}{n} - \frac{b \operatorname{CosIntegral}(bx^n) \sin(a)}{n} - \frac{b \cos(a) \operatorname{Si}(bx^n)}{n}$$

[Out]  $-\cos(a+b*x^n)/n/(x^n)-b*\cos(a)*\operatorname{Si}(b*x^n)/n-b*\operatorname{Ci}(b*x^n)*\sin(a)/n$

**Rubi** [A]

time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3461, 3378, 3384, 3380, 3383}

$$-\frac{b \sin(a) \operatorname{CosIntegral}(bx^n)}{n} - \frac{b \cos(a) \operatorname{Si}(bx^n)}{n} - \frac{x^{-n} \cos(a + bx^n)}{n}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^{(-1-n)}*\operatorname{Cos}[a + b*x^n], x]$

[Out]  $-(\operatorname{Cos}[a + b*x^n]/(n*x^n)) - (b*\operatorname{CosIntegral}[b*x^n]*\operatorname{Sin}[a])/n - (b*\operatorname{Cos}[a]*\operatorname{SinIntegral}[b*x^n])/n$

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c
+ d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

## Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

## Rubi steps

$$\begin{aligned} \int x^{-1-n} \cos(a + bx^n) dx &= \frac{\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2} dx, x, x^n\right)}{n} \\ &= -\frac{x^{-n} \cos(a + bx^n)}{n} - \frac{b \text{Subst}\left(\int \frac{\sin(a+bx)}{x} dx, x, x^n\right)}{n} \\ &= -\frac{x^{-n} \cos(a + bx^n)}{n} - \frac{(b \cos(a)) \text{Subst}\left(\int \frac{\sin(bx)}{x} dx, x, x^n\right)}{n} - \frac{(b \sin(a)) \text{Subst}\left(\int \frac{\cos(bx)}{x} dx, x, x^n\right)}{n} \\ &= -\frac{x^{-n} \cos(a + bx^n)}{n} - \frac{b \text{Ci}(bx^n) \sin(a)}{n} - \frac{b \cos(a) \text{Si}(bx^n)}{n} \end{aligned}$$

## Mathematica [A]

time = 0.10, size = 45, normalized size = 0.96

$$-\frac{x^{-n}(\cos(a + bx^n) + bx^n \text{CosIntegral}(bx^n) \sin(a) + bx^n \cos(a) \text{Si}(bx^n))}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Cos[a + b*x^n], x]
```

```
[Out] -((Cos[a + b*x^n] + b*x^n*CosIntegral[b*x^n]*Sin[a] + b*x^n*Cos[a]*SinIntegral[b*x^n])/(n*x^n))
```

## Maple [A]

time = 0.14, size = 45, normalized size = 0.96

method	result
default	$b \frac{\left(-\frac{\cos(a+bx^n)x^{-n}}{b} - \text{sinIntegral}(bx^n) \cos(a) - \text{cosineIntegral}(bx^n) \sin(a)\right)}{n}$
risch	$\frac{b e^{-ia} \pi \text{csgn}(bx^n)}{2n} - \frac{b e^{-ia} \text{sinIntegral}(bx^n)}{n} + \frac{i b e^{-ia} \text{expIntegral}(1, -ibx^n)}{2n} - \frac{i b e^{ia} \text{expIntegral}(1, -ibx^n)}{2n} - \frac{\cos(a+bx^n)x^{-n}}{n}$
meijerg	error in int/gproduct: numeric exception: division by zero\

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1-n)*cos(a+b*x^n),x,method=_RETURNVERBOSE)`

[Out] `1/n*b*(-cos(a+b*x^n)/b/(x^n)-Si(b*x^n)*cos(a)-Ci(b*x^n)*sin(a))`

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-n)*cos(a+b*x^n),x, algorithm="maxima")`

[Out] `integrate(x^(-n - 1)*cos(b*x^n + a), x)`

**Fricas** [A]

time = 0.38, size = 62, normalized size = 1.32

$$\frac{bx^n \operatorname{Ci}(bx^n) \sin(a) + bx^n \operatorname{Ci}(-bx^n) \sin(a) + 2bx^n \cos(a) \operatorname{Si}(bx^n) + 2 \cos(bx^n + a)}{2nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-n)*cos(a+b*x^n),x, algorithm="fricas")`

[Out] `-1/2*(b*x^n*cos_integral(b*x^n)*sin(a) + b*x^n*cos_integral(-b*x^n)*sin(a) + 2*b*x^n*cos(a)*sin_integral(b*x^n) + 2*cos(b*x^n + a))/(n*x^n)`

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1} \cos(a + bx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1-n)*cos(a+b*x**n),x)`

[Out] `Integral(x**(-n - 1)*cos(a + b*x**n), x)`

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cos(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*cos(b*x^n + a), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(a + b x^n)}{x^{n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)/x^(n + 1),x)
```

```
[Out] int(cos(a + b*x^n)/x^(n + 1), x)
```

### 3.80 $\int x^{-1-n} \cos^2(a + bx^n) dx$

**Optimal.** Leaf size=69

$$\frac{x^{-n}}{2n} - \frac{x^{-n} \cos(2(a + bx^n))}{2n} - \frac{b \operatorname{CosIntegral}(2bx^n) \sin(2a)}{n} - \frac{b \cos(2a) \operatorname{Si}(2bx^n)}{n}$$

[Out]  $-1/2/n/(x^n)^{-1/2} \cos(2*a+2*b*x^n)/n/(x^n) - b*\cos(2*a)*\operatorname{Si}(2*b*x^n)/n - b*\operatorname{Ci}(2*b*x^n)*\sin(2*a)/n$

**Rubi [A]**

time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3507, 3461, 3378, 3384, 3380, 3383}

$$-\frac{b \sin(2a) \operatorname{CosIntegral}(2bx^n)}{n} - \frac{b \cos(2a) \operatorname{Si}(2bx^n)}{n} - \frac{x^{-n} \cos(2(a + bx^n))}{2n} - \frac{x^{-n}}{2n}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^{(-1 - n)} \operatorname{Cos}[a + b*x^n]^2, x]$

[Out]  $-1/2*1/(n*x^n) - \operatorname{Cos}[2*(a + b*x^n)]/(2*n*x^n) - (b*\operatorname{CosIntegral}[2*b*x^n]*\operatorname{Sin}[2*a])/n - (b*\operatorname{Cos}[2*a]*\operatorname{SinIntegral}[2*b*x^n])/n$

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

NeQ[d\*e - c\*f, 0]

### Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

### Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*((e_.)*(x_)^(m_.), x
_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
 \int x^{-1-n} \cos^2(a + bx^n) dx &= \int \left( \frac{x^{-1-n}}{2} + \frac{1}{2} x^{-1-n} \cos(2a + 2bx^n) \right) dx \\
 &= -\frac{x^{-n}}{2n} + \frac{1}{2} \int x^{-1-n} \cos(2a + 2bx^n) dx \\
 &= -\frac{x^{-n}}{2n} + \frac{\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x^2} dx, x, x^n\right)}{2n} \\
 &= -\frac{x^{-n}}{2n} - \frac{x^{-n} \cos(2(a + bx^n))}{2n} - \frac{b \text{Subst}\left(\int \frac{\sin(2a+2bx)}{x} dx, x, x^n\right)}{n} \\
 &= -\frac{x^{-n}}{2n} - \frac{x^{-n} \cos(2(a + bx^n))}{2n} - \frac{(b \cos(2a)) \text{Subst}\left(\int \frac{\sin(2bx)}{x} dx, x, x^n\right)}{n} - \frac{(b \sin(2a)) \text{Subst}\left(\int \frac{\cos(2bx)}{x} dx, x, x^n\right)}{n} \\
 &= -\frac{x^{-n}}{2n} - \frac{x^{-n} \cos(2(a + bx^n))}{2n} - \frac{b \text{Ci}(2bx^n) \sin(2a)}{n} - \frac{b \cos(2a) \text{Si}(2bx^n)}{n}
 \end{aligned}$$

### Mathematica [A]

time = 0.20, size = 53, normalized size = 0.77

$$\frac{-x^{-n}(\cos^2(a + bx^n) + bx^n \text{CosIntegral}(2bx^n) \sin(2a) + bx^n \cos(2a) \text{Si}(2bx^n))}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Cos[a + b*x^n]^2, x]
```

[Out]  $-\left(\frac{\cos(a + b x^n)^2 + b x^n \operatorname{CosIntegral}[2 b x^n] \sin(2 a) + b x^n \cos(2 a) \operatorname{SinIntegral}[2 b x^n]}{n x^n}\right)$

**Maple [A]**

time = 0.11, size = 65, normalized size = 0.94

method	result
default	$-\frac{x^{-n}}{2n} + \frac{b \left( -\frac{\cos(2a+2bx^n)x^{-n}}{2b} - \operatorname{sinIntegral}(2bx^n) \cos(2a) - \operatorname{cosineIntegral}(2bx^n) \sin(2a) \right)}{n}$
risch	$-\frac{x^{-n}}{2n} + \frac{b e^{-2ia} \pi \operatorname{csgn}(bx^n)}{2n} - \frac{b e^{-2ia} \operatorname{sinIntegral}(2bx^n)}{n} + \frac{i b e^{-2ia} \operatorname{expIntegral}(1, -2ibx^n)}{2n} - \frac{i b e^{2ia} \operatorname{expIntegral}(1, -2ibx^n)}{2n}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1-n)*cos(a+b*x^n)^2,x,method=_RETURNVERBOSE)`

[Out]  $-1/2/n/(x^n)+1/n*b*(-1/2*\cos(2*a+2*b*x^n)/(x^n)/b-\operatorname{Si}(2*b*x^n)*\cos(2*a)-\operatorname{Ci}(2*b*x^n)*\sin(2*a))$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-n)*cos(a+b*x^n)^2,x, algorithm="maxima")`

[Out]  $1/2*(n*x^n*\operatorname{integrate}(\cos(2*b*x^n + 2*a)/(x*x^n), x) - 1)/(n*x^n)$

**Fricas [A]**

time = 0.38, size = 72, normalized size = 1.04

$$\frac{bx^n \operatorname{Ci}(2bx^n) \sin(2a) + bx^n \operatorname{Ci}(-2bx^n) \sin(2a) + 2bx^n \cos(2a) \operatorname{Si}(2bx^n) + 2 \cos(bx^n + a)^2}{2nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-n)*cos(a+b*x^n)^2,x, algorithm="fricas")`

[Out]  $-1/2*(b*x^n*\cos\_integral(2*b*x^n)*\sin(2*a) + b*x^n*\cos\_integral(-2*b*x^n)*\sin(2*a) + 2*b*x^n*\cos(2*a)*\sin\_integral(2*b*x^n) + 2*\cos(b*x^n + a)^2)/(n*x^n)$

**Sympy [F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(-1-n)\*cos(a+b\*x\*\*n)\*\*2,x)

[Out] Timed out

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n)\*cos(a+b\*x^n)^2,x, algorithm="giac")

[Out] integrate(x^(-n - 1)\*cos(b\*x^n + a)^2, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + bx^n)^2}{x^{n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^n)^2/x^(n + 1),x)

[Out] int(cos(a + b\*x^n)^2/x^(n + 1), x)



### 3.81 $\int x^{-1-n} \cos^3(a + bx^n) dx$

**Optimal.** Leaf size=113

$$\frac{3x^{-n} \cos(a + bx^n)}{4n} - \frac{x^{-n} \cos(3(a + bx^n))}{4n} - \frac{3b \operatorname{CosIntegral}(bx^n) \sin(a)}{4n} - \frac{3b \operatorname{CosIntegral}(3bx^n) \sin(3a)}{4n} - \frac{3b \operatorname{Ci}(bx^n) \sin(a)}{4n} - \frac{3b \operatorname{Ci}(3bx^n) \sin(3a)}{4n}$$

[Out]  $-3/4*\cos(a+b*x^n)/n/(x^n)-1/4*\cos(3*a+3*b*x^n)/n/(x^n)-3/4*b*\cos(a)*\operatorname{Si}(b*x^n)/n-3/4*b*\cos(3*a)*\operatorname{Si}(3*b*x^n)/n-3/4*b*\operatorname{Ci}(b*x^n)*\sin(a)/n-3/4*b*\operatorname{Ci}(3*b*x^n)*\sin(3*a)/n$

**Rubi [A]**

time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ ,

Rules used = {3507, 3461, 3378, 3384, 3380, 3383}

$$\frac{3b \sin(a) \operatorname{CosIntegral}(bx^n)}{4n} - \frac{3b \sin(3a) \operatorname{CosIntegral}(3bx^n)}{4n} - \frac{3b \cos(a) \operatorname{Si}(bx^n)}{4n} - \frac{3b \cos(3a) \operatorname{Si}(3bx^n)}{4n} - \frac{3x^{-n} \cos(a + bx^n)}{4n} - \frac{x^{-n} \cos(3(a + bx^n))}{4n}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^{(-1 - n)}*\operatorname{Cos}[a + b*x^n]^3, x]$

[Out]  $(-3*\operatorname{Cos}[a + b*x^n])/(4*n*x^n) - \operatorname{Cos}[3*(a + b*x^n)]/(4*n*x^n) - (3*b*\operatorname{CosIntegral}[b*x^n]*\operatorname{Sin}[a])/(4*n) - (3*b*\operatorname{CosIntegral}[3*b*x^n]*\operatorname{Sin}[3*a])/(4*n) - (3*b*\operatorname{Cos}[a]*\operatorname{SinIntegral}[b*x^n])/(4*n) - (3*b*\operatorname{Cos}[3*a]*\operatorname{SinIntegral}[3*b*x^n])/(4*n)$

**Rule 3378**

$\operatorname{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m + 1)}*(\operatorname{Sin}[e + f*x]/(d*(m + 1))), x] - \operatorname{Dist}[f/(d*(m + 1)), \operatorname{Int}[(c + d*x)^{(m + 1)}*\operatorname{Cos}[e + f*x], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \&\& \operatorname{LtQ}[m, -1]$

**Rule 3380**

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{SinIntegral}[e + f*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \&\& \operatorname{EqQ}[d*e - c*f, 0]$

**Rule 3383**

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CosIntegral}[e - \pi/2 + f*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \&\& \operatorname{EqQ}[d*(e - \pi/2) - c*f, 0]$

**Rule 3384**

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Sin}[c*(f/d) + f*x]/(c + d*x), x], x] + \operatorname{Dist}[\operatorname{Sin}[d*e - c*f]$

)/d], Int[Cos[c\*(f/d) + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

### Rule 3461

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol  
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p  
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(  
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(  
m + 1)/n], 0]))

### Rule 3507

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x  
\_Symbol] :> Int[ExpandTrigReduce[(e\*x)^m, (a + b\*Cos[c + d\*x^n])^p, x], x]  
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

### Rubi steps

$$\begin{aligned}
 \int x^{-1-n} \cos^3(a + bx^n) dx &= \int \left( \frac{3}{4} x^{-1-n} \cos(a + bx^n) + \frac{1}{4} x^{-1-n} \cos(3a + 3bx^n) \right) dx \\
 &= \frac{1}{4} \int x^{-1-n} \cos(3a + 3bx^n) dx + \frac{3}{4} \int x^{-1-n} \cos(a + bx^n) dx \\
 &= \frac{\text{Subst}\left(\int \frac{\cos(3a+3bx)}{x^2} dx, x, x^n\right)}{4n} + \frac{3\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2} dx, x, x^n\right)}{4n} \\
 &= -\frac{3x^{-n} \cos(a + bx^n)}{4n} - \frac{x^{-n} \cos(3(a + bx^n))}{4n} - \frac{(3b)\text{Subst}\left(\int \frac{\sin(a+bx)}{x} dx, x, x^n\right)}{4n} \\
 &= -\frac{3x^{-n} \cos(a + bx^n)}{4n} - \frac{x^{-n} \cos(3(a + bx^n))}{4n} - \frac{(3b \cos(a))\text{Subst}\left(\int \frac{\sin(bx)}{x} dx, x, x^n\right)}{4n} \\
 &= -\frac{3x^{-n} \cos(a + bx^n)}{4n} - \frac{x^{-n} \cos(3(a + bx^n))}{4n} - \frac{3b\text{Ci}(bx^n) \sin(a)}{4n} - \frac{3b\text{Ci}(3bx^n) \sin(3a)}{4n}
 \end{aligned}$$

### Mathematica [A]

time = 0.25, size = 95, normalized size = 0.84

$$-\frac{x^{-n}(3 \cos(a + bx^n) + \cos(3(a + bx^n))) + 3bx^n \text{CosIntegral}(bx^n) \sin(a) + 3bx^n \text{CosIntegral}(3bx^n) \sin(3a) + 3bx^n \cos(a) \text{Si}(bx^n) + 3bx^n \cos(3a) \text{Si}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n)\*Cos[a + b\*x^n]^3, x]

[Out]  $-1/4*(3*\text{Cos}[a + b*x^n] + \text{Cos}[3*(a + b*x^n)] + 3*b*x^n*\text{CosIntegral}[b*x^n]*\text{Sin}[a] + 3*b*x^n*\text{CosIntegral}[3*b*x^n]*\text{Sin}[3*a] + 3*b*x^n*\text{Cos}[a]*\text{SinIntegral}[b*x^n] + 3*b*x^n*\text{Cos}[3*a]*\text{SinIntegral}[3*b*x^n])/(n*x^n)$

**Maple [A]**

time = 0.14, size = 101, normalized size = 0.89

method	result
default	$\frac{3b \left( -\frac{\cos(a+bx^n)x^{-n}}{b} - \text{sinIntegral}(bx^n) \cos(a) - \text{cosineIntegral}(bx^n) \sin(a) \right)}{4n} + \frac{3b \left( -\frac{\cos(3a+3bx^n)x^{-n}}{3b} - \text{sinIntegral}(3bx^n) \cos(3a) \right)}{4n}$
risch	$\frac{3be^{-3ia}\pi \text{csgn}(bx^n)}{8n} - \frac{3be^{-3ia} \text{sinIntegral}(3bx^n)}{4n} + \frac{3ibe^{-3ia} \text{expIntegral}(1, -3ibx^n)}{8n} + \frac{3be^{-ia}\pi \text{csgn}(bx^n)}{8n} - \frac{3be^{-ia} \text{sinIntegral}(bx^n)}{4n}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1-n)*cos(a+b*x^n)^3,x,method=_RETURNVERBOSE)`

[Out]  $3/4/n*b*(-\cos(a+b*x^n)/b/(x^n)-\text{Si}(b*x^n)*\cos(a)-\text{Ci}(b*x^n)*\sin(a))+3/4/n*b*(-1/3*\cos(3*a+3*b*x^n)/(x^n)/b-\text{Si}(3*b*x^n)*\cos(3*a)-\text{Ci}(3*b*x^n)*\sin(3*a))$

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-n)*cos(a+b*x^n)^3,x, algorithm="maxima")`

[Out] `integrate(x^(-n - 1)*cos(b*x^n + a)^3, x)`

**Fricas [A]**

time = 0.41, size = 117, normalized size = 1.04

$$\frac{3bx^n \text{Ci}(3bx^n) \sin(3a) + 3bx^n \text{Ci}(-3bx^n) \sin(3a) + 3bx^n \text{Ci}(bx^n) \sin(a) + 3bx^n \text{Ci}(-bx^n) \sin(a) + 6bx^n \cos(3a) \text{Si}(3bx^n) + 6bx^n \cos(a) \text{Si}(bx^n) + 8 \cos(bx^n + a)^3}{8nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-n)*cos(a+b*x^n)^3,x, algorithm="fricas")`

[Out]  $-1/8*(3*b*x^n*\text{cos\_integral}(3*b*x^n)*\text{sin}(3*a) + 3*b*x^n*\text{cos\_integral}(-3*b*x^n)*\text{sin}(3*a) + 3*b*x^n*\text{cos\_integral}(b*x^n)*\text{sin}(a) + 3*b*x^n*\text{cos\_integral}(-b*x^n)*\text{sin}(a) + 6*b*x^n*\text{cos}(3*a)*\text{sin\_integral}(3*b*x^n) + 6*b*x^n*\text{cos}(a)*\text{sin\_integral}(b*x^n) + 8*\text{cos}(b*x^n + a)^3)/(n*x^n)$

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(-1-n)\*cos(a+b\*x\*\*n)\*\*3,x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n)\*cos(a+b\*x^n)^3,x, algorithm="giac")

[Out] integrate(x^(-n - 1)\*cos(b\*x^n + a)^3, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + bx^n)^3}{x^{n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x^n)^3/x^(n + 1),x)

[Out] int(cos(a + b\*x^n)^3/x^(n + 1), x)

### 3.82 $\int x^{-1-2n} \cos(a + bx^n) dx$

**Optimal.** Leaf size=78

$$-\frac{x^{-2n} \cos(a + bx^n)}{2n} - \frac{b^2 \cos(a) \operatorname{CosIntegral}(bx^n)}{2n} + \frac{bx^{-n} \sin(a + bx^n)}{2n} + \frac{b^2 \sin(a) \operatorname{Si}(bx^n)}{2n}$$

[Out]  $-1/2*b^2*Ci(b*x^n)*cos(a)/n-1/2*cos(a+b*x^n)/n/(x^{(2*n)})+1/2*b^2*Si(b*x^n)*sin(a)/n+1/2*b*sin(a+b*x^n)/n/(x^n)$

**Rubi [A]**

time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3461, 3378, 3384, 3380, 3383}

$$-\frac{b^2 \cos(a) \operatorname{CosIntegral}(bx^n)}{2n} + \frac{b^2 \sin(a) \operatorname{Si}(bx^n)}{2n} + \frac{bx^{-n} \sin(a + bx^n)}{2n} - \frac{x^{-2n} \cos(a + bx^n)}{2n}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(-1 - 2*n)}*\text{Cos}[a + b*x^n], x]$

[Out]  $-1/2*\text{Cos}[a + b*x^n]/(n*x^{(2*n)}) - (b^2*\text{Cos}[a]*\text{CosIntegral}[b*x^n])/(2*n) + (b*\text{Sin}[a + b*x^n])/(2*n*x^n) + (b^2*\text{Sin}[a]*\text{SinIntegral}[b*x^n])/(2*n)$

Rule 3378

$\text{Int}[\text{((c_.) + (d_.)*(x_))}^{(m_)}*\text{sin}[\text{(e_.) + (f_.)*(x_)}], x\_Symbol] \text{ :> } \text{Simp}[(c + d*x)^{(m + 1)}*(\text{Sin}[e + f*x]/(d*(m + 1))), x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\text{Cos}[e + f*x], x], x] \text{ /; } \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{LtQ}[m, -1]$

Rule 3380

$\text{Int}[\text{sin}[\text{(e_.) + (f_.)*(x_)}]/\text{((c_.) + (d_.)*(x_))}, x\_Symbol] \text{ :> } \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] \text{ /; } \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

Rule 3383

$\text{Int}[\text{sin}[\text{(e_.) + (f_.)*(x_)}]/\text{((c_.) + (d_.)*(x_))}, x\_Symbol] \text{ :> } \text{Simp}[\text{CosIntegral}[e - \text{Pi}/2 + f*x]/d, x] \text{ /; } \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{EqQ}[d*(e - \text{Pi}/2) - c*f, 0]$

Rule 3384

$\text{Int}[\text{sin}[\text{(e_.) + (f_.)*(x_)}]/\text{((c_.) + (d_.)*(x_))}, x\_Symbol] \text{ :> } \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[c*(f/d) + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[c*(f/d) + f*x]/(c + d*x), x], x] \text{ /; } \text{FreeQ}\{c, d, e, f\}, x \ \&\&$

NeQ[d\*e - c\*f, 0]

### Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol]
 := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x]
 /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

### Rubi steps

$$\begin{aligned}
 \int x^{-1-2n} \cos(a + bx^n) dx &= \frac{\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3} dx, x, x^n\right)}{n} \\
 &= -\frac{x^{-2n} \cos(a + bx^n)}{2n} - \frac{b \text{Subst}\left(\int \frac{\sin(a+bx)}{x^2} dx, x, x^n\right)}{2n} \\
 &= -\frac{x^{-2n} \cos(a + bx^n)}{2n} + \frac{bx^{-n} \sin(a + bx^n)}{2n} - \frac{b^2 \text{Subst}\left(\int \frac{\cos(a+bx)}{x} dx, x, x^n\right)}{2n} \\
 &= -\frac{x^{-2n} \cos(a + bx^n)}{2n} + \frac{bx^{-n} \sin(a + bx^n)}{2n} - \frac{(b^2 \cos(a)) \text{Subst}\left(\int \frac{\cos(bx)}{x} dx, x, x^n\right)}{2n} \\
 &= -\frac{x^{-2n} \cos(a + bx^n)}{2n} - \frac{b^2 \cos(a) \text{Ci}(bx^n)}{2n} + \frac{bx^{-n} \sin(a + bx^n)}{2n} + \frac{b^2 \sin(a) \text{Si}(bx^n)}{2n}
 \end{aligned}$$

### Mathematica [A]

time = 0.14, size = 70, normalized size = 0.90

$$\frac{x^{-2n}(\cos(a + bx^n) + b^2 x^{2n} \cos(a) \text{CosIntegral}(bx^n) - bx^n \sin(a + bx^n) - b^2 x^{2n} \sin(a) \text{Si}(bx^n))}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Cos[a + b*x^n], x]
```

```
[Out] -1/2*(Cos[a + b*x^n] + b^2*x^(2*n)*Cos[a]*CosIntegral[b*x^n] - b*x^n*Sin[a + b*x^n] - b^2*x^(2*n)*Sin[a]*SinIntegral[b*x^n])/(n*x^(2*n))
```

### Maple [A]

time = 0.13, size = 65, normalized size = 0.83

method	result
--------	--------

default	$\frac{b^2 \left( -\frac{x^{-2n} \cos(a+bx^n)}{2b^2} + \frac{\sin(a+bx^n)x^{-n}}{2b} + \frac{\sin \operatorname{Integral}(bx^n) \sin(a)}{2} - \frac{\operatorname{cosineIntegral}(bx^n) \cos(a)}{2} \right)}{n}$
risch	$-\frac{ib^2 e^{-ia} \pi \operatorname{csgn}(bx^n)}{4n} + \frac{ib^2 e^{-ia} \sin \operatorname{Integral}(bx^n)}{2n} + \frac{b^2 e^{-ia} \exp \operatorname{Integral}(1, -ibx^n)}{4n} + \frac{b^2 e^{ia} \exp \operatorname{Integral}(1, -ibx^n)}{4n} - \frac{x^{-2n} \cos(a)}{2n}$
meijerg	$b^2 \sqrt{\pi} \left( -\frac{x^{-2 \left( \frac{-1-2n}{2n} + \frac{1}{2n} \right)} n_2^{-\frac{-1-2n}{n} - \frac{1}{n}}}{\sqrt{\pi} b^2} + \frac{(-1)^{-\frac{-1-2n}{2n} - \frac{1}{2n}} \left( -\Psi \left( 1 - \frac{-1-2n}{2n} - \frac{1}{2n} \right) - \Psi \left( \frac{1}{2} - \frac{-1-2n}{2n} - \frac{1}{2n} \right) + 2n \ln(x) - 2 \ln(2) + \ln(b^2) \right)}{2 \sqrt{\pi} \Gamma \left( -\frac{-1-2n}{n} - \frac{1}{n} \right)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1-2*n)*cos(a+b*x^n),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{n} b^2 \left( -\frac{1}{2} \frac{1}{b^2} \frac{1}{(x^n)^2} \cos(a+b*x^n) + \frac{1}{2} \frac{\sin(a+b*x^n)}{b} \frac{1}{x^n} + \frac{1}{2} \operatorname{Si}(b*x^n) \right) \sin(a) - \frac{1}{2} \operatorname{Ci}(b*x^n) \cos(a)$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-2*n)*cos(a+b*x^n),x, algorithm="maxima")`

[Out] `integrate(x^(-2*n - 1)*cos(b*x^n + a), x)`

**Fricas** [A]

time = 0.37, size = 90, normalized size = 1.15

$$\frac{b^2 x^{2n} \cos(a) \operatorname{Ci}(bx^n) + b^2 x^{2n} \cos(a) \operatorname{Ci}(-bx^n) - 2 b^2 x^{2n} \sin(a) \operatorname{Si}(bx^n) - 2 b x^n \sin(bx^n + a) + 2 \cos(bx^n + a)}{4 n x^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-2*n)*cos(a+b*x^n),x, algorithm="fricas")`

[Out] 
$$\frac{-1/4 * (b^2 * x^{(2*n)} * \cos(a) * \cos\_integral(b*x^n) + b^2 * x^{(2*n)} * \cos(a) * \cos\_integral(-b*x^n) - 2 * b^2 * x^{(2*n)} * \sin(a) * \sin\_integral(b*x^n) - 2 * b * x^n * \sin(b*x^n + a) + 2 * \cos(b*x^n + a)) / (n * x^{(2*n)})}{n * x^{(2*n)}}$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1-2*n)*cos(a+b*x**n),x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1-2\*n)</sup>\*cos(a+b\*x<sup>n</sup>),x, algorithm="giac")

[Out] integrate(x<sup>(-2\*n - 1)</sup>\*cos(b\*x<sup>n</sup> + a), x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + b x^n)}{x^{2n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*x<sup>n</sup>)/x<sup>(2\*n + 1)</sup>,x)

[Out] int(cos(a + b\*x<sup>n</sup>)/x<sup>(2\*n + 1)</sup>, x)



### 3.83 $\int x^{-1-2n} \cos^2(a + bx^n) dx$

**Optimal.** Leaf size=95

$$\frac{x^{-2n}}{4n} - \frac{x^{-2n} \cos(2(a + bx^n))}{4n} - \frac{b^2 \cos(2a) \text{CosIntegral}(2bx^n)}{n} + \frac{bx^{-n} \sin(2(a + bx^n))}{2n} + \frac{b^2 \sin(2a) \text{Si}(2bx^n)}{n}$$

[Out]  $-1/4/n/(x^{(2*n)}) - b^2 * Ci(2*b*x^n) * cos(2*a) / n - 1/4 * cos(2*a + 2*b*x^n) / n / (x^{(2*n)}) + b^2 * Si(2*b*x^n) * sin(2*a) / n + 1/2 * b * sin(2*a + 2*b*x^n) / n / (x^n)$

**Rubi [A]**

time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3507, 3461, 3378, 3384, 3380, 3383}

$$-\frac{b^2 \cos(2a) \text{CosIntegral}(2bx^n)}{n} + \frac{b^2 \sin(2a) \text{Si}(2bx^n)}{n} + \frac{bx^{-n} \sin(2(a + bx^n))}{2n} - \frac{x^{-2n} \cos(2(a + bx^n))}{4n} - \frac{x^{-2n}}{4n}$$

Antiderivative was successfully verified.

[In] Int[x<sup>(-1 - 2\*n)</sup>\*Cos[a + b\*x<sup>n</sup>]<sup>2</sup>,x]

[Out]  $-1/4 * 1 / (n * x^{(2*n)}) - \text{Cos}[2*(a + b*x^n)] / (4*n*x^{(2*n)}) - (b^2 * \text{Cos}[2*a] * \text{CosIntegral}[2*b*x^n]) / n + (b * \text{Sin}[2*(a + b*x^n)]) / (2*n*x^n) + (b^2 * \text{Sin}[2*a] * \text{SinIntegral}[2*b*x^n]) / n$

**Rule 3378**

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[(c + d\*x)<sup>(m + 1)</sup>\*(Sin[e + f\*x]/(d\*(m + 1))), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)<sup>(m + 1)</sup>\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

**Rule 3380**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

**Rule 3383**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

**Rule 3384**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[c\*(f/d) + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)

)/d], Int[Cos[c\*(f/d) + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

### Rule 3461

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol  
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Cos[c + d\*x])^p  
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(  
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(  
m + 1)/n], 0]))

### Rule 3507

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x  
\_Symbol] := Int[ExpandTrigReduce[(e\*x)^m, (a + b\*Cos[c + d\*x^n])^p, x], x]  
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

### Rubi steps

$$\begin{aligned}
 \int x^{-1-2n} \cos^2(a + bx^n) dx &= \int \left( \frac{1}{2} x^{-1-2n} + \frac{1}{2} x^{-1-2n} \cos(2a + 2bx^n) \right) dx \\
 &= -\frac{x^{-2n}}{4n} + \frac{1}{2} \int x^{-1-2n} \cos(2a + 2bx^n) dx \\
 &= -\frac{x^{-2n}}{4n} + \frac{\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x^3} dx, x, x^n\right)}{2n} \\
 &= -\frac{x^{-2n}}{4n} - \frac{x^{-2n} \cos(2(a + bx^n))}{4n} - \frac{b \text{Subst}\left(\int \frac{\sin(2a+2bx)}{x^2} dx, x, x^n\right)}{2n} \\
 &= -\frac{x^{-2n}}{4n} - \frac{x^{-2n} \cos(2(a + bx^n))}{4n} + \frac{bx^{-n} \sin(2(a + bx^n))}{2n} - \frac{b^2 \text{Subst}\left(\int \frac{\cos(2a+2bx)}{x} dx, x, x^n\right)}{n} \\
 &= -\frac{x^{-2n}}{4n} - \frac{x^{-2n} \cos(2(a + bx^n))}{4n} + \frac{bx^{-n} \sin(2(a + bx^n))}{2n} - \frac{(b^2 \cos(2a)) \text{Subst}\left(\int \frac{1}{x} dx, x, x^n\right)}{n} \\
 &= -\frac{x^{-2n}}{4n} - \frac{x^{-2n} \cos(2(a + bx^n))}{4n} - \frac{b^2 \cos(2a) \text{Ci}(2bx^n)}{n} + \frac{bx^{-n} \sin(2(a + bx^n))}{2n}
 \end{aligned}$$

### Mathematica [A]

time = 0.23, size = 82, normalized size = 0.86

$$\frac{x^{-2n}(1 + \cos(2(a + bx^n))) + 4b^2x^{2n} \cos(2a) \text{CosIntegral}(2bx^n) - 2bx^n \sin(2(a + bx^n)) - 4b^2x^{2n} \sin(2a) \text{Si}(2bx^n)}{4n}$$

Antiderivative was successfully verified.

[In] Integrate[x<sup>(-1 - 2\*n)</sup>\*Cos[a + b\*x<sup>n</sup>]<sup>2</sup>,x]

[Out] 
$$\frac{-1/4*(1 + \text{Cos}[2*(a + b*x^n)]) + 4*b^2*x^{(2*n)}*\text{Cos}[2*a]*\text{CosIntegral}[2*b*x^n] - 2*b*x^n*\text{Sin}[2*(a + b*x^n)] - 4*b^2*x^{(2*n)}*\text{Sin}[2*a]*\text{SinIntegral}[2*b*x^n]}{(n*x^{(2*n)})}$$

**Maple** [A]

time = 0.12, size = 89, normalized size = 0.94

method	result
default	$-\frac{x^{-2n}}{4n} + \frac{2b^2 \left( -\frac{x^{-2n} \cos(2a+2b x^n)}{8b^2} + \frac{\sin(2a+2b x^n) x^{-n}}{4b} + \frac{\text{sinIntegral}(2b x^n) \sin(2a)}{2} - \frac{\text{cosineIntegral}(2b x^n) \cos(2a)}{2} \right)}{n}$
risch	$-\frac{x^{-2n}}{4n} - \frac{ib^2 e^{-2ia} \pi \text{csgn}(b x^n)}{2n} + \frac{ib^2 e^{-2ia} \text{sinIntegral}(2b x^n)}{n} + \frac{b^2 e^{-2ia} \text{expIntegral}(1, -2ib x^n)}{2n} + \frac{b^2 e^{2ia} \text{expIntegral}(1, -2ib x^n)}{2n}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>(-1-2\*n)</sup>\*cos(a+b\*x<sup>n</sup>)<sup>2</sup>,x,method=\_RETURNVERBOSE)

[Out] 
$$-1/4/(x^n)^2/n+2/n*b^2*(-1/8/(x^n)^2/b^2*\cos(2*a+2*b*x^n)+1/4*\sin(2*a+2*b*x^n)/(x^n)/b+1/2*Si(2*b*x^n)*\sin(2*a)-1/2*Ci(2*b*x^n)*\cos(2*a))$$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1-2\*n)</sup>\*cos(a+b\*x<sup>n</sup>)<sup>2</sup>,x, algorithm="maxima")

[Out] 
$$1/4*(2*n*x^{(2*n)}*\text{integrate}(\cos(2*b*x^n + 2*a)/(x*x^{(2*n)}), x) - 1)/(n*x^{(2*n)})$$

**Fricas** [A]

time = 0.38, size = 106, normalized size = 1.12

$$\frac{b^2 x^{2n} \cos(2a) \text{Ci}(2bx^n) + b^2 x^{2n} \cos(2a) \text{Ci}(-2bx^n) - 2b^2 x^{2n} \sin(2a) \text{Si}(2bx^n) - 2bx^n \cos(bx^n + a) \sin(bx^n + a) + \cos(bx^n + a)^2}{2nx^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1-2\*n)</sup>\*cos(a+b\*x<sup>n</sup>)<sup>2</sup>,x, algorithm="fricas")

[Out] 
$$-1/2*(b^2*x^{(2*n)}*\cos(2*a)*\cos\_integral(2*b*x^n) + b^2*x^{(2*n)}*\cos(2*a)*\cos\_integral(-2*b*x^n) - 2*b^2*x^{(2*n)}*\sin(2*a)*\sin\_integral(2*b*x^n) - 2*b*x^n*\cos(b*x^n + a)*\sin(b*x^n + a) + \cos(b*x^n + a)^2)/(n*x^{(2*n)})$$

**Sympy** [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1-2*n)*cos(a+b*x**n)**2,x)`

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-2*n)*cos(a+b*x^n)^2,x, algorithm="giac")`

[Out] `integrate(x^(-2*n - 1)*cos(b*x^n + a)^2, x)`

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + bx^n)^2}{x^{2n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x^n)^2/x^(2*n + 1),x)`

[Out] `int(cos(a + b*x^n)^2/x^(2*n + 1), x)`

### 3.84 $\int x^{-1-2n} \cos^3(a + bx^n) dx$

**Optimal.** Leaf size=165

$$\frac{3x^{-2n} \cos(a + bx^n)}{8n} - \frac{x^{-2n} \cos(3(a + bx^n))}{8n} - \frac{3b^2 \cos(a) \text{CosIntegral}(bx^n)}{8n} - \frac{9b^2 \cos(3a) \text{CosIntegral}(3bx^n)}{8n} +$$

[Out]  $-3/8*b^2*Ci(b*x^n)*\cos(a)/n-9/8*b^2*Ci(3*b*x^n)*\cos(3*a)/n-3/8*\cos(a+b*x^n)/n/(x^{(2*n)})-1/8*\cos(3*a+3*b*x^n)/n/(x^{(2*n)})+3/8*b^2*Si(b*x^n)*\sin(a)/n+9/8*b^2*Si(3*b*x^n)*\sin(3*a)/n+3/8*b*\sin(a+b*x^n)/n/(x^n)+3/8*b*\sin(3*a+3*b*x^n)/n/(x^n)$

**Rubi [A]**

time = 0.18, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3507, 3461, 3378, 3384, 3380, 3383}

$$-\frac{3b^2 \cos(a) \text{CosIntegral}(bx^n)}{8n} - \frac{9b^2 \cos(3a) \text{CosIntegral}(3bx^n)}{8n} + \frac{3b^2 \sin(a) \text{Si}(bx^n)}{8n} + \frac{9b^2 \sin(3a) \text{Si}(3bx^n)}{8n} + \frac{3bx^{-n} \sin(a + bx^n)}{8n} + \frac{3bx^{-n} \sin(3(a + bx^n))}{8n} - \frac{3x^{-2n} \cos(a + bx^n)}{8n} - \frac{x^{-2n} \cos(3(a + bx^n))}{8n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - 2\*n)\*Cos[a + b\*x^n]^3,x]

[Out]  $(-3*\text{Cos}[a + b*x^n])/(8*n*x^{(2*n)}) - \text{Cos}[3*(a + b*x^n)]/(8*n*x^{(2*n)}) - (3*b^2*\text{Cos}[a]*\text{CosIntegral}[b*x^n])/(8*n) - (9*b^2*\text{Cos}[3*a]*\text{CosIntegral}[3*b*x^n])/(8*n) + (3*b*\text{Sin}[a + b*x^n])/(8*n*x^n) + (3*b*\text{Sin}[3*(a + b*x^n)])/(8*n*x^n) + (3*b^2*\text{Sin}[a]*\text{SinIntegral}[b*x^n])/(8*n) + (9*b^2*\text{Sin}[3*a]*\text{SinIntegral}[3*b*x^n])/(8*n)$

**Rule 3378**

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*(Sin[e + f\*x]/(d\*(m + 1))), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

**Rule 3380**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

**Rule 3383**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

**Rule 3384**

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

### Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_)^(m_.), x
_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^{-1-2n} \cos^3(a + bx^n) dx &= \int \left( \frac{3}{4} x^{-1-2n} \cos(a + bx^n) + \frac{1}{4} x^{-1-2n} \cos(3a + 3bx^n) \right) dx \\
&= \frac{1}{4} \int x^{-1-2n} \cos(3a + 3bx^n) dx + \frac{3}{4} \int x^{-1-2n} \cos(a + bx^n) dx \\
&= \frac{\text{Subst}\left(\int \frac{\cos(3a+3bx)}{x^3} dx, x, x^n\right)}{4n} + \frac{3\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3} dx, x, x^n\right)}{4n} \\
&= -\frac{3x^{-2n} \cos(a + bx^n)}{8n} - \frac{x^{-2n} \cos(3(a + bx^n))}{8n} - \frac{(3b)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2} dx, x, x^n\right)}{8n} \\
&= -\frac{3x^{-2n} \cos(a + bx^n)}{8n} - \frac{x^{-2n} \cos(3(a + bx^n))}{8n} + \frac{3bx^{-n} \sin(a + bx^n)}{8n} + \frac{3bx^{-n}}{8n} \\
&= -\frac{3x^{-2n} \cos(a + bx^n)}{8n} - \frac{x^{-2n} \cos(3(a + bx^n))}{8n} + \frac{3bx^{-n} \sin(a + bx^n)}{8n} + \frac{3bx^{-n}}{8n} \\
&= -\frac{3x^{-2n} \cos(a + bx^n)}{8n} - \frac{x^{-2n} \cos(3(a + bx^n))}{8n} - \frac{3b^2 \cos(a)\text{Ci}(bx^n)}{8n} - \frac{9b^2 \cos(3a)\text{Si}(3bx^n)}{8n}
\end{aligned}$$

### Mathematica [A]

time = 0.38, size = 141, normalized size = 0.85

$$-\frac{x^{-2n}(3 \cos(a + bx^n) + \cos(3(a + bx^n)) + 3b^2 x^{2n} \cos(a) \text{CosIntegral}(bx^n) + 9b^2 x^{2n} \cos(3a) \text{CosIntegral}(3bx^n) - 3bx^n \sin(a + bx^n) - 3bx^n \sin(3(a + bx^n)) - 3b^2 x^{2n} \sin(a) \text{Si}(bx^n) - 9b^2 x^{2n} \sin(3a) \text{Si}(3bx^n))}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Cos[a + b*x^n]^3,x]
```

```
[Out] -1/8*(3*Cos[a + b*x^n] + Cos[3*(a + b*x^n)] + 3*b^2*x^(2*n)*Cos[a]*CosIntegral[b*x^n] + 9*b^2*x^(2*n)*Cos[3*a]*CosIntegral[3*b*x^n] - 3*b*x^n*Sin[a + b*x^n] - 3*b*x^n*Sin[3*(a + b*x^n)] - 3*b^2*x^(2*n)*Sin[a]*SinIntegral[b*x^n] - 9*b^2*x^(2*n)*Sin[3*a]*SinIntegral[3*b*x^n])/(n*x^(2*n))
```

**Maple [A]**

time = 0.16, size = 144, normalized size = 0.87

method	result
default	$\frac{3b^2 \left( -\frac{x^{-2n} \cos(a+bx^n)}{2b^2} + \frac{\sin(a+bx^n)x^{-n}}{2b} + \frac{\sinIntegral(bx^n) \sin(a)}{2} - \frac{\cosineIntegral(bx^n) \cos(a)}{2} \right)}{4n} + \frac{9b^2 \left( -\frac{\cos(3a+3bx^n)x^{-2n}}{18b^2} + \frac{\sin(3a+3bx^n)x^{-n}}{18b} + \frac{\sinIntegral(3bx^n) \sin(3a)}{2} - \frac{\cosineIntegral(3bx^n) \cos(3a)}{2} \right)}{4n}$
risch	$-\frac{9ib^2e^{-3ia}\pi \operatorname{csgn}(bx^n)}{16n} + \frac{9ib^2e^{-3ia} \sinIntegral(3bx^n)}{8n} + \frac{9b^2e^{-3ia} \expIntegral(1,-3ibx^n)}{16n} - \frac{3ib^2e^{-ia}\pi \operatorname{csgn}(bx^n)}{16n} + \frac{3ib^2e^{-ia} \sinIntegral(bx^n)}{8n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*cos(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/n*b^2*(-1/2/b^2/(x^n)^2*cos(a+b*x^n)+1/2*sin(a+b*x^n)/b/(x^n)+1/2*Si(b*x^n)*sin(a)-1/2*Ci(b*x^n)*cos(a))+9/4/n*b^2*(-1/18*cos(3*a+3*b*x^n)/(x^n)^2/b^2+1/6*sin(3*a+3*b*x^n)/(x^n)/b+1/2*Si(3*b*x^n)*sin(3*a)-1/2*Ci(3*b*x^n)*cos(3*a))
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a)^3, x)
```

**Fricas [A]**

time = 0.41, size = 167, normalized size = 1.01

$$\frac{9b^2x^{2n} \cos(3a) \operatorname{Ci}(3bx^n) + 3b^2x^{2n} \cos(a) \operatorname{Ci}(bx^n) + 3b^2x^{2n} \cos(a) \operatorname{Ci}(-bx^n) + 9b^2x^{2n} \cos(3a) \operatorname{Ci}(-3bx^n) - 24bx^n \cos(bx^n + a)^2 \sin(bx^n + a) - 18b^2x^{2n} \sin(3a) \operatorname{Si}(3bx^n) - 6b^2x^{2n} \sin(a) \operatorname{Si}(bx^n) + 8 \cos(bx^n + a)^3}{16nx^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(9*b^2*x^(2*n)*cos(3*a)*cos_integral(3*b*x^n) + 3*b^2*x^(2*n)*cos(a)*cos_integral(b*x^n) + 3*b^2*x^(2*n)*cos(a)*cos_integral(-b*x^n) + 9*b^2*x^(2*n)*cos(3*a)*cos_integral(-3*b*x^n) - 24*b*x^n*cos(b*x^n + a)^2*sin(b*x^n
```

```
+ a) - 18*b^2*x^(2*n)*sin(3*a)*sin_integral(3*b*x^n) - 6*b^2*x^(2*n)*sin(a)
*sin_integral(b*x^n) + 8*cos(b*x^n + a)^3/(n*x^(2*n))
```

**Sympy [F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*cos(a+b*x**n)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a)^3, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a + bx^n)^3}{x^{2n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)^3/x^(2*n + 1),x)
```

```
[Out] int(cos(a + b*x^n)^3/x^(2*n + 1), x)
```



### 3.85 $\int x^2 \cos((a + bx)^2) dx$

**Optimal.** Leaf size=99

$$\frac{a^2 \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b^3} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{2b^3} - \frac{a \sin((a + bx)^2)}{b^3} + \frac{(a + bx) \sin((a + bx)^2)}{2b^3}$$

[Out]  $-a*\sin((b*x+a)^2)/b^3+1/2*(b*x+a)*\sin((b*x+a)^2)/b^3+1/2*a^2*\operatorname{FresnelC}((b*x+a)*2^{(1/2)}/\operatorname{Pi}^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^3-1/4*\operatorname{FresnelS}((b*x+a)*2^{(1/2)}/\operatorname{Pi}^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^3$

**Rubi [A]**

time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3515, 3433, 3461, 2717, 3467, 3432}

$$\frac{\sqrt{\frac{\pi}{2}} a^2 \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b^3} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{2b^3} - \frac{a \sin((a + bx)^2)}{b^3} + \frac{(a + bx) \sin((a + bx)^2)}{2b^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*\operatorname{Cos}[(a + b*x)^2], x]$

[Out]  $(a^2*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{FresnelC}[\operatorname{Sqrt}[2/\operatorname{Pi}]*(a + b*x)])/b^3 - (\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{FresnelS}[\operatorname{Sqrt}[2/\operatorname{Pi}]*(a + b*x)])/(2*b^3) - (a*\operatorname{Sin}[(a + b*x)^2])/b^3 + ((a + b*x)*\operatorname{Sin}[(a + b*x)^2])/(2*b^3)$

Rule 2717

$\operatorname{Int}[\sin[\operatorname{Pi}/2 + (c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \operatorname{Simp}[\sin[c + d*x]/d, x] /;$   
FreeQ[{c, d}, x]

Rule 3432

$\operatorname{Int}[\sin[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[\operatorname{Pi}/2]/(f*\operatorname{Rt}[d, 2]))*\operatorname{FresnelS}[\operatorname{Sqrt}[2/\operatorname{Pi}]*\operatorname{Rt}[d, 2]*(e + f*x)], x] /;$  FreeQ[{d, e, f}, x]

Rule 3433

$\operatorname{Int}[\cos[(d_.)*((e_.) + (f_.)*(x_.))^2], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[\operatorname{Pi}/2]/(f*\operatorname{Rt}[d, 2]))*\operatorname{FresnelC}[\operatorname{Sqrt}[2/\operatorname{Pi}]*\operatorname{Rt}[d, 2]*(e + f*x)], x] /;$  FreeQ[{d, e, f}, x]

Rule 3461

$\operatorname{Int}[(a_.) + \cos[(c_.) + (d_.)*(x_.)^n]*(b_.)]^{(p_.)*(x_.)^m}, x\_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m + 1)/n] - 1)*(a + b*\cos[c + d*x])^p}], x]$

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

### Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] := Simp[e^(n - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

### Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Module[{k = If[FractionQ[n], Denominator[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
 \int x^2 \cos((a + bx)^2) dx &= \frac{\text{Subst}\left(\int (a^2 \cos(x^2) - 2ax \cos(x^2) + x^2 \cos(x^2)) dx, x, a + bx\right)}{b^3} \\
 &= \frac{\text{Subst}\left(\int x^2 \cos(x^2) dx, x, a + bx\right)}{b^3} - \frac{(2a)\text{Subst}\left(\int x \cos(x^2) dx, x, a + bx\right)}{b^3} + \frac{a^2 \text{Subst}\left(\int \cos(x^2) dx, x, a + bx\right)}{b^3} \\
 &= \frac{a^2 \sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b^3} + \frac{(a + bx) \sin((a + bx)^2)}{2b^3} - \frac{\text{Subst}\left(\int \sin(x^2) dx, x, a + bx\right)}{2b^3} \\
 &= \frac{a^2 \sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b^3} - \frac{\sqrt{\frac{\pi}{2}} S\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{2b^3} - \frac{a \sin((a + bx)^2)}{b^3} + \frac{(a + bx) \sin((a + bx)^2)}{2b^3}
 \end{aligned}$$

### Mathematica [A]

time = 0.29, size = 76, normalized size = 0.77

$$\frac{-2a^2 \sqrt{2\pi} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a + bx)\right) + \sqrt{2\pi} S\left(\sqrt{\frac{2}{\pi}}(a + bx)\right) + 2(a - bx) \sin((a + bx)^2)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[(a + b*x)^2], x]
```

[Out]  $-1/4*(-2*a^2*\text{Sqrt}[2*\text{Pi}]*\text{FresnelC}[\text{Sqrt}[2/\text{Pi}]*(a + b*x)] + \text{Sqrt}[2*\text{Pi}]*\text{FresnelS}[\text{Sqrt}[2/\text{Pi}]*(a + b*x)] + 2*(a - b*x)*\text{Sin}[(a + b*x)^2])/b^3$

**Maple [A]**

time = 0.17, size = 131, normalized size = 1.32

method	result
default	$\frac{x \sin(x^2 b^2 + 2abx + a^2)}{2b^2} - \frac{a \left( \frac{\sin(x^2 b^2 + 2abx + a^2)}{2b^2} - \frac{a \sqrt{2} \sqrt{\pi} \text{FresnelC}\left(\frac{\sqrt{2} (b^2 x + ab)}{\sqrt{\pi} \sqrt{b^2}}\right)}{2b \sqrt{b^2}} \right)}{b} - \frac{\sqrt{2} \sqrt{\pi} \text{S}\left(\frac{\sqrt{2} (b^2 x + ab)}{\sqrt{\pi} \sqrt{b^2}}\right)}{4b^2 \sqrt{b^2}}$
risch	$-\frac{a^2(-1)^{\frac{3}{4}} \sqrt{\pi} \text{erf}\left(b(-1)^{\frac{1}{4}} x + (-1)^{\frac{1}{4}} a\right)}{4b^3} - \frac{(-1)^{\frac{1}{4}} \sqrt{\pi} \text{erf}\left(b(-1)^{\frac{1}{4}} x + (-1)^{\frac{1}{4}} a\right)}{8b^3} - \frac{a^2 \sqrt{\pi} \text{erf}\left(-b\sqrt{-i} x + \frac{ia}{\sqrt{-i}}\right)}{4b^3 \sqrt{-i}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos((b*x+a)^2),x,method=_RETURNVERBOSE)`

[Out]  $1/2/b^2*x*\sin(b^2*x^2+2*a*b*x+a^2)-a/b*(1/2/b^2*\sin(b^2*x^2+2*a*b*x+a^2)-1/2*a/b*2^{(1/2)}*\text{Pi}^{(1/2)}/(b^2)^{(1/2)}*\text{FresnelC}(2^{(1/2)}/\text{Pi}^{(1/2)}/(b^2)^{(1/2)}*(b^2*x+a*b))-1/4/b^2*2^{(1/2)}*\text{Pi}^{(1/2)}/(b^2)^{(1/2)}*\text{FresnelS}(2^{(1/2)}/\text{Pi}^{(1/2)}/(b^2)^{(1/2)}*(b^2*x+a*b))$

**Maxima [C]** Result contains complex when optimal does not.

time = 0.78, size = 258, normalized size = 2.61

$4abx \left( -i e^{(b^2 x^2 + 2abx + a^2)} + i e^{(-b^2 x^2 - 2abx - a^2)} \right) + 4a^2 \left( -i e^{(b^2 x^2 + 2abx + a^2)} + i e^{(-b^2 x^2 - 2abx - a^2)} \right) - \sqrt{b^2 x^2 + 2abx + a^2} \left( (-i - 1) \sqrt{2} \sqrt{\pi} \left( \text{erf}\left(\frac{\sqrt{b^2 x^2 + 2abx + a^2}}{\sqrt{\pi}}\right) - 1 \right) + (i + 1) \sqrt{2} \sqrt{\pi} \left( \text{erf}\left(\frac{\sqrt{-b^2 x^2 - 2abx - a^2}}{\sqrt{\pi}}\right) - 1 \right) \right) a^2 + (i + 1) \sqrt{2} \Gamma\left(\frac{3}{2}, i b^2 x^2 + 2abx + a^2\right) - (i - 1) \sqrt{2} \Gamma\left(\frac{3}{2}, -i b^2 x^2 - 2abx - a^2\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos((b*x+a)^2),x, algorithm="maxima")`

[Out]  $-1/8*(4*a*b*x*(-I*e^{(I*b^2*x^2 + 2*I*a*b*x + I*a^2)} + I*e^{(-I*b^2*x^2 - 2*I*a*b*x - I*a^2)}) + 4*a^2*(-I*e^{(I*b^2*x^2 + 2*I*a*b*x + I*a^2)} + I*e^{(-I*b^2*x^2 - 2*I*a*b*x - I*a^2)}) - \text{sqrt}(b^2*x^2 + 2*a*b*x + a^2)*((-I - 1)*\text{sqrt}(2)*\text{sqrt}(\text{pi})*(\text{erf}(\text{sqrt}(I*b^2*x^2 + 2*I*a*b*x + I*a^2)) - 1) + (I + 1)*\text{sqrt}(2)*\text{sqrt}(\text{pi})*(\text{erf}(\text{sqrt}(-I*b^2*x^2 - 2*I*a*b*x - I*a^2)) - 1))*a^2 + (I + 1)*\text{sqrt}(2)*\text{gamma}(3/2, I*b^2*x^2 + 2*I*a*b*x + I*a^2) - (I - 1)*\text{sqrt}(2)*\text{gamma}(3/2, -I*b^2*x^2 - 2*I*a*b*x - I*a^2)))/(b^4*x + a*b^3)$

**Fricas [A]**

time = 0.36, size = 112, normalized size = 1.13

$$\frac{2 \sqrt{2} \pi a^2 \sqrt{\frac{b^2}{\pi}} \text{C}\left(\frac{\sqrt{2} (bx+a) \sqrt{\frac{b^2}{\pi}}}{b}\right) - \sqrt{2} \pi \sqrt{\frac{b^2}{\pi}} \text{S}\left(\frac{\sqrt{2} (bx+a) \sqrt{\frac{b^2}{\pi}}}{b}\right) + 2(b^2 x - ab) \sin(b^2 x^2 + 2abx + a^2)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos((b\*x+a)^2),x, algorithm="fricas")

[Out]  $\frac{1}{4}*(2*\sqrt{2}*\pi*a^2*\sqrt{b^2/\pi}*\text{fresnel\_cos}(\sqrt{2}*(b*x + a)*\sqrt{b^2/\pi})/b) - \sqrt{2}*\pi*\sqrt{b^2/\pi}*\text{fresnel\_sin}(\sqrt{2}*(b*x + a)*\sqrt{b^2/\pi})/b + 2*(b^2*x - a*b)*\sin(b^2*x^2 + 2*a*b*x + a^2))/b^4$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos(a^2 + 2abx + b^2x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cos((b\*x+a)\*\*2),x)

[Out] Integral(x\*\*2\*cos(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2), x)

**Giac** [C] Result contains complex when optimal does not.

time = 0.47, size = 159, normalized size = 1.61

$$-\frac{(i+1)\sqrt{2}\sqrt{\pi}(2a^2+i)\operatorname{erf}\left(\frac{1}{2}(i-\frac{1}{2})\sqrt{2}\left(x+\frac{a}{b}\right)|b\right)}{|b|} + \frac{4(i b(x+\frac{a}{b})-2i a)e^{(i b^2 x^2+2i a b x+i a^2)}}{b}}{16 b^2} - \frac{(i-1)\sqrt{2}\sqrt{\pi}(2a^2-i)\operatorname{erf}\left(-\frac{1}{2}(i+\frac{1}{2})\sqrt{2}\left(x+\frac{a}{b}\right)|b\right)}{|b|} + \frac{4(-i b(x+\frac{a}{b})+2i a)e^{(-i b^2 x^2-2i a b x-i a^2)}}{b}}{16 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos((b\*x+a)^2),x, algorithm="giac")

[Out]  $-1/16*((I + 1)*\sqrt{2}*\sqrt{\pi}*(2*a^2 + I)*\operatorname{erf}((1/2*I - 1/2)*\sqrt{2}*(x + a/b)*\operatorname{abs}(b))/\operatorname{abs}(b) + 4*(I*b*(x + a/b) - 2*I*a)*e^{(I*b^2*x^2 + 2*I*a*b*x + I*a^2)/b}/b^2 - 1/16*(-(I - 1)*\sqrt{2}*\sqrt{\pi}*(2*a^2 - I)*\operatorname{erf}(-(1/2*I + 1/2)*\sqrt{2}*(x + a/b)*\operatorname{abs}(b))/\operatorname{abs}(b) + 4*(-I*b*(x + a/b) + 2*I*a)*e^{(-I*b^2*x^2 - 2*I*a*b*x - I*a^2)/b}/b^2$

**Mupad** [B]

time = 0.15, size = 80, normalized size = 0.81

$$\frac{x \sin((a + b x)^2)}{2 b^2} - \frac{a \sin((a + b x)^2)}{2 b^3} - \frac{\sqrt{2} \sqrt{\pi} S\left(\frac{\sqrt{2}(a + b x)}{\sqrt{\pi}}\right)}{4 b^3} + \frac{\sqrt{2} a^2 \sqrt{\pi} C\left(\frac{\sqrt{2}(a + b x)}{\sqrt{\pi}}\right)}{2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cos((a + b\*x)^2),x)

[Out]  $(x*\sin((a + b*x)^2))/(2*b^2) - (a*\sin((a + b*x)^2))/(2*b^3) - (2^{(1/2)}*\pi^{(1/2)}*\text{fresnel}((2^{(1/2)}*(a + b*x))/\pi^{(1/2)}))/(4*b^3) + (2^{(1/2)}*a^2*\pi^{(1/2)}*\text{fresnel}((2^{(1/2)}*(a + b*x))/\pi^{(1/2)}))/(2*b^3)$

### 3.86 $\int x \cos((a + bx)^2) dx$

Optimal. Leaf size=47

$$-\frac{a\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b^2} + \frac{\sin((a + bx)^2)}{2b^2}$$

[Out]  $1/2*\sin((b*x+a)^2)/b^2-1/2*a*\operatorname{FresnelC}((b*x+a)*2^{(1/2)}/\operatorname{Pi}^{(1/2)})*2^{(1/2)*\operatorname{Pi}^{(1/2)}/b^2}$

**Rubi [A]**

time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {3515, 3433, 3461, 2717}

$$\frac{\sin((a + bx)^2)}{2b^2} - \frac{\sqrt{\frac{\pi}{2}} a \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b^2}$$

Antiderivative was successfully verified.

[In] `Int[x*Cos[(a + b*x)^2], x]`

[Out]  $-\left(\frac{a*\sqrt{\operatorname{Pi}/2}*\operatorname{FresnelC}[\sqrt{2/\operatorname{Pi}}*(a + b*x)]}{b^2}\right) + \frac{\operatorname{Sin}[(a + b*x)^2]}{(2*b^2)}$

Rule 2717

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`  
`FreeQ[{c, d}, x]`

Rule 3433

`Int[Cos[(d_.)*((e_.) + (f_.)*(x_.))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /;`  
`FreeQ[{d, e, f}, x]`

Rule 3461

`Int[((a_.) + Cos[(c_.) + (d_.)*(x_.)^(n_.)]*(b_.))^(p_.)*(x_.)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /;`  
`FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))`

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominator[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int x \cos((a + bx)^2) dx &= \frac{\text{Subst}\left(\int (-a \cos(x^2) + x \cos(x^2)) dx, x, a + bx\right)}{b^2} \\ &= \frac{\text{Subst}\left(\int x \cos(x^2) dx, x, a + bx\right)}{b^2} - \frac{a \text{Subst}\left(\int \cos(x^2) dx, x, a + bx\right)}{b^2} \\ &= -\frac{a \sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b^2} + \frac{\text{Subst}\left(\int \cos(x) dx, x, (a + bx)^2\right)}{2b^2} \\ &= -\frac{a \sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b^2} + \frac{\sin((a + bx)^2)}{2b^2} \end{aligned}$$

**Mathematica** [A]

time = 0.05, size = 42, normalized size = 0.89

$$\frac{-a\sqrt{2\pi} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a + bx)\right) + \sin((a + bx)^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Cos[(a + b\*x)^2], x]

[Out] (-(a\*Sqrt[2\*Pi]\*FresnelC[Sqrt[2/Pi]\*(a + b\*x)]) + Sin[(a + b\*x)^2])/(2\*b^2)

**Maple** [A]

time = 0.06, size = 63, normalized size = 1.34

method	result	size
default	$\frac{\sin(x^2b^2+2abx+a^2)}{2b^2} - \frac{a\sqrt{2}\sqrt{\pi}\text{FresnelC}\left(\frac{\sqrt{2}(b^2x+ab)}{\sqrt{\pi}\sqrt{b^2}}\right)}{2b\sqrt{b^2}}$	63

risch	$\frac{(-1)^{\frac{3}{4}} a \sqrt{\pi} \operatorname{erf}\left(b(-1)^{\frac{1}{4}} x + (-1)^{\frac{1}{4}} a\right)}{4b^2} + \frac{a \sqrt{\pi} \operatorname{erf}\left(-b\sqrt{-i} x + \frac{ia}{\sqrt{-i}}\right)}{4b^2 \sqrt{-i}} + \frac{\sin\left(\frac{(bx+a)^2}{2b^2}\right)}{2b^2}$	71
-------	--	----

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos((b*x+a)^2),x,method=_RETURNVERBOSE)`

[Out]  $\frac{1}{2} \sqrt{b^2 x^2 + 2 a b x + a^2} \sin\left(\frac{(b x + a)^2}{2 b^2}\right) - \frac{1}{2} \frac{a \sqrt{\pi}}{b^2} \operatorname{erf}\left(\frac{b x + a}{\sqrt{b^2 x^2 + 2 a b x + a^2}}\right) \sqrt{\pi}$

**Maxima** [C] Result contains complex when optimal does not.

time = 0.65, size = 199, normalized size = 4.23

$$\frac{2 b x \left( -i e^{(b^2 x^2 + 2 a b x + a^2)} + i e^{(-b^2 x^2 - 2 a b x - a^2)} \right) - \sqrt{b^2 x^2 + 2 a b x + a^2} \left( -(i-1) \sqrt{2} \sqrt{\pi} \left( \operatorname{erf}\left(\frac{\sqrt{i} \sqrt{b^2 x^2 + 2 a b x + a^2}}{\sqrt{2}}\right) - 1 \right) + (i+1) \sqrt{2} \sqrt{\pi} \left( \operatorname{erf}\left(\frac{\sqrt{-i} \sqrt{b^2 x^2 + 2 a b x + a^2}}{\sqrt{2}}\right) - 1 \right) \right) a + 2 a \left( -i e^{(b^2 x^2 + 2 a b x + a^2)} + i e^{(-b^2 x^2 - 2 a b x - a^2)} \right)}{8 (b^2 x + a b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos((b*x+a)^2),x, algorithm="maxima")`

[Out]  $\frac{1}{8} \left( 2 b^2 x^2 (-I e^{(I b^2 x^2 + 2 I a b x + I a^2)} + I e^{(-I b^2 x^2 - 2 I a b x - I a^2)}) - \sqrt{b^2 x^2 + 2 a b x + a^2} \left( -(I-1) \sqrt{2} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{I} \sqrt{b^2 x^2 + 2 I a b x + I a^2}}{\sqrt{2}}\right) - 1 \right) + (I+1) \sqrt{2} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{-I} \sqrt{b^2 x^2 + 2 I a b x + I a^2}}{\sqrt{2}}\right) - 1 \right) a + 2 a \left( -I e^{(I b^2 x^2 + 2 I a b x + I a^2)} + I e^{(-I b^2 x^2 - 2 I a b x - I a^2)} \right) \right) / (b^3 x + a b^2)$

**Fricas** [A]

time = 0.36, size = 63, normalized size = 1.34

$$\frac{\sqrt{2} \pi a \sqrt{\frac{b^2}{\pi}} C\left(\frac{\sqrt{2} (b x + a) \sqrt{\frac{b^2}{\pi}}}{b}\right) - b \sin(b^2 x^2 + 2 a b x + a^2)}{2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos((b*x+a)^2),x, algorithm="fricas")`

[Out]  $-\frac{1}{2} \sqrt{2} \pi a \sqrt{\frac{b^2}{\pi}} \operatorname{fresnel\_cos}\left(\sqrt{2} (b x + a) \sqrt{\frac{b^2}{\pi}}\right) - b \sin(b^2 x^2 + 2 a b x + a^2) / b^3$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x \cos(a^2 + 2 a b x + b^2 x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos((b\*x+a)\*\*2), x)

[Out] Integral(x\*cos(a\*\*2 + 2\*a\*b\*x + b\*\*2\*x\*\*2), x)

**Giac** [C] Result contains complex when optimal does not.

time = 0.57, size = 119, normalized size = 2.53

$$-\frac{(i+1)\sqrt{2}\sqrt{\pi}a\operatorname{erf}\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|}{|b|} + \frac{2ie^{(ib^2x^2+2i abx+ia^2)}}{b} - \frac{(i-1)\sqrt{2}\sqrt{\pi}a\operatorname{erf}\left(-\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|}{|b|} - \frac{2ie^{(-ib^2x^2-2i abx-ia^2)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos((b\*x+a)^2), x, algorithm="giac")

[Out]  $-1/8*(-(I + 1)*\sqrt{2}*\sqrt{\pi}*a*\operatorname{erf}((1/2*I - 1/2)*\sqrt{2}*(x + a/b)*\operatorname{abs}(b)))/\operatorname{abs}(b) + 2*I*e^{(I*b^2*x^2 + 2*I*a*b*x + I*a^2)/b}/b - 1/8*((I - 1)*\sqrt{2}*\sqrt{\pi}*a*\operatorname{erf}(-(1/2*I + 1/2)*\sqrt{2}*(x + a/b)*\operatorname{abs}(b)))/\operatorname{abs}(b) - 2*I*e^{(-I*b^2*x^2 - 2*I*a*b*x - I*a^2)/b}/b$

**Mupad** [B]

time = 0.07, size = 39, normalized size = 0.83

$$\frac{\sin((a + bx)^2)}{2b^2} - \frac{\sqrt{2} a \sqrt{\pi} C\left(\frac{\sqrt{2}(a+bx)}{\sqrt{\pi}}\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cos((a + b\*x)^2), x)

[Out]  $\sin((a + b*x)^2)/(2*b^2) - (2^{(1/2)}*a*\pi^{(1/2)}*\operatorname{fresnelc}((2^{(1/2)}*(a + b*x))/\pi^{(1/2)}))/(2*b^2)$



### 3.87 $\int \cos((a + bx)^2) dx$

Optimal. Leaf size=29

$$\frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b}$$

[Out]  $1/2*\operatorname{FresnelC}((b*x+a)*2^{(1/2)}/\operatorname{Pi}^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b$

Rubi [A]

time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3433}

$$\frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[(a + b*x)^2], x]$

[Out]  $(\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{FresnelC}[\operatorname{Sqrt}[2/\operatorname{Pi}]*(a + b*x)])/b$

Rule 3433

$\operatorname{Int}[\operatorname{Cos}[(d_.)*(e_.) + (f_.)*(x_.)]^2], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[\operatorname{Pi}/2]/(f*\operatorname{Rt}[d, 2]))*\operatorname{FresnelC}[\operatorname{Sqrt}[2/\operatorname{Pi}]*\operatorname{Rt}[d, 2]*(e + f*x)], x] /; \operatorname{FreeQ}\{d, e, f\}, x]$

Rubi steps

$$\int \cos((a + bx)^2) dx = \frac{\sqrt{\frac{\pi}{2}} C\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b}$$

Mathematica [A]

time = 0.01, size = 29, normalized size = 1.00

$$\frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a + bx)\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(a + b\*x)^2], x]

[Out] (Sqrt[Pi/2]\*FresnelC[Sqrt[2/Pi]\*(a + b\*x)])/b

**Maple** [A]

time = 0.06, size = 36, normalized size = 1.24

method	result	size
default	$\frac{\sqrt{2} \sqrt{\pi} \operatorname{FresnelC}\left(\frac{\sqrt{2} (b^2 x + ab)}{\sqrt{\pi} \sqrt{b^2}}\right)}{2\sqrt{b^2}}$	36
risch	$-\frac{\sqrt{\pi} (-1)^{\frac{3}{4}} \operatorname{erf}\left(b(-1)^{\frac{1}{4}} x + (-1)^{\frac{1}{4}} a\right)}{4b} - \frac{\sqrt{\pi} \operatorname{erf}\left(-b\sqrt{-i} x + \frac{ia}{\sqrt{-i}}\right)}{4b\sqrt{-i}}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos((b\*x+a)^2), x, method=\_RETURNVERBOSE)

[Out] 1/2\*2^(1/2)\*Pi^(1/2)/(b^2)^(1/2)\*FresnelC(2^(1/2)/Pi^(1/2)/(b^2)^(1/2)\*(b^2\*x+a\*b))

**Maxima** [C] Result contains complex when optimal does not.

time = 0.51, size = 84, normalized size = 2.90

$$\frac{\sqrt{\pi} \left( (i-1) \sqrt{2} \operatorname{erf}\left(-(-1)^{\frac{3}{4}}(ibx+ia)\right) - (i+1) \sqrt{2} \operatorname{erf}\left(\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2}(-ibx-ia)\right) + (i-1) \sqrt{2} \operatorname{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2}(-ibx-ia)\right) + (i+1) \sqrt{2} \operatorname{erf}\left(\frac{ibx+ia}{\sqrt{-i}}\right) \right)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((b\*x+a)^2), x, algorithm="maxima")

[Out] -1/16\*sqrt(pi)\*((I - 1)\*sqrt(2)\*erf(-(-1)^(3/4)\*(I\*b\*x + I\*a)) - (I + 1)\*sqrt(2)\*erf((1/2\*I + 1/2)\*sqrt(2)\*(-I\*b\*x - I\*a)) + (I - 1)\*sqrt(2)\*erf((1/2\*I - 1/2)\*sqrt(2)\*(-I\*b\*x - I\*a)) + (I + 1)\*sqrt(2)\*erf((I\*b\*x + I\*a)/sqrt(-I)))/b

**Fricas** [A]

time = 0.37, size = 40, normalized size = 1.38

$$\frac{\sqrt{2} \pi \sqrt{\frac{b^2}{\pi}} C\left(\frac{\sqrt{2} (bx+a) \sqrt{\frac{b^2}{\pi}}}{b}\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((b\*x+a)^2), x, algorithm="fricas")

[Out]  $\frac{1}{2}\sqrt{2}\pi\sqrt{b^2/\pi}\text{fresnel\_cos}(\sqrt{2}(b*x + a)\sqrt{b^2/\pi})/b/b^2$

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \cos((a + bx)^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((b*x+a)**2),x)`

[Out] `Integral(cos((a + b*x)**2), x)`

**Giac [C]** Result contains complex when optimal does not.

time = 0.44, size = 55, normalized size = 1.90

$$\frac{(i + 1) \sqrt{2} \sqrt{\pi} \operatorname{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \left(x + \frac{a}{b}\right) |b|\right)}{8 |b|} + \frac{(i - 1) \sqrt{2} \sqrt{\pi} \operatorname{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \left(x + \frac{a}{b}\right) |b|\right)}{8 |b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((b*x+a)^2),x, algorithm="giac")`

[Out]  $-(1/8*I + 1/8)*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}((1/2*I - 1/2)*\sqrt{2}*(x + a/b)*\operatorname{abs}(b))/\operatorname{abs}(b) + (1/8*I - 1/8)*\sqrt{2}*\sqrt{\pi}*\operatorname{erf}(-(1/2*I + 1/2)*\sqrt{2}*(x + a/b)*\operatorname{abs}(b))/\operatorname{abs}(b)$

**Mupad [B]**

time = 0.06, size = 32, normalized size = 1.10

$$\frac{\sqrt{2} \sqrt{\pi} C\left(\frac{\sqrt{2} b \sqrt{\frac{1}{b^2}} (a+bx)}{\sqrt{\pi}}\right) \sqrt{\frac{1}{b^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos((a + b*x)^2),x)`

[Out]  $(2^{(1/2)}*\pi^{(1/2)}*\operatorname{fresnelc}((2^{(1/2)}*b*(1/b^2)^{(1/2)}*(a + b*x))/\pi^{(1/2)})*(1/b^2)^{(1/2)}/2$

$$3.88 \quad \int \frac{\cos((a+bx)^2)}{x} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cos((a+bx)^2)}{x}, x\right)$$

[Out] Unintegrable(cos((b\*x+a)^2)/x,x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\cos((a+bx)^2)}{x} dx$$

Verification is not applicable to the result.

[In] Int[Cos[(a + b\*x)^2]/x,x]

[Out] Defer[Int][Cos[(a + b\*x)^2]/x, x]

Rubi steps

$$\int \frac{\cos((a+bx)^2)}{x} dx = \int \frac{\cos((a+bx)^2)}{x} dx$$

Mathematica [A]

time = 2.70, size = 0, normalized size = 0.00

$$\int \frac{\cos((a+bx)^2)}{x} dx$$

Verification is not applicable to the result.

[In] Integrate[Cos[(a + b\*x)^2]/x,x]

[Out] Integrate[Cos[(a + b\*x)^2]/x, x]

Maple [A]

time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\cos((bx+a)^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos((b*x+a)^2)/x,x)`

[Out] `int(cos((b*x+a)^2)/x,x)`

**Maxima** [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((b*x+a)^2)/x,x, algorithm="maxima")`

[Out] `integrate(cos((b*x + a)^2)/x, x)`

**Fricas** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((b*x+a)^2)/x,x, algorithm="fricas")`

[Out] `integral(cos(b^2*x^2 + 2*a*b*x + a^2)/x, x)`

**Sympy** [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a^2 + 2abx + b^2x^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((b*x+a)**2)/x,x)`

[Out] `Integral(cos(a**2 + 2*a*b*x + b**2*x**2)/x, x)`

**Giac** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((b*x+a)^2)/x,x, algorithm="giac")`

[Out] `integrate(cos((b*x + a)^2)/x, x)`

**Mupad** [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cos((a + bx)^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((a + b*x)^2)/x,x)
```

```
[Out] int(cos((a + b*x)^2)/x, x)
```

$$3.89 \quad \int \frac{\cos((a+bx)^2)}{x^2} dx$$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cos((a+bx)^2)}{x^2}, x\right)$$

[Out] Unintegrable(cos((b\*x+a)^2)/x^2, x)

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\cos((a+bx)^2)}{x^2} dx$$

Verification is not applicable to the result.

[In] Int[Cos[(a + b\*x)^2]/x^2, x]

[Out] Defer[Int][Cos[(a + b\*x)^2]/x^2, x]

Rubi steps

$$\int \frac{\cos((a+bx)^2)}{x^2} dx = \int \frac{\cos((a+bx)^2)}{x^2} dx$$

Mathematica [A]

time = 4.94, size = 0, normalized size = 0.00

$$\int \frac{\cos((a+bx)^2)}{x^2} dx$$

Verification is not applicable to the result.

[In] Integrate[Cos[(a + b\*x)^2]/x^2, x]

[Out] Integrate[Cos[(a + b\*x)^2]/x^2, x]

Maple [A]

time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\cos((bx+a)^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos((b*x+a)^2)/x^2,x)`

[Out] `int(cos((b*x+a)^2)/x^2,x)`

**Maxima** [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((b*x+a)^2)/x^2,x, algorithm="maxima")`

[Out] `integrate(cos((b*x + a)^2)/x^2, x)`

**Fricas** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((b*x+a)^2)/x^2,x, algorithm="fricas")`

[Out] `integral(cos(b^2*x^2 + 2*a*b*x + a^2)/x^2, x)`

**Sympy** [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a^2 + 2abx + b^2x^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((b*x+a)**2)/x**2,x)`

[Out] `Integral(cos(a**2 + 2*a*b*x + b**2*x**2)/x**2, x)`

**Giac** [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((b*x+a)^2)/x^2,x, algorithm="giac")`

[Out] `integrate(cos((b*x + a)^2)/x^2, x)`

**Mupad** [A]

time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cos((a + bx)^2)}{x^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((a + b*x)^2)/x^2,x)
```

```
[Out] int(cos((a + b*x)^2)/x^2, x)
```

### 3.90 $\int x^2 \cos(a + b\sqrt{c + dx}) dx$

**Optimal.** Leaf size=346

$$\frac{240 \cos(a + b\sqrt{c + dx})}{b^6 d^3} + \frac{24c \cos(a + b\sqrt{c + dx})}{b^4 d^3} + \frac{2c^2 \cos(a + b\sqrt{c + dx})}{b^2 d^3} - \frac{120(c + dx) \cos(a + b\sqrt{c + dx})}{b^4 d^3}$$

[Out] 240\*cos(a+b\*(d\*x+c)^(1/2))/b^6/d^3+24\*c\*cos(a+b\*(d\*x+c)^(1/2))/b^4/d^3+2\*c^2\*cos(a+b\*(d\*x+c)^(1/2))/b^2/d^3-120\*(d\*x+c)\*cos(a+b\*(d\*x+c)^(1/2))/b^4/d^3-12\*c\*(d\*x+c)\*cos(a+b\*(d\*x+c)^(1/2))/b^2/d^3+10\*(d\*x+c)^2\*cos(a+b\*(d\*x+c)^(1/2))/b^2/d^3-40\*(d\*x+c)^(3/2)\*sin(a+b\*(d\*x+c)^(1/2))/b^3/d^3-4\*c\*(d\*x+c)^(3/2)\*sin(a+b\*(d\*x+c)^(1/2))/b/d^3+2\*(d\*x+c)^(5/2)\*sin(a+b\*(d\*x+c)^(1/2))/b/d^3+240\*sin(a+b\*(d\*x+c)^(1/2))\*(d\*x+c)^(1/2)/b^5/d^3+24\*c\*sin(a+b\*(d\*x+c)^(1/2))\*(d\*x+c)^(1/2)/b^3/d^3+2\*c^2\*sin(a+b\*(d\*x+c)^(1/2))\*(d\*x+c)^(1/2)/b/d^3

**Rubi [A]**

time = 0.22, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3513, 3377, 2718}

$$\frac{240 \cos(a + b\sqrt{c + dx})}{b^6 d^3}, \frac{240 \sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{b^6 d^3}, \frac{120(c + dx) \cos(a + b\sqrt{c + dx})}{b^6 d^3}, \frac{24c \cos(a + b\sqrt{c + dx})}{b^4 d^3}, \frac{40(c + dx)^{3/2} \sin(a + b\sqrt{c + dx})}{b^4 d^3}, \frac{24c \sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{b^4 d^3}, \frac{2c^2 \cos(a + b\sqrt{c + dx})}{b^2 d^3}, \frac{10(c + dx)^2 \cos(a + b\sqrt{c + dx})}{b^2 d^3}, \frac{120(c + dx) \cos(a + b\sqrt{c + dx})}{b^4 d^3}, \frac{2c^2 \sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{b^2 d^3}, \frac{2(c + dx)^{5/2} \sin(a + b\sqrt{c + dx})}{b^2 d^3}, \frac{40(c + dx)^{3/2} \sin(a + b\sqrt{c + dx})}{b^2 d^3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Cos[a + b\*Sqrt[c + d\*x]],x]

[Out] (240\*Cos[a + b\*Sqrt[c + d\*x]]/(b^6\*d^3) + (24\*c\*Cos[a + b\*Sqrt[c + d\*x]]/(b^4\*d^3) + (2\*c^2\*Cos[a + b\*Sqrt[c + d\*x]]/(b^2\*d^3) - (120\*(c + d\*x)\*Cos[a + b\*Sqrt[c + d\*x]]/(b^4\*d^3) - (12\*c\*(c + d\*x)\*Cos[a + b\*Sqrt[c + d\*x]]/(b^2\*d^3) + (10\*(c + d\*x)^2\*Cos[a + b\*Sqrt[c + d\*x]]/(b^2\*d^3) + (240\*Sqrt[c + d\*x]\*Sin[a + b\*Sqrt[c + d\*x]]/(b^5\*d^3) + (24\*c\*Sqrt[c + d\*x]\*Sin[a + b\*Sqrt[c + d\*x]]/(b^3\*d^3) + (2\*c^2\*Sqrt[c + d\*x]\*Sin[a + b\*Sqrt[c + d\*x]]/(b\*d^3) - (40\*(c + d\*x)^(3/2)\*Sin[a + b\*Sqrt[c + d\*x]]/(b^3\*d^3) - (4\*c\*(c + d\*x)^(3/2)\*Sin[a + b\*Sqrt[c + d\*x]]/(b\*d^3) + (2\*(c + d\*x)^(5/2)\*Sin[a + b\*Sqrt[c + d\*x]]/(b\*d^3)

Rule 2718

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[-Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(-(c + d\*x)^m)\*(Cos[e + f\*x]/f), x] + Dist[d\*(m/f), Int[(c + d\*x)^(m - 1)\*Co

s[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3513

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((g\_.) + (h\_.)\*(x\_.))^(m\_.), x\_Symbol] := Dist[1/(n\*f), Subst[Int[ExpandIntegrand[(a + b\*Cos[c + d\*x])^p, x^(1/n - 1)\*(g - e\*(h/f) + h\*(x^(1/n)/f))^m, x], x, (e + f\*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p, 0] && IntegerQ[1/n]

### Rubi steps

$$\begin{aligned}
 \int x^2 \cos(a + b\sqrt{c + dx}) dx &= \frac{2 \text{Subst}\left(\int \left(\frac{c^2 x \cos(a+bx)}{d^2} - \frac{2cx^3 \cos(a+bx)}{d^2} + \frac{x^5 \cos(a+bx)}{d^2}\right) dx, x, \sqrt{c + dx}\right)}{d} \\
 &= \frac{2 \text{Subst}\left(\int x^5 \cos(a + bx) dx, x, \sqrt{c + dx}\right)}{d^3} - \frac{(4c) \text{Subst}\left(\int x^3 \cos(a + bx) dx, x, \sqrt{c + dx}\right)}{d^3} \\
 &= \frac{2c^2 \sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{bd^3} - \frac{4c(c + dx)^{3/2} \sin(a + b\sqrt{c + dx})}{bd^3} + \frac{10(c + dx)^{5/2} \cos(a + b\sqrt{c + dx})}{bd^3} \\
 &= \frac{2c^2 \cos(a + b\sqrt{c + dx})}{b^2 d^3} - \frac{12c(c + dx) \cos(a + b\sqrt{c + dx})}{b^2 d^3} + \frac{10(c + dx)^2 \sin(a + b\sqrt{c + dx})}{bd^3} \\
 &= \frac{2c^2 \cos(a + b\sqrt{c + dx})}{b^2 d^3} - \frac{12c(c + dx) \cos(a + b\sqrt{c + dx})}{b^2 d^3} + \frac{10(c + dx)^2 \sin(a + b\sqrt{c + dx})}{bd^3} \\
 &= \frac{24c \cos(a + b\sqrt{c + dx})}{b^4 d^3} + \frac{2c^2 \cos(a + b\sqrt{c + dx})}{b^2 d^3} - \frac{120(c + dx) \cos(a + b\sqrt{c + dx})}{b^4 d^3} \\
 &= \frac{24c \cos(a + b\sqrt{c + dx})}{b^4 d^3} + \frac{2c^2 \cos(a + b\sqrt{c + dx})}{b^2 d^3} - \frac{120(c + dx) \cos(a + b\sqrt{c + dx})}{b^4 d^3} \\
 &= \frac{240 \cos(a + b\sqrt{c + dx})}{b^6 d^3} + \frac{24c \cos(a + b\sqrt{c + dx})}{b^4 d^3} + \frac{2c^2 \cos(a + b\sqrt{c + dx})}{b^2 d^3}
 \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 0.77, size = 224, normalized size = 0.65

$$\frac{e^{-i(a+b\sqrt{c+dx})} (120 + 120ib\sqrt{c+dx} + ib^2 d^2 x^2 \sqrt{c+dx} - 4ib^3 \sqrt{c+dx} (2c + 5dx) - 12b^2 (4c + 5dx) + b^4 dx (4c + 5dx) + e^{i(a+b\sqrt{c+dx})} (120 - 120ib\sqrt{c+dx} - ib^2 d^2 x^2 \sqrt{c+dx} + 4ib^3 \sqrt{c+dx} (2c + 5dx) - 12b^2 (4c + 5dx) + b^4 dx (4c + 5dx)))}{b^6 d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Cos[a + b\*Sqrt[c + d\*x]],x]

[Out]  $(120 + (120*I)*b*\text{Sqrt}[c + d*x] + I*b^5*d^2*x^2*\text{Sqrt}[c + d*x] - (4*I)*b^3*\text{Sqrt}[c + d*x]*(2*c + 5*d*x) - 12*b^2*(4*c + 5*d*x) + b^4*d*x*(4*c + 5*d*x) + E^((2*I)*(a + b*\text{Sqrt}[c + d*x]))*(120 - (120*I)*b*\text{Sqrt}[c + d*x] - I*b^5*d^2*x^2*\text{Sqrt}[c + d*x] + (4*I)*b^3*\text{Sqrt}[c + d*x]*(2*c + 5*d*x) - 12*b^2*(4*c + 5*d*x) + b^4*d*x*(4*c + 5*d*x)))/(b^6*d^3*E^(I*(a + b*\text{Sqrt}[c + d*x])))$

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 824 vs.  $2(310) = 620$ .

time = 0.07, size = 825, normalized size = 2.38

method	result
derivativedivides	$\frac{-2ac^2 \sin(a+b\sqrt{dx+c}) + 2c^2 \left( \cos(a+b\sqrt{dx+c}) + (a+b\sqrt{dx+c}) \sin(a+b\sqrt{dx+c}) \right) + \frac{4a^3 c \sin(a+b\sqrt{dx+c})}{b^3}}{b^3}$
default	$\frac{-2ac^2 \sin(a+b\sqrt{dx+c}) + 2c^2 \left( \cos(a+b\sqrt{dx+c}) + (a+b\sqrt{dx+c}) \sin(a+b\sqrt{dx+c}) \right) + \frac{4a^3 c \sin(a+b\sqrt{dx+c})}{b^3}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)`

[Out]  $2/d^3/b^2*(-a*c^2*\sin(a+b*(d*x+c)^(1/2))+c^2*(\cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*\sin(a+b*(d*x+c)^(1/2)))+2/b^2*a^3*c*\sin(a+b*(d*x+c)^(1/2))-6/b^2*a^2*c*(\cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*\sin(a+b*(d*x+c)^(1/2)))+6/b^2*a*c*((a+b*(d*x+c)^(1/2))^2*\sin(a+b*(d*x+c)^(1/2))-2*\sin(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*\cos(a+b*(d*x+c)^(1/2)))-2/b^2*c*((a+b*(d*x+c)^(1/2))^3*\sin(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*\cos(a+b*(d*x+c)^(1/2))-6*\cos(a+b*(d*x+c)^(1/2))-6*(a+b*(d*x+c)^(1/2))*\sin(a+b*(d*x+c)^(1/2)))-1/b^4*a^5*\sin(a+b*(d*x+c)^(1/2))+5/b^4*a^4*(\cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*\sin(a+b*(d*x+c)^(1/2)))-10/b^4*a^3*((a+b*(d*x+c)^(1/2))^2*\sin(a+b*(d*x+c)^(1/2))-2*\sin(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*\cos(a+b*(d*x+c)^(1/2)))+10/b^4*a^2*((a+b*(d*x+c)^(1/2))^3*\sin(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*\cos(a+b*(d*x+c)^(1/2))-6*\cos(a+b*(d*x+c)^(1/2))-6*(a+b*(d*x+c)^(1/2))*\sin(a+b*(d*x+c)^(1/2)))-5/b^4*a*((a+b*(d*x+c)^(1/2))^4*\sin(a+b*(d*x+c)^(1/2))+4*(a+b*(d*x+c)^(1/2))^3*\cos(a+b*(d*x+c)^(1/2))-12*(a+b*(d*x+c)^(1/2))^2*\sin(a+b*(d*x+c)^(1/2))+24*\sin(a+b*(d*x+c)^(1/2))-24*(a+b*(d*x+c)^(1/2))*\cos(a+b*(d*x+c)^(1/2)))+1/b^4*((a+b*(d*x+c)^(1/2))^5*\sin(a+b*(d*x+c)^(1/2))+5*(a+b*(d*x+c)^(1/2))^4*\cos(a+b*(d*x+c)^(1/2))-20*(a+b*(d*x+c)^(1/2))^3*\sin(a+b*(d*x+c)^(1/2))-60*(a+b*(d*x+c)^(1/2))^2*\cos(a+b*(d*x+c)^(1/2))+120*\cos(a+b*(d*x+c)^(1/2))+120*(a+b*(d*x+c)^(1/2))*\sin(a+b*(d*x+c)^(1/2)))))$

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 672 vs.  $2(310) = 620$ .

time = 0.31, size = 672, normalized size = 1.94

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*(d\*x+c)^(1/2)),x, algorithm="maxima")

[Out] 
$$-2*(a*c^2*\sin(\sqrt{d*x+c}*b+a) - ((\sqrt{d*x+c}*b+a)*\sin(\sqrt{d*x+c}*b+a) + \cos(\sqrt{d*x+c}*b+a))*c^2 - 2*a^3*c*\sin(\sqrt{d*x+c}*b+a)/b^2 + 6*((\sqrt{d*x+c}*b+a)*\sin(\sqrt{d*x+c}*b+a) + \cos(\sqrt{d*x+c}*b+a))*a^2*c/b^2 + a^5*\sin(\sqrt{d*x+c}*b+a)/b^4 - 5*((\sqrt{d*x+c}*b+a)*\sin(\sqrt{d*x+c}*b+a) + \cos(\sqrt{d*x+c}*b+a))*a^4/b^4 - 6*(2*(\sqrt{d*x+c}*b+a)*\cos(\sqrt{d*x+c}*b+a) + ((\sqrt{d*x+c}*b+a)^2 - 2)*\sin(\sqrt{d*x+c}*b+a))*a*c/b^2 + 10*(2*(\sqrt{d*x+c}*b+a)*\cos(\sqrt{d*x+c}*b+a) + ((\sqrt{d*x+c}*b+a)^2 - 2)*\sin(\sqrt{d*x+c}*b+a))*a^3/b^4 + 2*(3*((\sqrt{d*x+c}*b+a)^2 - 2)*\cos(\sqrt{d*x+c}*b+a) + ((\sqrt{d*x+c}*b+a)^3 - 6*\sqrt{d*x+c}*b - 6*a)*\sin(\sqrt{d*x+c}*b+a))*c/b^2 - 10*(3*((\sqrt{d*x+c}*b+a)^2 - 2)*\cos(\sqrt{d*x+c}*b+a) + ((\sqrt{d*x+c}*b+a)^3 - 6*\sqrt{d*x+c}*b - 6*a)*\sin(\sqrt{d*x+c}*b+a))*a^2/b^4 + 5*(4*((\sqrt{d*x+c}*b+a)^3 - 6*\sqrt{d*x+c}*b - 6*a)*\cos(\sqrt{d*x+c}*b+a) + ((\sqrt{d*x+c}*b+a)^4 - 12*(\sqrt{d*x+c}*b+a)^2 + 24)*\sin(\sqrt{d*x+c}*b+a))*a/b^4 - (5*((\sqrt{d*x+c}*b+a)^4 - 12*(\sqrt{d*x+c}*b+a)^2 + 24)*\cos(\sqrt{d*x+c}*b+a) + ((\sqrt{d*x+c}*b+a)^5 - 20*(\sqrt{d*x+c}*b+a)^3 + 120*\sqrt{d*x+c}*b + 120*a)*\sin(\sqrt{d*x+c}*b+a))/b^4)/(b^2*d^3)$$

**Fricas** [A]

time = 0.35, size = 103, normalized size = 0.30

$$\frac{2 \left( (b^5 d^2 x^2 - 20 b^3 d x - 8 b^3 c + 120 b) \sqrt{d x + c} \sin(\sqrt{d x + c} b + a) + (5 b^4 d^2 x^2 - 48 b^2 c + 4 (b^4 c - 15 b^2) d x + 120) \cos(\sqrt{d x + c} b + a) \right)}{b^6 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*(d\*x+c)^(1/2)),x, algorithm="fricas")

[Out] 
$$2*((b^5*d^2*x^2 - 20*b^3*d*x - 8*b^3*c + 120*b)*\sqrt{d*x+c}*\sin(\sqrt{d*x+c}*b+a) + (5*b^4*d^2*x^2 - 48*b^2*c + 4*(b^4*c - 15*b^2)*d*x + 120)*\cos(\sqrt{d*x+c}*b+a))/(b^6*d^3)$$

**Sympy** [A]

time = 0.32, size = 269, normalized size = 0.78

$$\begin{cases} \frac{x^2 \cos(a)}{3} & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ \frac{x^2 \cos(a + b\sqrt{c})}{3} & \text{for } d = 0 \\ \frac{2a^2\sqrt{c+d} \sin(a + b\sqrt{c+d})}{3d} + \frac{8ac \cos(a + b\sqrt{c+d})}{3d^2} + \frac{10a^2 \cos(a + b\sqrt{c+d})}{3d^2} - \frac{16c\sqrt{c+d} \sin(a + b\sqrt{c+d})}{3d^2} - \frac{40c\sqrt{c+d} \sin(a + b\sqrt{c+d})}{3d^2} - \frac{96c \cos(a + b\sqrt{c+d})}{3d^2} - \frac{120c \cos(a + b\sqrt{c+d})}{3d^2} + \frac{240\sqrt{c+d} \sin(a + b\sqrt{c+d})}{3d^2} + \frac{240 \cos(a + b\sqrt{c+d})}{3d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cos(a+b\*(d\*x+c)\*\*(1/2)),x)

[Out] Piecewise((x\*\*3\*cos(a)/3, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x\*\*3\*cos(a + b\*sqrt(c))/3, Eq(d, 0)), (2\*x\*\*2\*sqrt(c + d\*x)\*sin(a + b\*sqrt(c + d\*x))/(b\*d) + 8\*c\*x\*cos(a + b\*sqrt(c + d\*x))/(b\*\*2\*d\*\*2) + 10\*x\*\*2\*cos(a + b\*sqrt(c + d\*x))/(b\*\*2\*d) - 16\*c\*sqrt(c + d\*x)\*sin(a + b\*sqrt(c + d\*x))/(b\*\*3\*d\*\*3) - 40\*x\*sqrt(c + d\*x)\*sin(a + b\*sqrt(c + d\*x))/(b\*\*3\*d\*\*2) - 96\*c\*cos(a + b\*sqrt(c + d\*x))/(b\*\*4\*d\*\*3) - 120\*x\*cos(a + b\*sqrt(c + d\*x))/(b\*\*4\*d\*\*2) + 240\*sqrt(c + d\*x)\*sin(a + b\*sqrt(c + d\*x))/(b\*\*5\*d\*\*3) + 240\*cos(a + b\*sqrt(c + d\*x))/(b\*\*6\*d\*\*3), True))

**Giac** [A]

time = 0.43, size = 480, normalized size = 1.39

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*(d\*x+c)^(1/2)),x, algorithm="giac")

[Out] 2\*((b^4\*c^2 - 6\*(sqrt(d\*x + c)\*b + a)^2\*b^2\*c + 12\*(sqrt(d\*x + c)\*b + a)\*a\*b^2\*c - 6\*a^2\*b^2\*c + 5\*(sqrt(d\*x + c)\*b + a)^4 - 20\*(sqrt(d\*x + c)\*b + a)^3\*a + 30\*(sqrt(d\*x + c)\*b + a)^2\*a^2 - 20\*(sqrt(d\*x + c)\*b + a)\*a^3 + 5\*a^4 + 12\*b^2\*c - 60\*(sqrt(d\*x + c)\*b + a)^2 + 120\*(sqrt(d\*x + c)\*b + a)\*a - 60\*a^2 + 120)\*cos(sqrt(d\*x + c)\*b + a)/(b^4\*d^2) + ((sqrt(d\*x + c)\*b + a)\*b^4\*c^2 - a\*b^4\*c^2 - 2\*(sqrt(d\*x + c)\*b + a)^3\*b^2\*c + 6\*(sqrt(d\*x + c)\*b + a)^2\*a\*b^2\*c - 6\*(sqrt(d\*x + c)\*b + a)\*a^2\*b^2\*c + 2\*a^3\*b^2\*c + (sqrt(d\*x + c)\*b + a)^5 - 5\*(sqrt(d\*x + c)\*b + a)^4\*a + 10\*(sqrt(d\*x + c)\*b + a)^3\*a^2 - 10\*(sqrt(d\*x + c)\*b + a)^2\*a^3 + 5\*(sqrt(d\*x + c)\*b + a)\*a^4 - a^5 + 12\*(sqrt(d\*x + c)\*b + a)\*b^2\*c - 12\*a\*b^2\*c - 20\*(sqrt(d\*x + c)\*b + a)^3 + 60\*(sqrt(d\*x + c)\*b + a)^2\*a - 60\*(sqrt(d\*x + c)\*b + a)\*a^2 + 20\*a^3 + 120\*sqrt(d\*x + c)\*b)\*sin(sqrt(d\*x + c)\*b + a)/(b^4\*d^2))/(b^2\*d)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \cos\left(a + b\sqrt{c + dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cos(a + b\*(c + d\*x)^(1/2)),x)

[Out] int(x^2\*cos(a + b\*(c + d\*x)^(1/2)), x)

### 3.91 $\int x \cos(a + b\sqrt{c + dx}) dx$

**Optimal.** Leaf size=167

$$-\frac{12 \cos(a + b\sqrt{c + dx})}{b^4 d^2} - \frac{2c \cos(a + b\sqrt{c + dx})}{b^2 d^2} + \frac{6(c + dx) \cos(a + b\sqrt{c + dx})}{b^2 d^2} - \frac{12\sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{b^3 d^2}$$

[Out]  $-12*\cos(a+b*(d*x+c)^{(1/2)})/b^4/d^2-2*c*\cos(a+b*(d*x+c)^{(1/2)})/b^2/d^2+6*(d*x+c)*\cos(a+b*(d*x+c)^{(1/2)})/b^2/d^2+2*(d*x+c)^{(3/2)}*\sin(a+b*(d*x+c)^{(1/2)})/b/d^2-12*\sin(a+b*(d*x+c)^{(1/2)})*(d*x+c)^{(1/2)}/b^3/d^2-2*c*\sin(a+b*(d*x+c)^{(1/2)})*(d*x+c)^{(1/2)}/b/d^2$

**Rubi [A]**

time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3513, 3377, 2718}

$$-\frac{12 \cos(a + b\sqrt{c + dx})}{b^4 d^2} - \frac{12\sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{b^3 d^2} + \frac{6(c + dx) \cos(a + b\sqrt{c + dx})}{b^2 d^2} - \frac{2c \cos(a + b\sqrt{c + dx})}{b^2 d^2} + \frac{2(c + dx)^{3/2} \sin(a + b\sqrt{c + dx})}{b d^2} - \frac{2c\sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{b d^2}$$

Antiderivative was successfully verified.

[In] Int[x\*Cos[a + b\*Sqrt[c + d\*x]],x]

[Out]  $(-12*\text{Cos}[a + b*\text{Sqrt}[c + d*x]])/(b^4*d^2) - (2*c*\text{Cos}[a + b*\text{Sqrt}[c + d*x]])/(b^2*d^2) + (6*(c + d*x)*\text{Cos}[a + b*\text{Sqrt}[c + d*x]])/(b^2*d^2) - (12*\text{Sqrt}[c + d*x]*\text{Sin}[a + b*\text{Sqrt}[c + d*x]])/(b^3*d^2) - (2*c*\text{Sqrt}[c + d*x]*\text{Sin}[a + b*\text{Sqrt}[c + d*x]])/(b*d^2) + (2*(c + d*x)^{(3/2)}*\text{Sin}[a + b*\text{Sqrt}[c + d*x]])/(b*d^2)$

**Rule 2718**

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

**Rule 3377**

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := Simp[(-(c + d\*x)^m)\*(Cos[e + f\*x]/f), x] + Dist[d\*(m/f), Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

**Rule 3513**

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_.))]^(n\_.))\*(b\_.)^(p\_.)\*((g\_.) + (h\_.)\*(x\_.))^(m\_.), x\_Symbol] := Dist[1/(n\*f), Subst[Int[ExpandIntegrand[(a + b\*Cos[c + d\*x])^p, x^(1/n - 1)\*(g - e\*(h/f) + h\*(x^(1/n)/f))^m, x], x, (e + f\*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned}
\int x \cos(a + b\sqrt{c + dx}) dx &= \frac{2\text{Subst}\left(\int \left(-\frac{cx \cos(a+bx)}{d} + \frac{x^3 \cos(a+bx)}{d}\right) dx, x, \sqrt{c + dx}\right)}{d} \\
&= \frac{2\text{Subst}\left(\int x^3 \cos(a + bx) dx, x, \sqrt{c + dx}\right)}{d^2} - \frac{(2c)\text{Subst}\left(\int x \cos(a + bx) dx, x, \sqrt{c + dx}\right)}{d^2} \\
&= -\frac{2c\sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{bd^2} + \frac{2(c + dx)^{3/2} \sin(a + b\sqrt{c + dx})}{bd^2} - \frac{6(c + dx) \cos(a + b\sqrt{c + dx})}{bd^2} \\
&= -\frac{2c \cos(a + b\sqrt{c + dx})}{b^2 d^2} + \frac{6(c + dx) \cos(a + b\sqrt{c + dx})}{b^2 d^2} - \frac{2c\sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{bd^2} \\
&= -\frac{2c \cos(a + b\sqrt{c + dx})}{b^2 d^2} + \frac{6(c + dx) \cos(a + b\sqrt{c + dx})}{b^2 d^2} - \frac{12\sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{bd^2} \\
&= -\frac{12 \cos(a + b\sqrt{c + dx})}{b^4 d^2} - \frac{2c \cos(a + b\sqrt{c + dx})}{b^2 d^2} + \frac{6(c + dx) \cos(a + b\sqrt{c + dx})}{b^2 d^2}
\end{aligned}$$

**Mathematica [A]**

time = 0.26, size = 71, normalized size = 0.43

$$\frac{2\left((-6 + b^2(2c + 3dx)) \cos(a + b\sqrt{c + dx}) + b\sqrt{c + dx}(-6 + b^2 dx) \sin(a + b\sqrt{c + dx})\right)}{b^4 d^2}$$

Antiderivative was successfully verified.

`[In] Integrate[x*Cos[a + b*Sqrt[c + d*x]],x]`

```
[Out] (2*((-6 + b^2*(2*c + 3*d*x))*Cos[a + b*Sqrt[c + d*x]] + b*Sqrt[c + d*x]*(-6 + b^2*d*x)*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

**Maple [A]**

time = 0.04, size = 299, normalized size = 1.79

method	result
derivativedivides	$ \frac{2ac \sin(a + b\sqrt{dx + c}) - 2c \left( \cos(a + b\sqrt{dx + c}) + (a + b\sqrt{dx + c}) \sin(a + b\sqrt{dx + c}) \right) - \frac{2a^3 \sin(a + b\sqrt{dx + c})}{b^2}}{b^4 d^2} $
default	$ \frac{2ac \sin(a + b\sqrt{dx + c}) - 2c \left( \cos(a + b\sqrt{dx + c}) + (a + b\sqrt{dx + c}) \sin(a + b\sqrt{dx + c}) \right) - \frac{2a^3 \sin(a + b\sqrt{dx + c})}{b^2}}{b^4 d^2} $



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)`

[Out]  $2/d^2/b^2*(a*c*\sin(a+b*(d*x+c)^{(1/2)})-c*(\cos(a+b*(d*x+c)^{(1/2)})+(a+b*(d*x+c)^{(1/2))*\sin(a+b*(d*x+c)^{(1/2)}))-1/b^2*a^3*\sin(a+b*(d*x+c)^{(1/2)})+3/b^2*a^2*(\cos(a+b*(d*x+c)^{(1/2)})+(a+b*(d*x+c)^{(1/2))*\sin(a+b*(d*x+c)^{(1/2)}))-3/b^2*a*((a+b*(d*x+c)^{(1/2)})^2*\sin(a+b*(d*x+c)^{(1/2)})-2*\sin(a+b*(d*x+c)^{(1/2)})+2*(a+b*(d*x+c)^{(1/2))*\cos(a+b*(d*x+c)^{(1/2)}))+1/b^2*((a+b*(d*x+c)^{(1/2)})^3*\sin(a+b*(d*x+c)^{(1/2)})+3*(a+b*(d*x+c)^{(1/2)})^2*\cos(a+b*(d*x+c)^{(1/2)})-6*\cos(a+b*(d*x+c)^{(1/2)})-6*(a+b*(d*x+c)^{(1/2))*\sin(a+b*(d*x+c)^{(1/2)}))$

**Maxima** [A]

time = 0.29, size = 263, normalized size = 1.57

$$2 \left( \frac{a \sin(\sqrt{dx+c}b+a) - ((\sqrt{dx+c}b+a) \sin(\sqrt{dx+c}b+a) + \cos(\sqrt{dx+c}b+a))c - \frac{a \sin(\sqrt{dx+c}b+a)}{b} + \frac{a^2((\sqrt{dx+c}b+a) \sin(\sqrt{dx+c}b+a) + \cos(\sqrt{dx+c}b+a))c}{b^2} - \frac{a^3((\sqrt{dx+c}b+a) \sin(\sqrt{dx+c}b+a) + \cos(\sqrt{dx+c}b+a))c}{b^3} + \frac{a^2((\sqrt{dx+c}b+a) \sin(\sqrt{dx+c}b+a) + \cos(\sqrt{dx+c}b+a))c}{b^2} - \frac{a^3((\sqrt{dx+c}b+a) \sin(\sqrt{dx+c}b+a) + \cos(\sqrt{dx+c}b+a))c}{b^3} + \frac{a^2((\sqrt{dx+c}b+a) \sin(\sqrt{dx+c}b+a) + \cos(\sqrt{dx+c}b+a))c}{b^2} - \frac{a^3((\sqrt{dx+c}b+a) \sin(\sqrt{dx+c}b+a) + \cos(\sqrt{dx+c}b+a))c}{b^3} \right) / b^4 d^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")`

[Out]  $2*(a*c*\sin(\sqrt{d*x+c}*b+a) - ((\sqrt{d*x+c}*b+a)*\sin(\sqrt{d*x+c}*b+a) + \cos(\sqrt{d*x+c}*b+a))*c - a^3*\sin(\sqrt{d*x+c}*b+a)/b^2 + 3*((\sqrt{d*x+c}*b+a)*\sin(\sqrt{d*x+c}*b+a) + \cos(\sqrt{d*x+c}*b+a))*a^2/b^2 - 3*(2*(\sqrt{d*x+c}*b+a)*\cos(\sqrt{d*x+c}*b+a) + ((\sqrt{d*x+c}*b+a)^2 - 2)*\sin(\sqrt{d*x+c}*b+a))*a/b^2 + (3*((\sqrt{d*x+c}*b+a)^2 - 2)*\cos(\sqrt{d*x+c}*b+a) + ((\sqrt{d*x+c}*b+a)^3 - 6*\sqrt{d*x+c}*b - 6*a)*\sin(\sqrt{d*x+c}*b+a))/b^2)/b^2*d^2$

**Fricas** [A]

time = 0.36, size = 67, normalized size = 0.40

$$\frac{2 \left( (b^3 dx - 6b) \sqrt{dx+c} \sin(\sqrt{dx+c}b+a) + (3b^2 dx + 2b^2 c - 6) \cos(\sqrt{dx+c}b+a) \right)}{b^4 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")`

[Out]  $2*((b^3*d*x - 6*b)*\sqrt{d*x+c}*\sin(\sqrt{d*x+c}*b+a) + (3*b^2*d*x + 2*b^2*c - 6)*\cos(\sqrt{d*x+c}*b+a))/(b^4*d^2)$

**Sympy** [A]

time = 0.18, size = 151, normalized size = 0.90

$$\begin{cases} \frac{x^2 \cos(a)}{2} & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ \frac{x^2 \cos(a+b\sqrt{c})}{2} & \text{for } d = 0 \\ \frac{2x\sqrt{c+dx} \sin(a+b\sqrt{c+dx})}{bd} + \frac{4c \cos(a+b\sqrt{c+dx})}{b^2 d^2} + \frac{6x \cos(a+b\sqrt{c+dx})}{b^2 d} - \frac{12\sqrt{c+dx} \sin(a+b\sqrt{c+dx})}{b^3 d^2} - \frac{12 \cos(a+b\sqrt{c+dx})}{b^4 d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*(d\*x+c)\*\*(1/2)),x)

[Out] Piecewise((x\*\*2\*cos(a)/2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x\*\*2\*cos(a + b\*sqrt(c))/2, Eq(d, 0)), (2\*x\*sqrt(c + d\*x)\*sin(a + b\*sqrt(c + d\*x))/(b\*d) + 4\*c\*cos(a + b\*sqrt(c + d\*x))/(b\*\*2\*d\*\*2) + 6\*x\*cos(a + b\*sqrt(c + d\*x))/(b\*\*2\*d) - 12\*sqrt(c + d\*x)\*sin(a + b\*sqrt(c + d\*x))/(b\*\*3\*d\*\*2) - 12\*cos(a + b\*sqrt(c + d\*x))/(b\*\*4\*d\*\*2), True))

**Giac** [A]

time = 0.41, size = 166, normalized size = 0.99

$$\frac{2 \left( \frac{(\sqrt{d}c-3(\sqrt{d}x+c) b+a)^2 + 6(\sqrt{d}x+c) b+a}{b^2} \cos(\sqrt{d}x+c) b+a \right) + \left( (\sqrt{d}x+c) b+a \right)^2 - 3(\sqrt{d}x+c) b+a}{b^2 d^2} \sin(\sqrt{d}x+c) b+a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*(d\*x+c)^(1/2)),x, algorithm="giac")

[Out] -2\*((b^2\*c - 3\*(sqrt(d\*x + c)\*b + a)^2 + 6\*(sqrt(d\*x + c)\*b + a)\*a - 3\*a^2 + 6)\*cos(sqrt(d\*x + c)\*b + a)/b^2 + ((sqrt(d\*x + c)\*b + a)\*b^2\*c - a\*b^2\*c - (sqrt(d\*x + c)\*b + a)^3 + 3\*(sqrt(d\*x + c)\*b + a)^2\*a - 3\*(sqrt(d\*x + c)\*b + a)\*a^2 + a^3 + 6\*sqrt(d\*x + c)\*b)\*sin(sqrt(d\*x + c)\*b + a)/b^2/(b^2\*d^2)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x \cos \left( a + b \sqrt{c + dx} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cos(a + b\*(c + d\*x)^(1/2)),x)

[Out] int(x\*cos(a + b\*(c + d\*x)^(1/2)), x)

### 3.92 $\int \cos(a + b\sqrt{c + dx}) dx$

Optimal. Leaf size=54

$$\frac{2 \cos(a + b\sqrt{c + dx})}{b^2 d} + \frac{2\sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{bd}$$

[Out]  $2*\cos(a+b*(d*x+c)^{(1/2)})/b^2/d+2*\sin(a+b*(d*x+c)^{(1/2)})*(d*x+c)^{(1/2)}/b/d$

Rubi [A]

time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3443, 3377, 2718}

$$\frac{2 \cos(a + b\sqrt{c + dx})}{b^2 d} + \frac{2\sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{bd}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Sqrt[c + d\*x]],x]

[Out]  $(2*\text{Cos}[a + b*\text{Sqrt}[c + d*x]])/(b^2*d) + (2*\text{Sqrt}[c + d*x]*\text{Sin}[a + b*\text{Sqrt}[c + d*x]])/(b*d)$

Rule 2718

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[-Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := Simp[(-(c + d\*x)^m)\*(Cos[e + f\*x]/f), x] + Dist[d\*(m/f), Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3443

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/(n\*f), Subst[Int[x^(1/n - 1)\*(a + b\*Cos[c + d\*x])^p, x], x, (e + f\*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned}
\int \cos\left(a + b\sqrt{c + dx}\right) dx &= \frac{2\text{Subst}\left(\int x \cos(a + bx) dx, x, \sqrt{c + dx}\right)}{d} \\
&= \frac{2\sqrt{c + dx} \sin\left(a + b\sqrt{c + dx}\right)}{bd} - \frac{2\text{Subst}\left(\int \sin(a + bx) dx, x, \sqrt{c + dx}\right)}{bd} \\
&= \frac{2 \cos\left(a + b\sqrt{c + dx}\right)}{b^2d} + \frac{2\sqrt{c + dx} \sin\left(a + b\sqrt{c + dx}\right)}{bd}
\end{aligned}$$

**Mathematica [A]**

time = 0.09, size = 48, normalized size = 0.89

$$\frac{2\left(\cos\left(a + b\sqrt{c + dx}\right) + b\sqrt{c + dx} \sin\left(a + b\sqrt{c + dx}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*Sqrt[c + d*x]],x]``[Out] (2*(Cos[a + b*Sqrt[c + d*x]] + b*Sqrt[c + d*x]*Sin[a + b*Sqrt[c + d*x]]))/(b^2*d)`**Maple [A]**

time = 0.04, size = 61, normalized size = 1.13

method	result	size
derivativedivides	$\frac{2 \cos\left(a+b\sqrt{dx+c}\right)+2\left(a+b\sqrt{dx+c}\right) \sin\left(a+b\sqrt{dx+c}\right)-2a \sin\left(a+b\sqrt{dx+c}\right)}{db^2}$	61
default	$\frac{2 \cos\left(a+b\sqrt{dx+c}\right)+2\left(a+b\sqrt{dx+c}\right) \sin\left(a+b\sqrt{dx+c}\right)-2a \sin\left(a+b\sqrt{dx+c}\right)}{db^2}$	61

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)``[Out] 2/d/b^2*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2))-a*sin(a+b*(d*x+c)^(1/2)))`**Maxima [A]**

time = 0.31, size = 60, normalized size = 1.11

$$\frac{2\left(\left(\sqrt{dx+c}b+a\right) \sin\left(\sqrt{dx+c}b+a\right)-a \sin\left(\sqrt{dx+c}b+a\right)+\cos\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")`

[Out]  $2*((\sqrt{d*x + c}*b + a)*\sin(\sqrt{d*x + c}*b + a) - a*\sin(\sqrt{d*x + c}*b + a) + \cos(\sqrt{d*x + c}*b + a))/(b^2*d)$

**Fricas** [A]

time = 0.38, size = 42, normalized size = 0.78

$$\frac{2 \left( \sqrt{dx + c} b \sin \left( \sqrt{dx + c} b + a \right) + \cos \left( \sqrt{dx + c} b + a \right) \right)}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")`

[Out]  $2*(\sqrt{d*x + c}*b*\sin(\sqrt{d*x + c}*b + a) + \cos(\sqrt{d*x + c}*b + a))/(b^2*d)$

**Sympy** [A]

time = 0.13, size = 65, normalized size = 1.20

$$\begin{cases} x \cos(a) & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ x \cos(a + b\sqrt{c}) & \text{for } d = 0 \\ \frac{2\sqrt{c + dx} \sin(a + b\sqrt{c + dx})}{bd} + \frac{2\cos(a + b\sqrt{c + dx})}{b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*(d*x+c)**(1/2)),x)`

[Out] `Piecewise((x*cos(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*cos(a + b*sqrt(c)), Eq(d, 0)), (2*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b*d) + 2*cos(a + b*sqrt(c + d*x))/(b**2*d), True))`

**Giac** [A]

time = 0.50, size = 42, normalized size = 0.78

$$\frac{2 \left( \sqrt{dx + c} b \sin \left( \sqrt{dx + c} b + a \right) + \cos \left( \sqrt{dx + c} b + a \right) \right)}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*(d*x+c)^(1/2)),x, algorithm="giac")`

[Out]  $2*(\sqrt{d*x + c}*b*\sin(\sqrt{d*x + c}*b + a) + \cos(\sqrt{d*x + c}*b + a))/(b^2*d)$

**Mupad [B]**

time = 0.34, size = 42, normalized size = 0.78

$$\frac{2 \left( \cos \left( a + b \sqrt{c + dx} \right) + b \sin \left( a + b \sqrt{c + dx} \right) \sqrt{c + dx} \right)}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*(c + d*x)^(1/2)),x)`

[Out] `(2*(cos(a + b*(c + d*x)^(1/2)) + b*sin(a + b*(c + d*x)^(1/2))*(c + d*x)^(1/2)))/(b^2*d)`

$$3.93 \quad \int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} dx$$

**Optimal.** Leaf size=126

$$\cos(a-b\sqrt{c}) \operatorname{CosIntegral}\left(b\left(\sqrt{c} + \sqrt{c+dx}\right)\right) + \cos(a+b\sqrt{c}) \operatorname{CosIntegral}\left(b\sqrt{c} - b\sqrt{c+dx}\right) - \sin(a-b\sqrt{c}) \operatorname{Si}\left(b\left(\sqrt{c} + \sqrt{c+dx}\right)\right) + \sin(a+b\sqrt{c}) \operatorname{Si}\left(b\sqrt{c} - b\sqrt{c+dx}\right)$$

```
[Out] Ci(b*(c^(1/2)+(d*x+c)^(1/2)))*cos(a-b*c^(1/2))+Ci(b*c^(1/2)-b*(d*x+c)^(1/2))*cos(a+b*c^(1/2))-Si(b*(c^(1/2)+(d*x+c)^(1/2)))*sin(a-b*c^(1/2))+Si(b*c^(1/2)-b*(d*x+c)^(1/2))*sin(a+b*c^(1/2))
```

**Rubi [A]**

time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3513, 3384, 3380, 3383}

$$\cos(a-b\sqrt{c}) \operatorname{CosIntegral}\left(b\left(\sqrt{c+dx} + \sqrt{c}\right)\right) + \cos(a+b\sqrt{c}) \operatorname{CosIntegral}\left(b\sqrt{c} - b\sqrt{c+dx}\right) - \sin(a-b\sqrt{c}) \operatorname{Si}\left(b\left(\sqrt{c} + \sqrt{c+dx}\right)\right) + \sin(a+b\sqrt{c}) \operatorname{Si}\left(b\sqrt{c} - b\sqrt{c+dx}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] Cos[a - b*Sqrt[c]]*CosIntegral[b*(Sqrt[c] + Sqrt[c + d*x])] + Cos[a + b*Sqrt[c]]*CosIntegral[b*Sqrt[c] - b*Sqrt[c + d*x]] - Sin[a - b*Sqrt[c]]*SinIntegral[b*(Sqrt[c] + Sqrt[c + d*x])] + Sin[a + b*Sqrt[c]]*SinIntegral[b*Sqrt[c] - b*Sqrt[c + d*x]]
```

**Rule 3380**

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

**Rule 3383**

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

**Rule 3384**

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

**Rule 3513**

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Dist[1/(n*f), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x], x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p, 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(a + b\sqrt{c + dx})}{x} dx &= \frac{2 \operatorname{Subst}\left(\int \left(-\frac{d \cos(a+bx)}{2(\sqrt{c}-x)} + \frac{d \cos(a+bx)}{2(\sqrt{c}+x)}\right) dx, x, \sqrt{c+dx}\right)}{d} \\ &= -\operatorname{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{c}-x} dx, x, \sqrt{c+dx}\right) + \operatorname{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{c}+x} dx, x, \sqrt{c+dx}\right) \\ &= \cos(a - b\sqrt{c}) \operatorname{Subst}\left(\int \frac{\cos(b\sqrt{c} + bx)}{\sqrt{c} + x} dx, x, \sqrt{c+dx}\right) - \cos(a + b\sqrt{c}) \operatorname{Subst}\left(\int \frac{\cos(b\sqrt{c} + bx)}{\sqrt{c} - x} dx, x, \sqrt{c+dx}\right) \\ &= \cos(a - b\sqrt{c}) \operatorname{Ci}\left(b\left(\sqrt{c} + \sqrt{c+dx}\right)\right) + \cos(a + b\sqrt{c}) \operatorname{Ci}\left(b\sqrt{c} - b\sqrt{c+dx}\right) \end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.  
time = 0.70, size = 145, normalized size = 1.15

$$\frac{1}{2} e^{-i(a+b\sqrt{c})} \left( \operatorname{Ei}\left(-ib\left(-\sqrt{c} + \sqrt{c+dx}\right)\right) + e^{2i(a+b\sqrt{c})} \operatorname{Ei}\left(ib\left(-\sqrt{c} + \sqrt{c+dx}\right)\right) + e^{2ib\sqrt{c}} \operatorname{Ei}\left(-ib\left(\sqrt{c} + \sqrt{c+dx}\right)\right) + e^{2ia} \operatorname{Ei}\left(ib\left(\sqrt{c} + \sqrt{c+dx}\right)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Sqrt[c + d\*x]]/x,x]

[Out] (ExpIntegralEi[(-I)\*b\*(-Sqrt[c] + Sqrt[c + d\*x])] + E^((2\*I)\*(a + b\*Sqrt[c]))\*ExpIntegralEi[I\*b\*(-Sqrt[c] + Sqrt[c + d\*x])] + E^((2\*I)\*b\*Sqrt[c])\*ExpIntegralEi[(-I)\*b\*(Sqrt[c] + Sqrt[c + d\*x])] + E^((2\*I)\*a)\*ExpIntegralEi[I\*b\*(Sqrt[c] + Sqrt[c + d\*x])])/(2\*E^(I\*(a + b\*Sqrt[c])))

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(102) = 204.  
time = 0.07, size = 271, normalized size = 2.15

method	result
derivativedivides	$\frac{b\left(a+b\sqrt{c}\right)\left(\sin\operatorname{Integral}\left(b\sqrt{c}-b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{c}\right)+\cosine\operatorname{Integral}\left(b\sqrt{dx+c}-b\sqrt{c}\right)\cos\left(a+b\sqrt{c}\right)\right)}{\sqrt{c}}$



default	$\frac{b \left( a + b \sqrt{c} \right) \left( \operatorname{Si} \left( b \sqrt{c} - b \sqrt{dx + c} \right) \sin \left( a + b \sqrt{c} \right) + \operatorname{Ci} \left( b \sqrt{dx + c} - b \sqrt{c} \right) \cos \left( a + b \sqrt{c} \right) \right)}{\sqrt{c}}$
---------	---

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*(d*x+c)^(1/2))/x,x,method=_RETURNVERBOSE)`

[Out] 
$$\frac{2}{b^2} \left( \frac{1}{2} b (a + b \sqrt{c}) / c^{1/2} \left( \operatorname{Si}(b \sqrt{c} - b \sqrt{dx + c}) \sin(a + b \sqrt{c}) + \operatorname{Ci}(b \sqrt{dx + c} - b \sqrt{c}) \cos(a + b \sqrt{c}) \right) - \frac{1}{2} b (a - b \sqrt{c}) / c^{1/2} \left( -\operatorname{Si}(b \sqrt{dx + c} - b \sqrt{c}) \sin(a - b \sqrt{c}) + \operatorname{Ci}(b \sqrt{dx + c} - b \sqrt{c}) \cos(a - b \sqrt{c}) \right) - b^2 a \left( \frac{1}{2} b / c^{1/2} \left( \operatorname{Si}(b \sqrt{c} - b \sqrt{dx + c}) \sin(a + b \sqrt{c}) + \operatorname{Ci}(b \sqrt{dx + c} - b \sqrt{c}) \cos(a + b \sqrt{c}) \right) - \frac{1}{2} b / c^{1/2} \left( -\operatorname{Si}(b \sqrt{dx + c} - b \sqrt{c}) \sin(a - b \sqrt{c}) + \operatorname{Ci}(b \sqrt{dx + c} - b \sqrt{c}) \cos(a - b \sqrt{c}) \right) \right) \right)$$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*(d*x+c)^(1/2))/x,x, algorithm="maxima")`

[Out] `integrate(cos(sqrt(d*x + c)*b + a)/x, x)`

**Fricas** [C] Result contains complex when optimal does not.

time = 0.37, size = 149, normalized size = 1.18

$$\frac{1}{2} \operatorname{Ei} \left( i \sqrt{dx + c} b - \sqrt{-b^2 c} \right) e^{(ia + \sqrt{-b^2 c})} + \frac{1}{2} \operatorname{Ei} \left( i \sqrt{dx + c} b + \sqrt{-b^2 c} \right) e^{(ia - \sqrt{-b^2 c})} + \frac{1}{2} \operatorname{Ei} \left( -i \sqrt{dx + c} b - \sqrt{-b^2 c} \right) e^{(-ia + \sqrt{-b^2 c})} + \frac{1}{2} \operatorname{Ei} \left( -i \sqrt{dx + c} b + \sqrt{-b^2 c} \right) e^{(-ia - \sqrt{-b^2 c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*(d*x+c)^(1/2))/x,x, algorithm="fricas")`

[Out] 
$$\frac{1}{2} \operatorname{Ei} \left( I \sqrt{dx + c} b - \sqrt{-b^2 c} \right) e^{(I a + \sqrt{-b^2 c})} + \frac{1}{2} \operatorname{Ei} \left( I \sqrt{dx + c} b + \sqrt{-b^2 c} \right) e^{(I a - \sqrt{-b^2 c})} + \frac{1}{2} \operatorname{Ei} \left( -I \sqrt{dx + c} b - \sqrt{-b^2 c} \right) e^{(-I a + \sqrt{-b^2 c})} + \frac{1}{2} \operatorname{Ei} \left( -I \sqrt{dx + c} b + \sqrt{-b^2 c} \right) e^{(-I a - \sqrt{-b^2 c})}$$

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos \left( a + b \sqrt{c + dx} \right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*(d\*x+c)\*\*(1/2))/x,x)

[Out] Integral(cos(a + b\*sqrt(c + d\*x))/x, x)

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*(d\*x+c)^(1/2))/x,x, algorithm="giac")

[Out] integrate(cos(sqrt(d\*x + c)\*b + a)/x, x)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a + b\sqrt{c + dx}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*(c + d\*x)^(1/2))/x,x)

[Out] int(cos(a + b\*(c + d\*x)^(1/2))/x, x)

$$3.94 \quad \int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2} dx$$

**Optimal.** Leaf size=184

$$-\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} + \frac{bd\operatorname{CosIntegral}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\sin\left(a-b\sqrt{c}\right)}{2\sqrt{c}} - \frac{bd\operatorname{CosIntegral}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}}$$

[Out]  $-\cos(a+b*(d*x+c)^{(1/2)})/x+1/2*b*d*\cos(a-b*c^{(1/2)})*Si(b*(c^{(1/2)}+(d*x+c)^{(1/2)}))/c^{(1/2)}+1/2*b*d*\cos(a+b*c^{(1/2)})*Si(b*c^{(1/2)}-b*(d*x+c)^{(1/2)})/c^{(1/2)}+1/2*b*d*Ci(b*(c^{(1/2)}+(d*x+c)^{(1/2)}))*\sin(a-b*c^{(1/2)})/c^{(1/2)}-1/2*b*d*Ci(b*c^{(1/2)}-b*(d*x+c)^{(1/2)})*\sin(a+b*c^{(1/2)})/c^{(1/2)}$

**Rubi [A]**

time = 0.25, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3513, 3423, 3414, 3384, 3380, 3383}

$$\frac{bd\sin(a-b\sqrt{c})\operatorname{CosIntegral}\left(b\left(\sqrt{c+dx}+\sqrt{c}\right)\right)}{2\sqrt{c}} - \frac{bd\sin(a+b\sqrt{c})\operatorname{CosIntegral}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}} + \frac{bd\cos(a-b\sqrt{c})\operatorname{Si}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} + \frac{bd\cos(a+b\sqrt{c})\operatorname{Si}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}} - \frac{\cos(a+b\sqrt{c+dx})}{x}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Sqrt[c + d\*x]]/x^2,x]

[Out]  $-(\operatorname{Cos}[a+b\sqrt{c+dx}]/x) + (b*d*\operatorname{CosIntegral}[b*(\sqrt{c}+\sqrt{c+dx})]*\operatorname{Sin}[a-b\sqrt{c}])/(2*\sqrt{c}) - (b*d*\operatorname{CosIntegral}[b*\sqrt{c}-b*\sqrt{c+dx}]*\operatorname{Sin}[a+b\sqrt{c}])/(2*\sqrt{c}) + (b*d*\operatorname{Cos}[a-b\sqrt{c}]*\operatorname{SinIntegral}[b*(\sqrt{c}+\sqrt{c+dx})])/(2*\sqrt{c}) + (b*d*\operatorname{Cos}[a+b\sqrt{c}]*\operatorname{SinIntegral}[b*\sqrt{c}-b*\sqrt{c+dx}])/(2*\sqrt{c})$

**Rule 3380**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

**Rule 3383**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) - c\*f, 0]

**Rule 3384**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[c\*(f/d) + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[c\*(f/d) + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&

NeQ[d\*e - c\*f, 0]

#### Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] := Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

#### Rule 3423

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_
), x_Symbol] := Simp[e^m*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1))),
x] + Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sin[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
negerQ[n] || GtQ[e, 0])
```

#### Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_)^(n_))]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_)^(m_.), x_Symbol] := Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
0] && IntegerQ[1/n]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\cos(a + b\sqrt{c + dx})}{x^2} dx &= \frac{2\text{Subst}\left(\int \frac{x \cos(a+bx)}{\left(-\frac{c}{d} + \frac{x^2}{d}\right)^2} dx, x, \sqrt{c + dx}\right)}{d} \\
&= -\frac{\cos(a + b\sqrt{c + dx})}{x} - b\text{Subst}\left(\int \frac{\sin(a + bx)}{-\frac{c}{d} + \frac{x^2}{d}} dx, x, \sqrt{c + dx}\right) \\
&= -\frac{\cos(a + b\sqrt{c + dx})}{x} - b\text{Subst}\left(\int \left(-\frac{d \sin(a + bx)}{2\sqrt{c}(\sqrt{c} - x)} - \frac{d \sin(a + bx)}{2\sqrt{c}(\sqrt{c} + x)}\right) dx, x, \sqrt{c + dx}\right) \\
&= -\frac{\cos(a + b\sqrt{c + dx})}{x} + \frac{(bd)\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{c} - x} dx, x, \sqrt{c + dx}\right)}{2\sqrt{c}} + \frac{(bd)\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{c} + x} dx, x, \sqrt{c + dx}\right)}{2\sqrt{c}} \\
&= -\frac{\cos(a + b\sqrt{c + dx})}{x} + \frac{(bd \cos(a - b\sqrt{c})) \text{Subst}\left(\int \frac{\sin(b\sqrt{c} + bx)}{\sqrt{c} + x} dx, x, \sqrt{c + dx}\right)}{2\sqrt{c}} \\
&= -\frac{\cos(a + b\sqrt{c + dx})}{x} + \frac{bd\text{Ci}\left(b\left(\sqrt{c} + \sqrt{c + dx}\right)\right) \sin(a - b\sqrt{c})}{2\sqrt{c}} - \frac{bd\text{Ci}\left(b\left(\sqrt{c} - \sqrt{c + dx}\right)\right) \sin(a - b\sqrt{c})}{2\sqrt{c}}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.23, size = 240, normalized size = 1.30

$$\frac{i\left(e^{-ia}\left(2i\sqrt{c}e^{-ib\sqrt{c+dx}} - bde^{-ib\sqrt{c}}x\text{Ei}\left(-ib\left(-\sqrt{c} + \sqrt{c+dx}\right)\right) + bde^{ib\sqrt{c}}x\text{Ei}\left(-ib\left(\sqrt{c} + \sqrt{c+dx}\right)\right)\right) + e^{i(a-b\sqrt{c})}\left(2i\sqrt{c}e^{ib\sqrt{c}}x\text{Ei}\left(ib\left(-\sqrt{c} + \sqrt{c+dx}\right)\right) - bdx\text{Ei}\left(ib\left(\sqrt{c} + \sqrt{c+dx}\right)\right)\right)\right)}{4\sqrt{c}x}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Sqrt[c + d\*x]]/x^2,x]

[Out] ((I/4)\*(((2\*I)\*Sqrt[c])/E^(I\*b\*Sqrt[c + d\*x]) - (b\*d\*x\*ExpIntegralEi[(-I)\*b\*(-Sqrt[c] + Sqrt[c + d\*x])])/E^(I\*b\*Sqrt[c]) + b\*d\*E^(I\*b\*Sqrt[c])\*x\*ExpIntegralEi[(-I)\*b\*(Sqrt[c] + Sqrt[c + d\*x])])/E^(I\*a) + E^(I\*(a - b\*Sqrt[c])))\*((2\*I)\*Sqrt[c]\*E^(I\*b\*(Sqrt[c] + Sqrt[c + d\*x])) + b\*d\*E^((2\*I)\*b\*Sqrt[c])\*x\*ExpIntegralEi[I\*b\*(-Sqrt[c] + Sqrt[c + d\*x])] - b\*d\*x\*ExpIntegralEi[I\*b\*(Sqrt[c] + Sqrt[c + d\*x])]))/(Sqrt[c]\*x)

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 713 vs. 2(142) = 284.

time = 0.08, size = 714, normalized size = 3.88

method	result
--------	--------

derivativedivides	$2d \frac{\cos(a+b\sqrt{dx+c}) \left( -\frac{b^2 a (a+b\sqrt{dx+c})}{2c} + \frac{b^2 (-cb^2+a^2)}{2c} \right)}{-cb^2+a^2-2(a+b\sqrt{dx+c})_{a+(a+b\sqrt{dx+c})^2}} - \frac{ba \left( \text{sinIntegral}(b\sqrt{c}-b\sqrt{dx+c}) \right) \sin(a+b\sqrt{dx+c})}{(a+b\sqrt{dx+c})^2}$
default	$2d \frac{\cos(a+b\sqrt{dx+c}) \left( -\frac{b^2 a (a+b\sqrt{dx+c})}{2c} + \frac{b^2 (-cb^2+a^2)}{2c} \right)}{-cb^2+a^2-2(a+b\sqrt{dx+c})_{a+(a+b\sqrt{dx+c})^2}} - \frac{ba \left( \text{sinIntegral}(b\sqrt{c}-b\sqrt{dx+c}) \right) \sin(a+b\sqrt{dx+c})}{(a+b\sqrt{dx+c})^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*(d*x+c)^(1/2))/x^2,x,method=_RETURNVERBOSE)`

[Out]  $2*d/b^2*(\cos(a+b*(d*x+c)^{(1/2)})*(-1/2*b^2*a/c*(a+b*(d*x+c)^{(1/2)})+1/2*b^2*(-b^2*c+a^2)/c)/(-c*b^2+a^2-2*(a+b*(d*x+c)^{(1/2)})*a+(a+b*(d*x+c)^{(1/2)})^2)-1/4*b*a/c^{(3/2)}*(\text{Si}(b*c^{(1/2)}-b*(d*x+c)^{(1/2)})*\sin(a+b*c^{(1/2)})+\text{Ci}(b*(d*x+c)^{(1/2)}-b*c^{(1/2)})*\cos(a+b*c^{(1/2)}))+1/4*b*a/c^{(3/2)}*(-\text{Si}(b*(d*x+c)^{(1/2)}+b*c^{(1/2)})*\sin(a-b*c^{(1/2)})+\text{Ci}(b*(d*x+c)^{(1/2)}+b*c^{(1/2)})*\cos(a-b*c^{(1/2)}))+1/4*b*(-c*b^2+a^2-(a+b*c^{(1/2)})*a)/c^{(3/2)}*(-\text{Si}(b*c^{(1/2)}-b*(d*x+c)^{(1/2)})*\cos(a+b*c^{(1/2)})+\text{Ci}(b*(d*x+c)^{(1/2)}-b*c^{(1/2)})*\sin(a+b*c^{(1/2)}))-1/4*b*(-c*b^2+a^2-(a-b*c^{(1/2)})*a)/c^{(3/2)}*(\text{Si}(b*(d*x+c)^{(1/2)}+b*c^{(1/2)})*\cos(a-b*c^{(1/2)})+\text{Ci}(b*(d*x+c)^{(1/2)}+b*c^{(1/2)})*\sin(a-b*c^{(1/2)}))-a*b^4*(\cos(a+b*(d*x+c)^{(1/2)})*(-1/2/c/b^2*(a+b*(d*x+c)^{(1/2)})+1/2*a/c/b^2)/(-c*b^2+a^2-2*(a+b*(d*x+c)^{(1/2)})*a+(a+b*(d*x+c)^{(1/2)})^2)-1/4/c^{(3/2)}/b^3*(\text{Si}(b*c^{(1/2)}-b*(d*x+c)^{(1/2)})*\sin(a+b*c^{(1/2)})+\text{Ci}(b*(d*x+c)^{(1/2)}-b*c^{(1/2)})*\cos(a+b*c^{(1/2)}))+1/4/c^{(3/2)}/b^3*(-\text{Si}(b*(d*x+c)^{(1/2)}+b*c^{(1/2)})*\sin(a-b*c^{(1/2)})+\text{Ci}(b*(d*x+c)^{(1/2)}+b*c^{(1/2)})*\cos(a-b*c^{(1/2)}))-1/4/c/b^2*(-\text{Si}(b*c^{(1/2)}-b*(d*x+c)^{(1/2)})*\cos(a+b*c^{(1/2)})+\text{Ci}(b*(d*x+c)^{(1/2)}-b*c^{(1/2)})*\sin(a+b*c^{(1/2)}))-1/4/c/b^2*(\text{Si}(b*(d*x+c)^{(1/2)}+b*c^{(1/2)})*\cos(a-b*c^{(1/2)})+\text{Ci}(b*(d*x+c)^{(1/2)}+b*c^{(1/2)})*\sin(a-b*c^{(1/2)}))$

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="maxima")`

[Out] `integrate(cos(sqrt(d*x + c)*b + a)/x^2, x)`

**Fricas** [C] Result contains complex when optimal does not.

time = 0.40, size = 210, normalized size = 1.14

$$\frac{\sqrt{-b^2c} dx Ei(i\sqrt{dx+c} b - \sqrt{-b^2c}) e^{(i a + \sqrt{-b^2c})} - \sqrt{-b^2c} dx Ei(i\sqrt{dx+c} b + \sqrt{-b^2c}) e^{(i a - \sqrt{-b^2c})} + \sqrt{-b^2c} dx Ei(-i\sqrt{dx+c} b - \sqrt{-b^2c}) e^{(-i a + \sqrt{-b^2c})} - \sqrt{-b^2c} dx Ei(-i\sqrt{dx+c} b + \sqrt{-b^2c}) e^{(-i a - \sqrt{-b^2c})} - 4 c \cos(\sqrt{dx+c} b + a)}{4 c x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*(d\*x+c)^(1/2))/x^2,x, algorithm="fricas")

[Out] 1/4\*(sqrt(-b^2\*c)\*d\*x\*Ei(I\*sqrt(d\*x + c)\*b - sqrt(-b^2\*c))\*e^(I\*a + sqrt(-b^2\*c)) - sqrt(-b^2\*c)\*d\*x\*Ei(I\*sqrt(d\*x + c)\*b + sqrt(-b^2\*c))\*e^(I\*a - sqrt(-b^2\*c)) + sqrt(-b^2\*c)\*d\*x\*Ei(-I\*sqrt(d\*x + c)\*b - sqrt(-b^2\*c))\*e^(-I\*a + sqrt(-b^2\*c)) - sqrt(-b^2\*c)\*d\*x\*Ei(-I\*sqrt(d\*x + c)\*b + sqrt(-b^2\*c))\*e^(-I\*a - sqrt(-b^2\*c)) - 4\*c\*cos(sqrt(d\*x + c)\*b + a))/(c\*x)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + b\sqrt{c + dx}\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*(d\*x+c)\*\*(1/2))/x\*\*2,x)

[Out] Integral(cos(a + b\*sqrt(c + d\*x))/x\*\*2, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*(d\*x+c)^(1/2))/x^2,x, algorithm="giac")

[Out] integrate(cos(sqrt(d\*x + c)\*b + a)/x^2, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a + b\sqrt{c + dx}\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*(c + d\*x)^(1/2))/x^2,x)

[Out] int(cos(a + b\*(c + d\*x)^(1/2))/x^2, x)

### 3.95 $\int x^2 \cos \left( a + b\sqrt[3]{c + dx} \right) dx$

**Optimal.** Leaf size=537

$$\frac{720c \cos \left( a + b\sqrt[3]{c + dx} \right)}{b^6 d^3} - \frac{120960 \sqrt[3]{c + dx} \cos \left( a + b\sqrt[3]{c + dx} \right)}{b^8 d^3} + \frac{6c^2 \sqrt[3]{c + dx} \cos \left( a + b\sqrt[3]{c + dx} \right)}{b^2 d^3} + \dots$$

[Out]  $-720*c*\cos(a+b*(d*x+c)^{(1/3)})/b^6/d^3-120960*(d*x+c)^{(1/3)}*\cos(a+b*(d*x+c)^{(1/3)})/b^8/d^3+6*c^2*(d*x+c)^{(1/3)}*\cos(a+b*(d*x+c)^{(1/3)})/b^2/d^3+360*c*(d*x+c)^{(2/3)}*\cos(a+b*(d*x+c)^{(1/3)})/b^4/d^3+20160*(d*x+c)*\cos(a+b*(d*x+c)^{(1/3)})/b^6/d^3-30*c*(d*x+c)^{(4/3)}*\cos(a+b*(d*x+c)^{(1/3)})/b^2/d^3-1008*(d*x+c)^{(5/3)}*\cos(a+b*(d*x+c)^{(1/3)})/b^4/d^3+24*(d*x+c)^{(7/3)}*\cos(a+b*(d*x+c)^{(1/3)})/b^2/d^3+120960*\sin(a+b*(d*x+c)^{(1/3)})/b^9/d^3-6*c^2*\sin(a+b*(d*x+c)^{(1/3)})/b^3/d^3-720*c*(d*x+c)^{(1/3)}*\sin(a+b*(d*x+c)^{(1/3)})/b^5/d^3-60480*(d*x+c)^{(2/3)}*\sin(a+b*(d*x+c)^{(1/3)})/b^7/d^3+3*c^2*(d*x+c)^{(2/3)}*\sin(a+b*(d*x+c)^{(1/3)})/b/d^3+120*c*(d*x+c)*\sin(a+b*(d*x+c)^{(1/3)})/b^3/d^3+5040*(d*x+c)^{(4/3)}*\sin(a+b*(d*x+c)^{(1/3)})/b^5/d^3-6*c*(d*x+c)^{(5/3)}*\sin(a+b*(d*x+c)^{(1/3)})/b/d^3-168*(d*x+c)^2*\sin(a+b*(d*x+c)^{(1/3)})/b^3/d^3+3*(d*x+c)^{(8/3)}*\sin(a+b*(d*x+c)^{(1/3)})/b/d^3$

**Rubi [A]**

time = 0.36, antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3513, 3377, 2717, 2718}

Antiderivative was successfully verified.

[In] Int[x^2\*Cos[a + b\*(c + d\*x)^(1/3)],x]

[Out]  $(-720*c*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^6*d^3) - (120960*(c + d*x)^{(1/3)}*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^8*d^3) + (6*c^2*(c + d*x)^{(1/3)}*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^2*d^3) + (360*c*(c + d*x)^{(2/3)}*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^4*d^3) + (20160*(c + d*x)*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^6*d^3) - (30*c*(c + d*x)^{(4/3)}*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^2*d^3) - (1008*(c + d*x)^{(5/3)}*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^4*d^3) + (24*(c + d*x)^{(7/3)}*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^2*d^3) + (120960*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b^9*d^3) - (6*c^2*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b^3*d^3) - (720*c*(c + d*x)^{(1/3)}*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b^5*d^3) - (60480*(c + d*x)^{(2/3)}*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b^7*d^3) + (3*c^2*(c + d*x)^{(2/3)}*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b*d^3) + (120*c*(c + d*x)*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b^3*d^3) + (5040*(c + d*x)^{(4/3)}*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b^5*d^3) - (6*c*(c + d*x)^{(5/3)}*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b*d^3) - (168*(c + d*x)^2*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b^3*d^3) + (3*(c + d*x)^{(8/3)}*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b*d^3)$



Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-
(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))]^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] := Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cos(a + b\sqrt[3]{c + dx}) dx &= \frac{3 \text{Subst}\left(\int \left(\frac{c^2 x^2 \cos(a+bx)}{d^2} - \frac{2cx^5 \cos(a+bx)}{d^2} + \frac{x^8 \cos(a+bx)}{d^2}\right) dx, x, \sqrt[3]{c + dx}\right)}{d} \\
&= \frac{3 \text{Subst}\left(\int x^8 \cos(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^3} - \frac{(6c) \text{Subst}\left(\int x^5 \cos(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^3} \\
&= \frac{3c^2(c + dx)^{2/3} \sin(a + b\sqrt[3]{c + dx})}{bd^3} - \frac{6c(c + dx)^{5/3} \sin(a + b\sqrt[3]{c + dx})}{bd^3} + \\
&= \frac{6c^2 \sqrt[3]{c + dx} \cos(a + b\sqrt[3]{c + dx})}{b^2 d^3} - \frac{30c(c + dx)^{4/3} \cos(a + b\sqrt[3]{c + dx})}{b^2 d^3} + \\
&= \frac{6c^2 \sqrt[3]{c + dx} \cos(a + b\sqrt[3]{c + dx})}{b^2 d^3} - \frac{30c(c + dx)^{4/3} \cos(a + b\sqrt[3]{c + dx})}{b^2 d^3} + \\
&= \frac{6c^2 \sqrt[3]{c + dx} \cos(a + b\sqrt[3]{c + dx})}{b^2 d^3} + \frac{360c(c + dx)^{2/3} \cos(a + b\sqrt[3]{c + dx})}{b^4 d^3} \\
&= \frac{6c^2 \sqrt[3]{c + dx} \cos(a + b\sqrt[3]{c + dx})}{b^2 d^3} + \frac{360c(c + dx)^{2/3} \cos(a + b\sqrt[3]{c + dx})}{b^4 d^3} \\
&= -\frac{720c \cos(a + b\sqrt[3]{c + dx})}{b^6 d^3} + \frac{6c^2 \sqrt[3]{c + dx} \cos(a + b\sqrt[3]{c + dx})}{b^2 d^3} + \frac{360c(c + dx)^{2/3} \cos(a + b\sqrt[3]{c + dx})}{b^4 d^3} \\
&= -\frac{720c \cos(a + b\sqrt[3]{c + dx})}{b^6 d^3} + \frac{6c^2 \sqrt[3]{c + dx} \cos(a + b\sqrt[3]{c + dx})}{b^2 d^3} + \frac{360c(c + dx)^{2/3} \cos(a + b\sqrt[3]{c + dx})}{b^4 d^3} \\
&= -\frac{720c \cos(a + b\sqrt[3]{c + dx})}{b^6 d^3} - \frac{120960 \sqrt[3]{c + dx} \cos(a + b\sqrt[3]{c + dx})}{b^8 d^3} + \frac{6c^2(c + dx)^{2/3} \cos(a + b\sqrt[3]{c + dx})}{b^4 d^3} \\
&= -\frac{720c \cos(a + b\sqrt[3]{c + dx})}{b^6 d^3} - \frac{120960 \sqrt[3]{c + dx} \cos(a + b\sqrt[3]{c + dx})}{b^8 d^3} + \frac{6c^2(c + dx)^{2/3} \cos(a + b\sqrt[3]{c + dx})}{b^4 d^3}
\end{aligned}$$

**Mathematica [C]** Result contains complex when optimal does not.

time = 1.04, size = 382, normalized size = 0.71

$$\frac{3c^2 \sqrt[3]{c + dx} \cos(a + b\sqrt[3]{c + dx})}{b^2 d^3} - \frac{360c(c + dx)^{2/3} \cos(a + b\sqrt[3]{c + dx})}{b^4 d^3} - \frac{720c \cos(a + b\sqrt[3]{c + dx})}{b^6 d^3} - \frac{120960 \sqrt[3]{c + dx} \cos(a + b\sqrt[3]{c + dx})}{b^8 d^3} + \frac{6c^2(c + dx)^{2/3} \cos(a + b\sqrt[3]{c + dx})}{b^4 d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Cos[a + b\*(c + d\*x)^(1/3)],x]

[Out] (3\*((-40320\*I)\*(-1 + E^((2\*I)\*(a + b\*(c + d\*x)^(1/3)))) - 40320\*b\*(1 + E^((2\*I)\*(a + b\*(c + d\*x)^(1/3))))\*(c + d\*x)^(1/3) + (20160\*I)\*b^2\*(-1 + E^((2\*I)\*(a + b\*(c + d\*x)^(1/3))))\*(c + d\*x)^(2/3) - I\*b^8\*d^2\*(-1 + E^((2\*I)\*(a + b\*(c + d\*x)^(1/3))))\*x^2\*(c + d\*x)^(2/3) + 2\*b^7\*d\*(1 + E^((2\*I)\*(a + b\*(c + d\*x)^(1/3))))\*(c + d\*x)^(2/3) - 360c\*(c + d\*x)^(2/3)\*cos(a + b\*(c + d\*x)^(1/3)))/b^4/d^3

$$(c + d*x)^{(1/3)})) * x * (c + d*x)^{(1/3)} * (3*c + 4*d*x) - (240*I) * b^4 * (-1 + E^{((2*I)*(a + b*(c + d*x)^{(1/3))})}) * (c + d*x)^{(1/3)} * (6*c + 7*d*x) - 24*b^5 * (1 + E^{((2*I)*(a + b*(c + d*x)^{(1/3))})}) * (c + d*x)^{(2/3)} * (9*c + 14*d*x) + 240*b^3 * (1 + E^{((2*I)*(a + b*(c + d*x)^{(1/3))})}) * (27*c + 28*d*x) + (2*I) * b^6 * (-1 + E^{((2*I)*(a + b*(c + d*x)^{(1/3))})}) * (9*c^2 + 36*c*d*x + 28*d^2*x^2)) / (2*b^9 * d^3 * E^{(I*(a + b*(c + d*x)^{(1/3))})})$$

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 1808 vs.  $2(477) = 954$ .

time = 0.06, size = 1809, normalized size = 3.37

method	result	size
derivativedivides	Expression too large to display	1809
default	Expression too large to display	1809

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)`

[Out]  $3/d^3/b^3 * (a^2*c^2*\sin(a+b*(d*x+c)^{(1/3)}) - 2*a*c^2*(\cos(a+b*(d*x+c)^{(1/3)})) + (a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)})) + c^2*((a+b*(d*x+c)^{(1/3)})^2*\sin(a+b*(d*x+c)^{(1/3)}) - 2*\sin(a+b*(d*x+c)^{(1/3)}) + 2*(a+b*(d*x+c)^{(1/3)})*\cos(a+b*(d*x+c)^{(1/3)})) + 2/b^3*a^5*c*\sin(a+b*(d*x+c)^{(1/3)}) - 10/b^3*a^4*c*(\cos(a+b*(d*x+c)^{(1/3)})) + (a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)})) + 20/b^3*a^3*c*((a+b*(d*x+c)^{(1/3)})^2*\sin(a+b*(d*x+c)^{(1/3)}) - 2*\sin(a+b*(d*x+c)^{(1/3)}) + 2*(a+b*(d*x+c)^{(1/3)})*\cos(a+b*(d*x+c)^{(1/3)})) - 20/b^3*a^2*c*((a+b*(d*x+c)^{(1/3)})^3*\sin(a+b*(d*x+c)^{(1/3)}) + 3*(a+b*(d*x+c)^{(1/3)})^2*\cos(a+b*(d*x+c)^{(1/3)}) - 6*\cos(a+b*(d*x+c)^{(1/3)}) - 6*(a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)})) + 10/b^3*a*c*((a+b*(d*x+c)^{(1/3)})^4*\sin(a+b*(d*x+c)^{(1/3)}) + 4*(a+b*(d*x+c)^{(1/3)})^3*\cos(a+b*(d*x+c)^{(1/3)}) - 12*(a+b*(d*x+c)^{(1/3)})^2*\sin(a+b*(d*x+c)^{(1/3)}) + 24*\sin(a+b*(d*x+c)^{(1/3)}) - 24*(a+b*(d*x+c)^{(1/3)})*\cos(a+b*(d*x+c)^{(1/3)})) - 2/b^3*c*((a+b*(d*x+c)^{(1/3)})^5*\sin(a+b*(d*x+c)^{(1/3)}) + 5*(a+b*(d*x+c)^{(1/3)})^4*\cos(a+b*(d*x+c)^{(1/3)}) - 20*(a+b*(d*x+c)^{(1/3)})^3*\sin(a+b*(d*x+c)^{(1/3)}) - 60*(a+b*(d*x+c)^{(1/3)})^2*\cos(a+b*(d*x+c)^{(1/3)}) + 120*\cos(a+b*(d*x+c)^{(1/3)}) + 120*(a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)})) + 1/b^6*a^8*\sin(a+b*(d*x+c)^{(1/3)}) - 8/b^6*a^7*(\cos(a+b*(d*x+c)^{(1/3)}) + (a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)})) + 28/b^6*a^6*((a+b*(d*x+c)^{(1/3)})^2*\sin(a+b*(d*x+c)^{(1/3)}) - 2*\sin(a+b*(d*x+c)^{(1/3)}) + 2*(a+b*(d*x+c)^{(1/3)})*\cos(a+b*(d*x+c)^{(1/3)})) - 56/b^6*a^5*((a+b*(d*x+c)^{(1/3)})^3*\sin(a+b*(d*x+c)^{(1/3)}) + 3*(a+b*(d*x+c)^{(1/3)})^2*\cos(a+b*(d*x+c)^{(1/3)}) - 6*\cos(a+b*(d*x+c)^{(1/3)}) - 6*(a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)})) + 70/b^6*a^4*((a+b*(d*x+c)^{(1/3)})^4*\sin(a+b*(d*x+c)^{(1/3)}) + 4*(a+b*(d*x+c)^{(1/3)})^3*\cos(a+b*(d*x+c)^{(1/3)}) - 12*(a+b*(d*x+c)^{(1/3)})^2*\sin(a+b*(d*x+c)^{(1/3)}) + 24*\sin(a+b*(d*x+c)^{(1/3)}) - 24*(a+b*(d*x+c)^{(1/3)})*\cos(a+b*(d*x+c)^{(1/3)})) - 56/b^6*a^3*((a+b*(d*x+c)^{(1/3)})^5*\sin(a+b*(d*x+c)^{(1/3)}) + 5*(a+b*(d*x+c)^{(1/3)})^4*\cos(a+b*(d*x+c)^{(1/3)}) - 20*(a+b*(d*x+c)^{(1/3)})^3*\sin(a+b*(d*x+c)^{(1/3)}) - 60*(a+b*(d*x+c)^{(1/3)})^2*\cos(a+b*(d*x+c)^{(1/3)}) + 120*\cos(a+b*(d*x+c)^{(1/3)}) +$

$$120*(a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)}))+28/b^6*a^2*((a+b*(d*x+c)^{(1/3)})^6*\sin(a+b*(d*x+c)^{(1/3)}+6*(a+b*(d*x+c)^{(1/3)})^5*\cos(a+b*(d*x+c)^{(1/3)})-30*(a+b*(d*x+c)^{(1/3)})^4*\sin(a+b*(d*x+c)^{(1/3)})-120*(a+b*(d*x+c)^{(1/3)})^3*\cos(a+b*(d*x+c)^{(1/3)}+360*(a+b*(d*x+c)^{(1/3)})^2*\sin(a+b*(d*x+c)^{(1/3)})-720*\sin(a+b*(d*x+c)^{(1/3)}+720*(a+b*(d*x+c)^{(1/3)})*\cos(a+b*(d*x+c)^{(1/3)}))-8/b^6*a*((a+b*(d*x+c)^{(1/3)})^7*\sin(a+b*(d*x+c)^{(1/3)}+7*(a+b*(d*x+c)^{(1/3)})^6*\cos(a+b*(d*x+c)^{(1/3)})-42*(a+b*(d*x+c)^{(1/3)})^5*\sin(a+b*(d*x+c)^{(1/3)})-210*(a+b*(d*x+c)^{(1/3)})^4*\cos(a+b*(d*x+c)^{(1/3)}+840*(a+b*(d*x+c)^{(1/3)})^3*\sin(a+b*(d*x+c)^{(1/3)}+2520*(a+b*(d*x+c)^{(1/3)})^2*\cos(a+b*(d*x+c)^{(1/3)})-5040*\cos(a+b*(d*x+c)^{(1/3)}-5040*(a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)}))+1/b^6*((a+b*(d*x+c)^{(1/3)})^8*\sin(a+b*(d*x+c)^{(1/3)}+8*(a+b*(d*x+c)^{(1/3)})^7*\cos(a+b*(d*x+c)^{(1/3)})-56*(a+b*(d*x+c)^{(1/3)})^6*\sin(a+b*(d*x+c)^{(1/3)})-336*(a+b*(d*x+c)^{(1/3)})^5*\cos(a+b*(d*x+c)^{(1/3)}+1680*(a+b*(d*x+c)^{(1/3)})^4*\sin(a+b*(d*x+c)^{(1/3)}+6720*(a+b*(d*x+c)^{(1/3)})^3*\cos(a+b*(d*x+c)^{(1/3)})-20160*(a+b*(d*x+c)^{(1/3)})^2*\sin(a+b*(d*x+c)^{(1/3)}+40320*\sin(a+b*(d*x+c)^{(1/3)})-40320*(a+b*(d*x+c)^{(1/3)})*\cos(a+b*(d*x+c)^{(1/3)))))$$

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 1349 vs.  $2(477) = 954$ .

time = 0.34, size = 1349, normalized size = 2.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")`

[Out]  $3*(a^2*c^2*\sin((d*x + c)^{(1/3)}*b + a) - 2*(((d*x + c)^{(1/3)}*b + a)*\sin((d*x + c)^{(1/3)}*b + a) + \cos((d*x + c)^{(1/3)}*b + a))*a*c^2 + 2*a^5*c*\sin((d*x + c)^{(1/3)}*b + a)/b^3 - 10*(((d*x + c)^{(1/3)}*b + a)*\sin((d*x + c)^{(1/3)}*b + a) + \cos((d*x + c)^{(1/3)}*b + a))*a^4*c/b^3 + (2*(((d*x + c)^{(1/3)}*b + a)*\cos((d*x + c)^{(1/3)}*b + a) + (((d*x + c)^{(1/3)}*b + a)^2 - 2)*\sin((d*x + c)^{(1/3)}*b + a))*c^2 + a^8*\sin((d*x + c)^{(1/3)}*b + a)/b^6 - 8*(((d*x + c)^{(1/3)}*b + a)*\sin((d*x + c)^{(1/3)}*b + a) + \cos((d*x + c)^{(1/3)}*b + a))*a^7/b^6 + 20*(2*(((d*x + c)^{(1/3)}*b + a)*\cos((d*x + c)^{(1/3)}*b + a) + (((d*x + c)^{(1/3)}*b + a)^2 - 2)*\sin((d*x + c)^{(1/3)}*b + a))*a^3*c/b^3 + 28*(2*(((d*x + c)^{(1/3)}*b + a)*\cos((d*x + c)^{(1/3)}*b + a) + (((d*x + c)^{(1/3)}*b + a)^2 - 2)*\sin((d*x + c)^{(1/3)}*b + a))*a^6/b^6 - 20*(3*(((d*x + c)^{(1/3)}*b + a)^2 - 2)*\cos((d*x + c)^{(1/3)}*b + a) + (((d*x + c)^{(1/3)}*b + a)^3 - 6*(d*x + c)^{(1/3)}*b - 6*a)*\sin((d*x + c)^{(1/3)}*b + a))*a^2*c/b^3 - 56*(3*(((d*x + c)^{(1/3)}*b + a)^2 - 2)*\cos((d*x + c)^{(1/3)}*b + a) + (((d*x + c)^{(1/3)}*b + a)^3 - 6*(d*x + c)^{(1/3)}*b - 6*a)*\sin((d*x + c)^{(1/3)}*b + a))*a^5/b^6 + 10*(4*(((d*x + c)^{(1/3)}*b + a)^3 - 6*(d*x + c)^{(1/3)}*b - 6*a)*\cos((d*x + c)^{(1/3)}*b + a) + ((d*x + c)^{(1/3)}*b + a)^4 - 12*((d*x + c)^{(1/3)}*b + a)^2 + 24)*\sin((d*x + c)^{(1/3)}*b + a))*a*c/b^3 + 70*(4*(((d*x + c)^{(1/3)}*b + a)^3 - 6*(d*x + c)^{(1/3)}*b - 6*a)*\cos((d*x + c)^{(1/3)}*b + a) + (((d*x + c)^{(1/3)}*b + a)^4 - 12*(($

```

d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x + c)^(1/3)*b + a))*a^4/b^6 - 2*(5*((
(d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)
^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b + a)^3 +
120*(d*x + c)^(1/3)*b + 120*a)*sin((d*x + c)^(1/3)*b + a))*c/b^3 - 56*(5*((
(d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)
^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b + a)^3
+ 120*(d*x + c)^(1/3)*b + 120*a)*sin((d*x + c)^(1/3)*b + a))*a^3/b^6 + 28*(
6*(((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b + a)^3 + 120*(d*x + c)
^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^6 -
30*((d*x + c)^(1/3)*b + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*sin((d
*x + c)^(1/3)*b + a))*a^2/b^6 - 8*(7*(((d*x + c)^(1/3)*b + a)^6 - 30*((d*x
+ c)^(1/3)*b + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*cos((d*x + c)^(1
/3)*b + a) + (((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c)^(1/3)*b + a)^5 + 84
0*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b - 5040*a)*sin((d*x + c)
^(1/3)*b + a))*a/b^6 + (8*(((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c)^(1/3)
*b + a)^5 + 840*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b - 5040*a)
*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^8 - 56*((d*x + c)^(
1/3)*b + a)^6 + 1680*((d*x + c)^(1/3)*b + a)^4 - 20160*((d*x + c)^(1/3)*b +
a)^2 + 40320)*sin((d*x + c)^(1/3)*b + a))/b^6)/(b^3*d^3)

```

**Fricas** [A]

time = 0.39, size = 182, normalized size = 0.34

$$\frac{3 \left( (3360 b^3 dx + 3240 b^3 c - 12(14 b^5 dx + 9 b^5 c)(dx + c)^3 + (4 b^7 dx^2 + 3 b^7 c dx - 20160 b)(dx + c)^3) \cos((dx + c)^{1/3} b + a) - (56 b^6 dx^2 + 72 b^6 c dx + 18 b^6 c^2 - (b^8 dx^2 - 20160 b^2)(dx + c)^3 - 240(7 b^4 dx + 6 b^4 c)(dx + c)^3 - 40320) \sin((dx + c)^{1/3} b + a) \right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*(d\*x+c)^(1/3)),x, algorithm="fricas")

```

[Out] 3*(2*(3360*b^3*d*x + 3240*b^3*c - 12*(14*b^5*d*x + 9*b^5*c)*(d*x + c)^(2/3)
+ (4*b^7*d^2*x^2 + 3*b^7*c*d*x - 20160*b)*(d*x + c)^(1/3))*cos((d*x + c)^(
1/3)*b + a) - (56*b^6*d^2*x^2 + 72*b^6*c*d*x + 18*b^6*c^2 - (b^8*d^2*x^2 -
20160*b^2)*(d*x + c)^(2/3) - 240*(7*b^4*d*x + 6*b^4*c)*(d*x + c)^(1/3) - 40
320)*sin((d*x + c)^(1/3)*b + a))/b^9*d^3)

```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos\left(a + b\sqrt[3]{c + dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cos(a+b\*(d\*x+c)\*\*(1/3)),x)

```

[Out] Integral(x**2*cos(a + b*(c + d*x)**(1/3)), x)

```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. 2(477) = 954.

time = 0.42, size = 1104, normalized size = 2.06

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*(d\*x+c)^(1/3)),x, algorithm="giac")

[Out] 
$$3*(2*((d*x + c)^{(1/3)}*b + a)*b^6*c^2 - a*b^6*c^2 - 5*((d*x + c)^{(1/3)}*b + a)^4*b^3*c + 20*((d*x + c)^{(1/3)}*b + a)^3*a*b^3*c - 30*((d*x + c)^{(1/3)}*b + a)^2*a^2*b^3*c + 20*((d*x + c)^{(1/3)}*b + a)*a^3*b^3*c - 5*a^4*b^3*c + 4*((d*x + c)^{(1/3)}*b + a)^7 - 28*((d*x + c)^{(1/3)}*b + a)^6*a + 84*((d*x + c)^{(1/3)}*b + a)^5*a^2 - 140*((d*x + c)^{(1/3)}*b + a)^4*a^3 + 140*((d*x + c)^{(1/3)}*b + a)^3*a^4 - 84*((d*x + c)^{(1/3)}*b + a)^2*a^5 + 28*((d*x + c)^{(1/3)}*b + a)*a^6 - 4*a^7 + 60*((d*x + c)^{(1/3)}*b + a)^2*b^3*c - 120*((d*x + c)^{(1/3)}*b + a)*a*b^3*c + 60*a^2*b^3*c - 168*((d*x + c)^{(1/3)}*b + a)^5 + 840*((d*x + c)^{(1/3)}*b + a)^4*a - 1680*((d*x + c)^{(1/3)}*b + a)^3*a^2 + 1680*((d*x + c)^{(1/3)}*b + a)^2*a^3 - 840*((d*x + c)^{(1/3)}*b + a)*a^4 + 168*a^5 - 120*b^3*c + 3360*((d*x + c)^{(1/3)}*b + a)^3 - 10080*((d*x + c)^{(1/3)}*b + a)^2*a + 10080*((d*x + c)^{(1/3)}*b + a)*a^2 - 3360*a^3 - 20160*(d*x + c)^{(1/3)}*b*cos((d*x + c)^{(1/3)}*b + a)/(b^8*d^2) + (((d*x + c)^{(1/3)}*b + a)^2*b^6*c^2 - 2*((d*x + c)^{(1/3)}*b + a)*a*b^6*c^2 + a^2*b^6*c^2 - 2*((d*x + c)^{(1/3)}*b + a)^5*b^3*c + 10*((d*x + c)^{(1/3)}*b + a)^4*a*b^3*c - 20*((d*x + c)^{(1/3)}*b + a)^3*a^2*b^3*c + 20*((d*x + c)^{(1/3)}*b + a)^2*a^3*b^3*c - 10*((d*x + c)^{(1/3)}*b + a)*a^4*b^3*c + 2*a^5*b^3*c + ((d*x + c)^{(1/3)}*b + a)^8 - 8*((d*x + c)^{(1/3)}*b + a)^7*a + 28*((d*x + c)^{(1/3)}*b + a)^6*a^2 - 56*((d*x + c)^{(1/3)}*b + a)^5*a^3 + 70*((d*x + c)^{(1/3)}*b + a)^4*a^4 - 56*((d*x + c)^{(1/3)}*b + a)^3*a^5 + 28*((d*x + c)^{(1/3)}*b + a)^2*a^6 - 8*((d*x + c)^{(1/3)}*b + a)*a^7 + a^8 - 2*b^6*c^2 + 40*((d*x + c)^{(1/3)}*b + a)^3*b^3*c - 120*((d*x + c)^{(1/3)}*b + a)^2*a*b^3*c + 120*((d*x + c)^{(1/3)}*b + a)*a^2*b^3*c - 40*a^3*b^3*c - 56*((d*x + c)^{(1/3)}*b + a)^6 + 336*((d*x + c)^{(1/3)}*b + a)^5*a - 840*((d*x + c)^{(1/3)}*b + a)^4*a^2 + 1120*((d*x + c)^{(1/3)}*b + a)^3*a^3 - 840*((d*x + c)^{(1/3)}*b + a)^2*a^4 + 336*((d*x + c)^{(1/3)}*b + a)*a^5 - 56*a^6 - 240*((d*x + c)^{(1/3)}*b + a)*b^3*c + 240*a*b^3*c + 1680*((d*x + c)^{(1/3)}*b + a)^4 - 6720*((d*x + c)^{(1/3)}*b + a)^3*a + 10080*((d*x + c)^{(1/3)}*b + a)^2*a^2 - 6720*((d*x + c)^{(1/3)}*b + a)*a^3 + 1680*a^4 - 20160*((d*x + c)^{(1/3)}*b + a)^2 + 40320*((d*x + c)^{(1/3)}*b + a)*a - 20160*a^2 + 40320)*sin((d*x + c)^{(1/3)}*b + a)/(b^8*d^2))/(b*d)$$

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \cos\left(a + b(c + dx)^{1/3}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cos(a + b\*(c + d\*x)^(1/3)),x)

[Out] int(x^2\*cos(a + b\*(c + d\*x)^(1/3)), x)

### 3.96 $\int x \cos \left( a + b\sqrt[3]{c + dx} \right) dx$

**Optimal.** Leaf size=261

$$\frac{360 \cos \left( a + b\sqrt[3]{c + dx} \right)}{b^6 d^2} - \frac{6c\sqrt[3]{c + dx} \cos \left( a + b\sqrt[3]{c + dx} \right)}{b^2 d^2} - \frac{180(c + dx)^{2/3} \cos \left( a + b\sqrt[3]{c + dx} \right)}{b^4 d^2} + \frac{15(c + dx)^{5/3} \cos \left( a + b\sqrt[3]{c + dx} \right)}{b^2 d^2}$$

[Out]  $360*\cos(a+b*(d*x+c)^{(1/3)})/b^6/d^2-6*c*(d*x+c)^{(1/3)}*\cos(a+b*(d*x+c)^{(1/3)})/b^2/d^2-180*(d*x+c)^{(2/3)}*\cos(a+b*(d*x+c)^{(1/3)})/b^4/d^2+15*(d*x+c)^{(4/3)}*\cos(a+b*(d*x+c)^{(1/3)})/b^2/d^2+6*c*\sin(a+b*(d*x+c)^{(1/3)})/b^3/d^2+360*(d*x+c)^{(1/3)}*\sin(a+b*(d*x+c)^{(1/3)})/b^5/d^2-3*c*(d*x+c)^{(2/3)}*\sin(a+b*(d*x+c)^{(1/3)})/b/d^2-60*(d*x+c)*\sin(a+b*(d*x+c)^{(1/3)})/b^3/d^2+3*(d*x+c)^{(5/3)}*\sin(a+b*(d*x+c)^{(1/3)})/b/d^2$

**Rubi [A]**

time = 0.16, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3513, 3377, 2717, 2718}

$$\frac{360 \cos \left( a + b\sqrt[3]{c + dx} \right)}{b^6 d^2} + \frac{360 \sqrt[3]{c + dx} \sin \left( a + b\sqrt[3]{c + dx} \right)}{b^5 d^2} - \frac{180(c + dx)^{2/3} \cos \left( a + b\sqrt[3]{c + dx} \right)}{b^4 d^2} - \frac{60(c + dx) \sin \left( a + b\sqrt[3]{c + dx} \right)}{b^3 d^2} + \frac{6c \sin \left( a + b\sqrt[3]{c + dx} \right)}{b^2 d^2} + \frac{15(c + dx)^{4/3} \cos \left( a + b\sqrt[3]{c + dx} \right)}{b^2 d^2} - \frac{6c \sqrt[3]{c + dx} \cos \left( a + b\sqrt[3]{c + dx} \right)}{b^2 d^2} + \frac{3(c + dx)^{5/3} \sin \left( a + b\sqrt[3]{c + dx} \right)}{b^2 d^2} - \frac{3(c + dx)^{2/3} \sin \left( a + b\sqrt[3]{c + dx} \right)}{b^2 d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*\text{Cos}[a + b*(c + d*x)^{(1/3)}], x]$

[Out]  $(360*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^6*d^2) - (6*c*(c + d*x)^{(1/3)}*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^2*d^2) - (180*(c + d*x)^{(2/3)}*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^4*d^2) + (15*(c + d*x)^{(4/3)}*\text{Cos}[a + b*(c + d*x)^{(1/3)}])/(b^2*d^2) + (6*c*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b^3*d^2) + (360*(c + d*x)^{(1/3)}*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b^5*d^2) - (3*c*(c + d*x)^{(2/3)}*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b*d^2) - (60*(c + d*x)*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b^3*d^2) + (3*(c + d*x)^{(5/3)}*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b*d^2)$

**Rule 2717**

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$   
FreeQ[{c, d}, x]

**Rule 2718**

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{Cos}[c + d*x]/d, x] /;$  FreeQ[{c, d}, x]

**Rule 3377**

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[d*(m/f), \text{Int}[(c + d*x)^{(m-1)}*\text{Co}$

`s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

### Rule 3513

`Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p, 0] && IntegerQ[1/n]`

### Rubi steps

$$\begin{aligned}
 \int x \cos\left(a + b\sqrt[3]{c + dx}\right) dx &= \frac{3 \operatorname{Subst}\left(\int \left(-\frac{cx^2 \cos(a+bx)}{d} + \frac{x^5 \cos(a+bx)}{d}\right) dx, x, \sqrt[3]{c + dx}\right)}{d} \\
 &= \frac{3 \operatorname{Subst}\left(\int x^5 \cos(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^2} - \frac{(3c) \operatorname{Subst}\left(\int x^2 \cos(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^2} \\
 &= -\frac{3c(c + dx)^{2/3} \sin\left(a + b\sqrt[3]{c + dx}\right)}{bd^2} + \frac{3(c + dx)^{5/3} \sin\left(a + b\sqrt[3]{c + dx}\right)}{bd^2} \\
 &= -\frac{6c\sqrt[3]{c + dx} \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^2d^2} + \frac{15(c + dx)^{4/3} \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^2d^2} \\
 &= -\frac{6c\sqrt[3]{c + dx} \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^2d^2} + \frac{15(c + dx)^{4/3} \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^2d^2} + \\
 &= -\frac{6c\sqrt[3]{c + dx} \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^2d^2} - \frac{180(c + dx)^{2/3} \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^4d^2} + \\
 &= -\frac{6c\sqrt[3]{c + dx} \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^2d^2} - \frac{180(c + dx)^{2/3} \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^4d^2} + \\
 &= \frac{360 \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^6d^2} - \frac{6c\sqrt[3]{c + dx} \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^2d^2} - \frac{180(c + dx)}{b^4d^2}
 \end{aligned}$$

### Mathematica [A]

time = 0.40, size = 117, normalized size = 0.45

$$\frac{3\left(\left(120 - 60b^2(c + dx)^{2/3} + b^4\sqrt[3]{c + dx}(3c + 5dx)\right) \cos\left(a + b\sqrt[3]{c + dx}\right) + b\left(120\sqrt[3]{c + dx} + b^4dx(c + dx)^{2/3} - 2b^2(9c + 10dx)\right) \sin\left(a + b\sqrt[3]{c + dx}\right)\right)}{b^6d^2}$$

Antiderivative was successfully verified.

`[In] Integrate[x*Cos[a + b*(c + d*x)^(1/3)], x]`



[Out]  $(3*((120 - 60*b^2*(c + d*x)^{(2/3)} + b^4*(c + d*x)^{(1/3)}*(3*c + 5*d*x))*\text{Cos}[a + b*(c + d*x)^{(1/3)}] + b*(120*(c + d*x)^{(1/3)} + b^4*d*x*(c + d*x)^{(2/3)} - 2*b^2*(9*c + 10*d*x))*\text{Sin}[a + b*(c + d*x)^{(1/3)}])/(b^6*d^2)$

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 654 vs.  $2(231) = 462$ .

time = 0.04, size = 655, normalized size = 2.51 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)`

[Out]  $3/d^2/b^3*(-a^2*c*\sin(a+b*(d*x+c)^{(1/3)})+2*a*c*(\cos(a+b*(d*x+c)^{(1/3)})+(a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)}))-c*((a+b*(d*x+c)^{(1/3)})^2*\sin(a+b*(d*x+c)^{(1/3)})-2*\sin(a+b*(d*x+c)^{(1/3)})+2*(a+b*(d*x+c)^{(1/3)})*\cos(a+b*(d*x+c)^{(1/3)}))-1/b^3*a^5*\sin(a+b*(d*x+c)^{(1/3)})+5/b^3*a^4*(\cos(a+b*(d*x+c)^{(1/3)})+(a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)}))-10/b^3*a^3*((a+b*(d*x+c)^{(1/3)})^2*\sin(a+b*(d*x+c)^{(1/3)})-2*\sin(a+b*(d*x+c)^{(1/3)})+2*(a+b*(d*x+c)^{(1/3)})*\cos(a+b*(d*x+c)^{(1/3)}))+10/b^3*a^2*((a+b*(d*x+c)^{(1/3)})^3*\sin(a+b*(d*x+c)^{(1/3)})+3*(a+b*(d*x+c)^{(1/3)})^2*\cos(a+b*(d*x+c)^{(1/3)})-6*\cos(a+b*(d*x+c)^{(1/3)})-6*(a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)}))-5/b^3*a*((a+b*(d*x+c)^{(1/3)})^4*\sin(a+b*(d*x+c)^{(1/3)})+4*(a+b*(d*x+c)^{(1/3)})^3*\cos(a+b*(d*x+c)^{(1/3)})-12*(a+b*(d*x+c)^{(1/3)})^2*\sin(a+b*(d*x+c)^{(1/3)})+24*\sin(a+b*(d*x+c)^{(1/3)})-24*(a+b*(d*x+c)^{(1/3)})*\cos(a+b*(d*x+c)^{(1/3)}))+1/b^3*((a+b*(d*x+c)^{(1/3)})^5*\sin(a+b*(d*x+c)^{(1/3)})+5*(a+b*(d*x+c)^{(1/3)})^4*\cos(a+b*(d*x+c)^{(1/3)})-20*(a+b*(d*x+c)^{(1/3)})^3*\sin(a+b*(d*x+c)^{(1/3)})-60*(a+b*(d*x+c)^{(1/3)})^2*\cos(a+b*(d*x+c)^{(1/3)})+120*\cos(a+b*(d*x+c)^{(1/3)})+120*(a+b*(d*x+c)^{(1/3)})*\sin(a+b*(d*x+c)^{(1/3)}))$

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 523 vs.  $2(231) = 462$ .

time = 0.33, size = 523, normalized size = 2.00

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Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")`

[Out]  $-3*(a^2*c*\sin((d*x + c)^{(1/3)}*b + a) - 2*(((d*x + c)^{(1/3)}*b + a)*\sin((d*x + c)^{(1/3)}*b + a) + \cos((d*x + c)^{(1/3)}*b + a))*a*c + a^5*\sin((d*x + c)^{(1/3)}*b + a)/b^3 - 5*(((d*x + c)^{(1/3)}*b + a)*\sin((d*x + c)^{(1/3)}*b + a) + \cos((d*x + c)^{(1/3)}*b + a))*a^4/b^3 + (2*(((d*x + c)^{(1/3)}*b + a)*\cos((d*x + c)^{(1/3)}*b + a) + (((d*x + c)^{(1/3)}*b + a)^2 - 2)*\sin((d*x + c)^{(1/3)}*b + a))*c + 10*(2*(((d*x + c)^{(1/3)}*b + a)*\cos((d*x + c)^{(1/3)}*b + a) + (((d*x + c)^{(1/3)}*b + a)^2 - 2)*\sin((d*x + c)^{(1/3)}*b + a))*a^3/b^3 - 10*(3*(((d*x + c)^{(1/3)}*b + a)^2 - 2)*\cos((d*x + c)^{(1/3)}*b + a) + (((d*x + c)^{(1/3)}*b + a)^3 - 6*(d*x + c)^{(1/3)}*b - 6*a)*\sin((d*x + c)^{(1/3)}*b + a))*a^2/b^3 + 5*(4*$

$$\frac{((d*x + c)^{(1/3)*b + a})^3 - 6*(d*x + c)^{(1/3)*b - 6*a)*\cos((d*x + c)^{(1/3)*b + a}) + (((d*x + c)^{(1/3)*b + a})^4 - 12*((d*x + c)^{(1/3)*b + a})^2 + 24)*\sin((d*x + c)^{(1/3)*b + a))*a/b^3 - (5*((d*x + c)^{(1/3)*b + a})^4 - 12*((d*x + c)^{(1/3)*b + a})^2 + 24)*\cos((d*x + c)^{(1/3)*b + a}) + (((d*x + c)^{(1/3)*b + a})^5 - 20*((d*x + c)^{(1/3)*b + a})^3 + 120*(d*x + c)^{(1/3)*b + 120*a)*\sin((d*x + c)^{(1/3)*b + a))/b^3)/(b^3*d^2)}$$

**Fricas** [A]

time = 0.38, size = 110, normalized size = 0.42

$$\frac{3 \left( (60(dx+c)^{\frac{2}{3}}b^2 - (5b^4dx + 3b^4c)(dx+c)^{\frac{1}{3}} - 120) \cos((dx+c)^{\frac{1}{3}}b+a) - ((dx+c)^{\frac{2}{3}}b^5dx - 20b^3dx - 18b^3c + 120(dx+c)^{\frac{1}{3}}b) \sin((dx+c)^{\frac{1}{3}}b+a) \right)}{b^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*(d\*x+c)^(1/3)),x, algorithm="fricas")

[Out] -3\*((60\*(d\*x + c)^(2/3)\*b^2 - (5\*b^4\*d\*x + 3\*b^4\*c)\*(d\*x + c)^(1/3) - 120)\*cos((d\*x + c)^(1/3)\*b + a) - ((d\*x + c)^(2/3)\*b^5\*d\*x - 20\*b^3\*d\*x - 18\*b^3\*c + 120\*(d\*x + c)^(1/3)\*b)\*sin((d\*x + c)^(1/3)\*b + a))/(b^6\*d^2)

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x \cos \left( a + b\sqrt[3]{c + dx} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*(d\*x+c)\*\*(1/3)),x)

[Out] Integral(x\*cos(a + b\*(c + d\*x)\*\*(1/3)), x)

**Giac** [A]

time = 0.48, size = 370, normalized size = 1.42

$$\frac{3 \left( (60(dx+c)^{\frac{2}{3}}b^2 - (5b^4dx + 3b^4c)(dx+c)^{\frac{1}{3}} - 120) \cos((dx+c)^{\frac{1}{3}}b+a) - ((dx+c)^{\frac{2}{3}}b^5dx - 20b^3dx - 18b^3c + 120(dx+c)^{\frac{1}{3}}b) \sin((dx+c)^{\frac{1}{3}}b+a) \right)}{b^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*(d\*x+c)^(1/3)),x, algorithm="giac")

[Out] -3\*((2\*((d\*x + c)^(1/3)\*b + a)\*b^3\*c - 2\*a\*b^3\*c - 5\*((d\*x + c)^(1/3)\*b + a)^4 + 20\*((d\*x + c)^(1/3)\*b + a)^3\*a - 30\*((d\*x + c)^(1/3)\*b + a)^2\*a^2 + 20\*((d\*x + c)^(1/3)\*b + a)\*a^3 - 5\*a^4 + 60\*((d\*x + c)^(1/3)\*b + a)^2 - 120\*((d\*x + c)^(1/3)\*b + a)\*a + 60\*a^2 - 120)\*cos((d\*x + c)^(1/3)\*b + a)/b^5 + (((d\*x + c)^(1/3)\*b + a)^2\*b^3\*c - 2\*((d\*x + c)^(1/3)\*b + a)\*a\*b^3\*c + a^2\*b^3\*c - ((d\*x + c)^(1/3)\*b + a)^5 + 5\*((d\*x + c)^(1/3)\*b + a)^4\*a - 10\*((d\*x + c)^(1/3)\*b + a)^3\*a^2 + 10\*((d\*x + c)^(1/3)\*b + a)^2\*a^3 - 5\*((d\*x + c)^(1/3)\*b + a)\*a^4 + a^5 - 2\*b^3\*c + 20\*((d\*x + c)^(1/3)\*b + a)^3 - 60\*((d\*x + c)^(1/3)\*b + a)^2)\*sin((d\*x + c)^(1/3)\*b + a)/b^5

+ c)<sup>(1/3)</sup>\*b + a)<sup>2</sup>\*a + 60\*((d\*x + c)<sup>(1/3)</sup>\*b + a)\*a<sup>2</sup> - 20\*a<sup>3</sup> - 120\*(d\*x + c)<sup>(1/3)</sup>\*b)\*sin((d\*x + c)<sup>(1/3)</sup>\*b + a)/b<sup>5</sup>)/(b\*d<sup>2</sup>)

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int x \cos\left(a + b(c + dx)^{1/3}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cos(a + b\*(c + d\*x)<sup>(1/3)</sup>),x)

[Out] int(x\*cos(a + b\*(c + d\*x)<sup>(1/3)</sup>), x)

### 3.97 $\int \cos \left( a + b\sqrt[3]{c + dx} \right) dx$

**Optimal.** Leaf size=85

$$\frac{6\sqrt[3]{c+dx} \cos \left( a + b\sqrt[3]{c+dx} \right)}{b^2d} - \frac{6 \sin \left( a + b\sqrt[3]{c+dx} \right)}{b^3d} + \frac{3(c+dx)^{2/3} \sin \left( a + b\sqrt[3]{c+dx} \right)}{bd}$$

[Out]  $6*(d*x+c)^{(1/3)}*\cos(a+b*(d*x+c)^{(1/3)})/b^2/d-6*\sin(a+b*(d*x+c)^{(1/3)})/b^3/d+3*(d*x+c)^{(2/3)}*\sin(a+b*(d*x+c)^{(1/3)})/b/d$

**Rubi [A]**

time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3443, 3377, 2717}

$$-\frac{6 \sin \left( a + b\sqrt[3]{c+dx} \right)}{b^3d} + \frac{6\sqrt[3]{c+dx} \cos \left( a + b\sqrt[3]{c+dx} \right)}{b^2d} + \frac{3(c+dx)^{2/3} \sin \left( a + b\sqrt[3]{c+dx} \right)}{bd}$$

Antiderivative was successfully verified.

[In] `Int[Cos[a + b*(c + d*x)^(1/3)], x]`

[Out]  $(6*(c + d*x)^{(1/3)}*\cos[a + b*(c + d*x)^{(1/3)}])/(b^2*d) - (6*\sin[a + b*(c + d*x)^{(1/3)}])/(b^3*d) + (3*(c + d*x)^{(2/3)}*\sin[a + b*(c + d*x)^{(1/3)}])/(b*d)$

Rule 2717

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`  
`FreeQ[{c, d}, x]`

Rule 3377

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(- (c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /;`  
`FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 3443

`Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))]^(n_)]*(b_.))^(p_.), x_Symbol] := Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x, (e + f*x)^n], x] /;`  
`FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[1/n]`

Rubi steps

$$\begin{aligned}
\int \cos\left(a + b\sqrt[3]{c + dx}\right) dx &= \frac{3\text{Subst}\left(\int x^2 \cos(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d} \\
&= \frac{3(c + dx)^{2/3} \sin\left(a + b\sqrt[3]{c + dx}\right)}{bd} - \frac{6\text{Subst}\left(\int x \sin(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{bd} \\
&= \frac{6\sqrt[3]{c + dx} \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^2d} + \frac{3(c + dx)^{2/3} \sin\left(a + b\sqrt[3]{c + dx}\right)}{bd} - \frac{6\text{Subst}\left(\int \sin(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{bd} \\
&= \frac{6\sqrt[3]{c + dx} \cos\left(a + b\sqrt[3]{c + dx}\right)}{b^2d} - \frac{6 \sin\left(a + b\sqrt[3]{c + dx}\right)}{b^3d} + \frac{3(c + dx)^{2/3} \sin\left(a + b\sqrt[3]{c + dx}\right)}{bd}
\end{aligned}$$

**Mathematica [A]**

time = 0.13, size = 65, normalized size = 0.76

$$\frac{6b\sqrt[3]{c + dx} \cos\left(a + b\sqrt[3]{c + dx}\right) + 3(-2 + b^2(c + dx)^{2/3}) \sin\left(a + b\sqrt[3]{c + dx}\right)}{b^3d}$$

Antiderivative was successfully verified.

`[In] Integrate[Cos[a + b*(c + d*x)^(1/3)], x]`

```
[Out] (6*b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)] + 3*(-2 + b^2*(c + d*x)^(2/3))*Sin[a + b*(c + d*x)^(1/3)]/(b^3*d)
```

**Maple [A]**

time = 0.04, size = 131, normalized size = 1.54

method	result
derivativedivides	$\frac{3a^2 \sin\left(a + b(dx + c)^{\frac{1}{3}}\right) - 6a \left(\cos\left(a + b(dx + c)^{\frac{1}{3}}\right) + \left(a + b(dx + c)^{\frac{1}{3}}\right) \sin\left(a + b(dx + c)^{\frac{1}{3}}\right)\right) + 3\left(a + b(dx + c)^{\frac{1}{3}}\right)^2 \sin\left(a + b(dx + c)^{\frac{1}{3}}\right)}{db^3}$
default	$\frac{3a^2 \sin\left(a + b(dx + c)^{\frac{1}{3}}\right) - 6a \left(\cos\left(a + b(dx + c)^{\frac{1}{3}}\right) + \left(a + b(dx + c)^{\frac{1}{3}}\right) \sin\left(a + b(dx + c)^{\frac{1}{3}}\right)\right) + 3\left(a + b(dx + c)^{\frac{1}{3}}\right)^2 \sin\left(a + b(dx + c)^{\frac{1}{3}}\right)}{db^3}$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(a+b*(d*x+c)^(1/3)), x, method=_RETURNVERBOSE)`

```
[Out] 3/d/b^3*(a^2*sin(a+b*(d*x+c)^(1/3))-2*a*(cos(a+b*(d*x+c)^(1/3))+(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-2*sin(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3))
```

**Maxima [A]**

time = 0.28, size = 118, normalized size = 1.39

$$\frac{3\left(a^2 \sin\left((dx + c)^{\frac{1}{3}}b + a\right) - 2\left(\left((dx + c)^{\frac{1}{3}}b + a\right) \sin\left((dx + c)^{\frac{1}{3}}b + a\right) + \cos\left((dx + c)^{\frac{1}{3}}b + a\right)\right)a + 2\left((dx + c)^{\frac{1}{3}}b + a\right) \cos\left((dx + c)^{\frac{1}{3}}b + a\right) + \left(\left((dx + c)^{\frac{1}{3}}b + a\right)^2 - 2\right) \sin\left((dx + c)^{\frac{1}{3}}b + a\right)\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*(d\*x+c)^(1/3)),x, algorithm="maxima")

[Out]  $3*(a^2*\sin((d*x + c)^{(1/3)*b + a}) - 2*((d*x + c)^{(1/3)*b + a})*\sin((d*x + c)^{(1/3)*b + a}) + \cos((d*x + c)^{(1/3)*b + a})*a + 2*((d*x + c)^{(1/3)*b + a})*\cos((d*x + c)^{(1/3)*b + a}) + (((d*x + c)^{(1/3)*b + a})^2 - 2)*\sin((d*x + c)^{(1/3)*b + a))/(b^3*d)$

**Fricas** [A]

time = 0.42, size = 57, normalized size = 0.67

$$\frac{3 \left( 2(dx+c)^{\frac{1}{3}} b \cos \left( (dx+c)^{\frac{1}{3}} b + a \right) + \left( (dx+c)^{\frac{2}{3}} b^2 - 2 \right) \sin \left( (dx+c)^{\frac{1}{3}} b + a \right) \right)}{b^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*(d\*x+c)^(1/3)),x, algorithm="fricas")

[Out]  $3*(2*(d*x + c)^{(1/3)*b}*\cos((d*x + c)^{(1/3)*b + a}) + ((d*x + c)^{(2/3)*b} - 2)*\sin((d*x + c)^{(1/3)*b + a}))/b^3*d$

**Sympy** [A]

time = 0.24, size = 94, normalized size = 1.11

$$\begin{cases} x \cos(a) & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ x \cos(a + b\sqrt[3]{c}) & \text{for } d = 0 \\ \frac{3(c+dx)^{\frac{2}{3}} \sin\left(\frac{a+b\sqrt[3]{c+dx}}{bd}\right)}{bd} + \frac{6\sqrt[3]{c+dx} \cos\left(\frac{a+b\sqrt[3]{c+dx}}{bd}\right)}{b^2 d} - \frac{6 \sin\left(\frac{a+b\sqrt[3]{c+dx}}{bd}\right)}{b^3 d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*(d\*x+c)\*\*(1/3)),x)

[Out] Piecewise((x\*cos(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x\*cos(a + b\*c\*\*(1/3)), Eq(d, 0)), (3\*(c + d\*x)\*\*(2/3)\*sin(a + b\*(c + d\*x)\*\*(1/3))/(b\*d) + 6\*(c + d\*x)\*\*(1/3)\*cos(a + b\*(c + d\*x)\*\*(1/3))/(b\*\*2\*d) - 6\*sin(a + b\*(c + d\*x)\*\*(1/3))/(b\*\*3\*d), True))

**Giac** [A]

time = 0.41, size = 81, normalized size = 0.95

$$\frac{3 \left( \frac{2(dx+c)^{\frac{1}{3}} \cos \left( (dx+c)^{\frac{1}{3}} b + a \right)}{b} + \frac{\left( (dx+c)^{\frac{1}{3}} b + a \right)^2 - 2 \left( (dx+c)^{\frac{1}{3}} b + a \right) a + a^2 - 2}{b^2} \sin \left( (dx+c)^{\frac{1}{3}} b + a \right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*(d\*x+c)^(1/3)),x, algorithm="giac")

```
[Out] 3*(2*(d*x + c)^(1/3)*cos((d*x + c)^(1/3)*b + a)/b + (((d*x + c)^(1/3)*b + a)
)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*sin((d*x + c)^(1/3)*b + a)/b^2
)/(b*d)
```

**Mupad [B]**

time = 0.38, size = 68, normalized size = 0.80

$$\frac{6b \cos\left(a + b(c + dx)^{1/3}\right) (c + dx)^{1/3} - 6 \sin\left(a + b(c + dx)^{1/3}\right) + 3b^2 \sin\left(a + b(c + dx)^{1/3}\right) (c + dx)^{2/3}}{b^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] (6*b*cos(a + b*(c + d*x)^(1/3))*(c + d*x)^(1/3) - 6*sin(a + b*(c + d*x)^(1/3))
+ 3*b^2*sin(a + b*(c + d*x)^(1/3))*(c + d*x)^(2/3))/(b^3*d)
```

$$3.98 \quad \int \frac{\cos\left(a + b\sqrt[3]{c + dx}\right)}{x} dx$$

**Optimal.** Leaf size=234

$$\cos\left(a + b\sqrt[3]{c}\right) \operatorname{CosIntegral}\left(b\sqrt[3]{c} - b\sqrt[3]{c + dx}\right) + \cos\left(a + (-1)^{2/3}b\sqrt[3]{c}\right) \operatorname{CosIntegral}\left((-1)^{2/3}b\sqrt[3]{c} - b\sqrt[3]{c + dx}\right)$$

```
[Out] Ci(b*c^(1/3)-b*(d*x+c)^(1/3))*cos(a+b*c^(1/3))+Ci((-1)^(1/3)*b*c^(1/3)+b*(d*x+c)^(1/3))*cos(a-(-1)^(1/3)*b*c^(1/3))+Ci((-1)^(2/3)*b*c^(1/3)-b*(d*x+c)^(1/3))*cos(a+(-1)^(2/3)*b*c^(1/3))+Si(b*c^(1/3)-b*(d*x+c)^(1/3))*sin(a+b*c^(1/3))-Si((-1)^(1/3)*b*c^(1/3)+b*(d*x+c)^(1/3))*sin(a-(-1)^(1/3)*b*c^(1/3))+Si((-1)^(2/3)*b*c^(1/3)-b*(d*x+c)^(1/3))*sin(a+(-1)^(2/3)*b*c^(1/3))
```

**Rubi [A]**

time = 0.33, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3513, 3384, 3380, 3383}

$\cos(a + b\sqrt[3]{c}) \operatorname{CosIntegral}(b\sqrt[3]{c} - b\sqrt[3]{c + dx}) + \cos(a + (-1)^{2/3}b\sqrt[3]{c}) \operatorname{CosIntegral}((-1)^{2/3}b\sqrt[3]{c} - b\sqrt[3]{c + dx}) + \cos(a - \sqrt[3]{-1}b\sqrt[3]{c}) \operatorname{CosIntegral}(b\sqrt[3]{c} + \sqrt[3]{-1}b\sqrt[3]{c}) + \sin(a + b\sqrt[3]{c}) \operatorname{Si}(b\sqrt[3]{c} - b\sqrt[3]{c + dx}) + \sin(a + (-1)^{2/3}b\sqrt[3]{c}) \operatorname{Si}((-1)^{2/3}b\sqrt[3]{c} - b\sqrt[3]{c + dx}) - \sin(a - \sqrt[3]{-1}b\sqrt[3]{c}) \operatorname{Si}(\sqrt[3]{-1}b\sqrt[3]{c} + \sqrt[3]{c + dx})$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] Cos[a + b*c^(1/3)]*CosIntegral[b*c^(1/3) - b*(c + d*x)^(1/3)] + Cos[a + (-1)^(2/3)*b*c^(1/3)]*CosIntegral[(-1)^(2/3)*b*c^(1/3) - b*(c + d*x)^(1/3)] + Cos[a - (-1)^(1/3)*b*c^(1/3)]*CosIntegral[(-1)^(1/3)*b*c^(1/3) + b*(c + d*x)^(1/3)] + Sin[a + b*c^(1/3)]*SinIntegral[b*c^(1/3) - b*(c + d*x)^(1/3)] + Sin[a + (-1)^(2/3)*b*c^(1/3)]*SinIntegral[(-1)^(2/3)*b*c^(1/3) - b*(c + d*x)^(1/3)] - Sin[a - (-1)^(1/3)*b*c^(1/3)]*SinIntegral[(-1)^(1/3)*b*c^(1/3) + b*(c + d*x)^(1/3)]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)
```



)/d], Int[Cos[c\*(f/d) + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

### Rule 3513

Int[((a\_.) + Cos[(c\_.) + (d\_.)\*((e\_.) + (f\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((g\_.  
. ) + (h\_.)\*(x\_.))^(m\_.), x\_Symbol] := Dist[1/(n\*f), Subst[Int[ExpandIntegran  
d[(a + b\*Cos[c + d\*x])^p, x^(1/n - 1)\*(g - e\*(h/f) + h\*(x^(1/n)/f))^m, x],  
x], x, (e + f\*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,  
0] && IntegerQ[1/n]

### Rubi steps

$$\int \frac{\cos\left(a + b\sqrt[3]{c + dx}\right)}{x} dx = \frac{3 \text{Subst}\left(\int \left(\frac{-d \cos(a+bx)}{3(\sqrt[3]{c} - x)} - \frac{d \cos(a+bx)}{3(-\sqrt[3]{-1} \sqrt[3]{c} - x)} - \frac{d \cos(a+bx)}{3((-1)^{2/3} \sqrt[3]{c} - x)}\right) dx, x, \sqrt[3]{c} - x\right)}{d}$$

$$= -\text{Subst}\left(\int \frac{\cos(a + bx)}{\sqrt[3]{c} - x} dx, x, \sqrt[3]{c + dx}\right) - \text{Subst}\left(\int \frac{\cos(a + bx)}{-\sqrt[3]{-1} \sqrt[3]{c} - x} dx, x, \sqrt[3]{c + dx}\right)$$

$$= -\left(\cos(a + b\sqrt[3]{c}) \text{Subst}\left(\int \frac{\cos(b\sqrt[3]{c} - bx)}{\sqrt[3]{c} - x} dx, x, \sqrt[3]{c + dx}\right)\right) - \cos(a + b\sqrt[3]{c})$$

$$= \cos(a + b\sqrt[3]{c}) \text{Ci}\left(b\sqrt[3]{c} - b\sqrt[3]{c + dx}\right) + \cos(a + (-1)^{2/3} b\sqrt[3]{c}) \text{Ci}\left((-1)^{2/3} b\sqrt[3]{c} - b\sqrt[3]{c + dx}\right)$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 0.38, size = 243, normalized size = 1.04

$\frac{1}{2}(\text{RootSum}[-\#1^3, \cos(a + b\#1) \text{CosIntegral}(b(\sqrt[3]{c + dx} - \#1)) - \text{CosIntegral}(b(\sqrt[3]{c} - \#1))] \sin(a + b\#1) - \cos(a + b\#1) \text{Si}(b(\sqrt[3]{c + dx} - \#1)) - \sin(a + b\#1) \text{Si}(b(\sqrt[3]{c} - \#1))] x) + \text{RootSum}[-\#1^3, \cos(a + b\#1) \text{CosIntegral}(b(\sqrt[3]{c + dx} - \#1)) + \text{CosIntegral}(b(\sqrt[3]{c} - \#1))] \sin(a + b\#1) + \cos(a + b\#1) \text{Si}(b(\sqrt[3]{c + dx} - \#1)) - \sin(a + b\#1) \text{Si}(b(\sqrt[3]{c} - \#1))] x) / 2$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*(c + d\*x)^(1/3)]/x,x]

[Out] (RootSum[c - #1^3 & , Cos[a + b\*#1]\*CosIntegral[b\*((c + d\*x)^(1/3) - #1]] - I\*CosIntegral[b\*((c + d\*x)^(1/3) - #1)]\*Sin[a + b\*#1] - I\*Cos[a + b\*#1]\*SinIntegral[b\*((c + d\*x)^(1/3) - #1]] - Sin[a + b\*#1]\*SinIntegral[b\*((c + d\*x)^(1/3) - #1]] & ] + RootSum[c - #1^3 & , Cos[a + b\*#1]\*CosIntegral[b\*((c + d\*x)^(1/3) - #1]] + I\*CosIntegral[b\*((c + d\*x)^(1/3) - #1)]\*Sin[a + b\*#1] + I\*Cos[a + b\*#1]\*SinIntegral[b\*((c + d\*x)^(1/3) - #1]] - Sin[a + b\*#1]\*SinIntegral[b\*((c + d\*x)^(1/3) - #1]] & ])/2

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.06, size = 279, normalized size = 1.19

method	result
derivativedivides	$b^3 a^2 \left( \frac{\sum_{-R1=\text{RootOf}(-b^3 c + Z^3 - 3a Z^2 + 3a^2 Z - a^3)} \text{sinIntegral}\left(-b(dx+c)^{\frac{1}{3}} + \_R1 - a\right) \sin(\_R1) + \text{cosineIntegral}\left(b(dx+c)^{\frac{1}{3}} + \_R1 - a\right) \cos(\_R1)}{\_R1^2 - 2\_R1 a + a^2} \right)$
default	$b^3 a^2 \left( \frac{\sum_{-R1=\text{RootOf}(-b^3 c + Z^3 - 3a Z^2 + 3a^2 Z - a^3)} \text{sinIntegral}\left(-b(dx+c)^{\frac{1}{3}} + \_R1 - a\right) \sin(\_R1) + \text{cosineIntegral}\left(b(dx+c)^{\frac{1}{3}} + \_R1 - a\right) \cos(\_R1)}{\_R1^2 - 2\_R1 a + a^2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*(d*x+c)^(1/3))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/3*b^3*a^2*sum(1/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_R1-a)*sin(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3))-2/3*b^3*a*sum(_R1/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_R1-a)*sin(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3))+1/3*b^3*sum(_R1^2/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_R1-a)*sin(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3)))
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x,x, algorithm="maxima")
```

```
[Out] integrate(cos((d*x + c)^(1/3)*b + a)/x, x)
```

**Fricas [C]** Result contains complex when optimal does not.

time = 0.42, size = 287, normalized size = 1.23

$$\frac{1}{2} \text{Ei}\left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(-\sqrt{3}-1\right)\right) e^{\frac{1}{2} \sqrt{3} \left(-\sqrt{3}-1\right) \left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(-\sqrt{3}-1\right)\right)} + \frac{1}{2} \text{Ei}\left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(-\sqrt{3}-1\right)\right) e^{\frac{1}{2} \sqrt{3} \left(-\sqrt{3}-1\right) \left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(-\sqrt{3}-1\right)\right)} + \frac{1}{2} \text{Ei}\left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(\sqrt{3}-1\right)\right) e^{\frac{1}{2} \sqrt{3} \left(\sqrt{3}-1\right) \left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(\sqrt{3}-1\right)\right)} + \frac{1}{2} \text{Ei}\left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(\sqrt{3}-1\right)\right) e^{\frac{1}{2} \sqrt{3} \left(\sqrt{3}-1\right) \left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(\sqrt{3}-1\right)\right)} + \frac{1}{2} \text{Ei}\left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(-\sqrt{3}+1\right)\right) e^{\frac{1}{2} \sqrt{3} \left(-\sqrt{3}+1\right) \left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(-\sqrt{3}+1\right)\right)} + \frac{1}{2} \text{Ei}\left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(-\sqrt{3}+1\right)\right) e^{\frac{1}{2} \sqrt{3} \left(-\sqrt{3}+1\right) \left(\frac{(dx+c)^{1/3}}{b} + \frac{1}{2} \sqrt{3} \left(-\sqrt{3}+1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x,x, algorithm="fricas")
```

```
[Out] 1/2*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*b^3*c)^(1/3)*(I*sqrt(3) + 1) + I*a) + 1/2*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) + 1) - I*a) + 1/2*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) + 1) + I*a) + 1/2*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*b^3*c)^(1/3)*(-I*sqrt(3) + 1) - I*a)
```

) + 1) - I\*a) + 1/2\*Ei(I\*(d\*x + c)^(1/3)\*b + (I\*b^3\*c)^(1/3))\*e^(I\*a - (I\*b^3\*c)^(1/3)) + 1/2\*Ei(-I\*(d\*x + c)^(1/3)\*b + (-I\*b^3\*c)^(1/3))\*e^(-I\*a - (-I\*b^3\*c)^(1/3))

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + b\sqrt[3]{c + dx}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*(d\*x+c)\*\*(1/3))/x,x)

[Out] Integral(cos(a + b\*(c + d\*x)\*\*(1/3))/x, x)

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*(d\*x+c)^(1/3))/x,x, algorithm="giac")

[Out] integrate(cos((d\*x + c)^(1/3)\*b + a)/x, x)

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a + b(c + dx)^{1/3}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b\*(c + d\*x)^(1/3))/x,x)

[Out] int(cos(a + b\*(c + d\*x)^(1/3))/x, x)

$$3.99 \quad \int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2} dx$$

**Optimal.** Leaf size=332

$$\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{bd\text{CosIntegral}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\sin\left(a+b\sqrt[3]{c}\right)}{3c^{2/3}} + \frac{\sqrt[3]{-1}bd\text{CosIntegral}\left(\sqrt[3]{-1}b\sqrt[3]{c}\right)}{3c^{2/3}}$$

[Out]  $-\cos(a+b*(d*x+c)^{(1/3)})/x+1/3*b*d*\cos(a+b*c^{(1/3)})*Si(b*c^{(1/3)}-b*(d*x+c)^{(1/3)})/c^{(2/3)}+1/3*(-1)^{(2/3)}*b*d*\cos(a+(-1)^{(2/3)}*b*c^{(1/3)})*Si((-1)^{(2/3)}*b*c^{(1/3)}-b*(d*x+c)^{(1/3)})/c^{(2/3)}+1/3*(-1)^{(1/3)}*b*d*\cos(a-(-1)^{(1/3)}*b*c^{(1/3)})*Si((-1)^{(1/3)}*b*c^{(1/3)}+b*(d*x+c)^{(1/3)})/c^{(2/3)}-1/3*b*d*Ci(b*c^{(1/3)}-b*(d*x+c)^{(1/3)})*\sin(a+b*c^{(1/3)})/c^{(2/3)}+1/3*(-1)^{(1/3)}*b*d*Ci((-1)^{(1/3)}*b*c^{(1/3)}+b*(d*x+c)^{(1/3)})*\sin(a-(-1)^{(1/3)}*b*c^{(1/3)})/c^{(2/3)}-1/3*(-1)^{(2/3)}*b*d*Ci((-1)^{(2/3)}*b*c^{(1/3)}-b*(d*x+c)^{(1/3)})*\sin(a+(-1)^{(2/3)}*b*c^{(1/3)})/c^{(2/3)}$

**Rubi [A]**

time = 0.53, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3513, 3423, 3414, 3384, 3380, 3383}

$\frac{\text{Min}(a+k\sqrt{c})\text{CosIntegral}(b\sqrt{c-k\sqrt{c+dx}})}{3c^{2/3}} - \frac{\sqrt{-1}\text{Min}(a-\sqrt{-1}k\sqrt{c})\text{CosIntegral}(b\sqrt{-1}k\sqrt{c+dx})}{3c^{2/3}} - \frac{(-1)^{1/3}\text{Min}(a+(-1)^{1/3}k\sqrt{c})\text{CosIntegral}((-1)^{1/3}k\sqrt{c-k\sqrt{c+dx}})}{3c^{2/3}} - \frac{\text{Max}(a+k\sqrt{c})\text{Si}(b\sqrt{c-k\sqrt{c+dx}})}{3c^{2/3}} + \frac{(-1)^{1/3}\text{Max}(a+(-1)^{1/3}k\sqrt{c})\text{Si}((-1)^{1/3}k\sqrt{c-k\sqrt{c+dx}})}{3c^{2/3}} - \frac{\sqrt{-1}\text{Max}(a-\sqrt{-1}k\sqrt{c})\text{Si}(\sqrt{-1}k\sqrt{c+dx})}{3c^{2/3}} - \frac{\cos(a+k\sqrt{c+dx})}{x}$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*(c + d\*x)^(1/3)]/x^2,x]

[Out]  $-(\text{Cos}[a + b*(c + d*x)^{(1/3)}]/x) - (b*d*\text{CosIntegral}[b*c^{(1/3)} - b*(c + d*x)^{(1/3)}]*\text{Sin}[a + b*c^{(1/3)}])/(3*c^{(2/3)}) + ((-1)^{(1/3)}*b*d*\text{CosIntegral}[(-1)^{(1/3)}*b*c^{(1/3)} + b*(c + d*x)^{(1/3)}]*\text{Sin}[a - (-1)^{(1/3)}*b*c^{(1/3)}])/(3*c^{(2/3)}) - ((-1)^{(2/3)}*b*d*\text{CosIntegral}[(-1)^{(2/3)}*b*c^{(1/3)} - b*(c + d*x)^{(1/3)}]*\text{Sin}[a + (-1)^{(2/3)}*b*c^{(1/3)}])/(3*c^{(2/3)}) + (b*d*\text{Cos}[a + b*c^{(1/3)}]*\text{SinIntegral}[b*c^{(1/3)} - b*(c + d*x)^{(1/3)}])/(3*c^{(2/3)}) + ((-1)^{(2/3)}*b*d*\text{Cos}[a + (-1)^{(2/3)}*b*c^{(1/3)}]*\text{SinIntegral}[(-1)^{(2/3)}*b*c^{(1/3)} - b*(c + d*x)^{(1/3)}])/(3*c^{(2/3)}) + ((-1)^{(1/3)}*b*d*\text{Cos}[a - (-1)^{(1/3)}*b*c^{(1/3)}]*\text{SinIntegral}[(-1)^{(1/3)}*b*c^{(1/3)} + b*(c + d*x)^{(1/3)}])/(3*c^{(2/3)})$

**Rule 3380**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[SinIntegral[e + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*e - c\*f, 0]

**Rule 3383**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CosIntegral[e - Pi/2 + f\*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d\*(e - Pi/2) -

$c*f, 0]$

#### Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

#### Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] := Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

#### Rule 3423

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_
), x_Symbol] := Simp[e^m*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1))),
x] + Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sin[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
negerQ[n] || GtQ[e, 0])
```

#### Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] := Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
0] && IntegerQ[1/n]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(a + b\sqrt[3]{c + dx}\right)}{x^2} dx &= \frac{3\text{Subst}\left(\int \frac{x^2 \cos(a+bx)}{\left(-\frac{c}{d} + \frac{x^3}{d}\right)^2} dx, x, \sqrt[3]{c + dx}\right)}{d} \\
&= -\frac{\cos\left(a + b\sqrt[3]{c + dx}\right)}{x} - b\text{Subst}\left(\int \frac{\sin(a + bx)}{-\frac{c}{d} + \frac{x^3}{d}} dx, x, \sqrt[3]{c + dx}\right) \\
&= -\frac{\cos\left(a + b\sqrt[3]{c + dx}\right)}{x} - b\text{Subst}\left(\int \left(-\frac{d \sin(a + bx)}{3c^{2/3}(\sqrt[3]{c} - x)} - \frac{d \sin(a + bx)}{3c^{2/3}(\sqrt[3]{c} + \sqrt[3]{-c})}\right) dx, x, \sqrt[3]{c + dx}\right) \\
&= -\frac{\cos\left(a + b\sqrt[3]{c + dx}\right)}{x} + \frac{(bd)\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt[3]{c} - x} dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}} + \frac{(bd)\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt[3]{c} + x} dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&= -\frac{\cos\left(a + b\sqrt[3]{c + dx}\right)}{x} - \frac{(bd \cos(a + b\sqrt[3]{c})) \text{Subst}\left(\int \frac{\sin(b\sqrt[3]{c} - bx)}{\sqrt[3]{c} - x} dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&= -\frac{\cos\left(a + b\sqrt[3]{c + dx}\right)}{x} - \frac{bd\text{Ci}\left(b\sqrt[3]{c} - b\sqrt[3]{c + dx}\right) \sin(a + b\sqrt[3]{c})}{3c^{2/3}} + \frac{\sqrt[3]{-1} bd \text{Ci}\left(b\sqrt[3]{c} + b\sqrt[3]{c + dx}\right) \sin(a + b\sqrt[3]{c})}{3c^{2/3}}
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 0.58, size = 138, normalized size = 0.42

$$-\frac{\cos\left(a + b\sqrt[3]{c + dx}\right)}{x} - \frac{1}{6} ibd\text{RootSum}\left[c - \#1^3 \&, \frac{e^{-ia - ib\#1} \text{Ei}\left(-ib\left(\sqrt[3]{c + dx} - \#1\right)\right)}{\#1^2}\right] + \frac{1}{6} ibd\text{RootSum}\left[c - \#1^3 \&, \frac{e^{ia + ib\#1} \text{Ei}\left(ib\left(\sqrt[3]{c + dx} - \#1\right)\right)}{\#1^2}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*(c + d\*x)^(1/3)]/x^2,x]

[Out] -(Cos[a + b\*(c + d\*x)^(1/3)]/x) - (I/6)\*b\*d\*RootSum[c - #1^3 &, (E^((-I)\*a - I\*b\*#1)\*ExpIntegralEi[(-I)\*b\*((c + d\*x)^(1/3) - #1)])/#1^2 & ] + (I/6)\*b\*d\*RootSum[c - #1^3 &, (E^(I\*a + I\*b\*#1)\*ExpIntegralEi[I\*b\*((c + d\*x)^(1/3) - #1)])/#1^2 & ]

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.09, size = 931, normalized size = 2.80

method	result	size
derivativedivides	Expression too large to display	931
default	Expression too large to display	931



```
[Out] -1/12*(2*(I*b^3*c)^(1/3)*d*x*Ei(I*(d*x + c)^(1/3)*b + (I*b^3*c)^(1/3))*e^(I
*a - (I*b^3*c)^(1/3)) + 2*(-I*b^3*c)^(1/3)*d*x*Ei(-I*(d*x + c)^(1/3)*b + (-
I*b^3*c)^(1/3))*e^(-I*a - (-I*b^3*c)^(1/3)) - (I*b^3*c)^(1/3)*(I*sqrt(3)*d*
x + d*x)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e^(
1/2*(I*b^3*c)^(1/3)*(I*sqrt(3) + 1) + I*a) - (-I*b^3*c)^(1/3)*(I*sqrt(3)*d*
x + d*x)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e
^(1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) + 1) - I*a) - (I*b^3*c)^(1/3)*(-I*sqrt(3)
*d*x + d*x)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(I*sqrt(3) - 1))*e
^(1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) + 1) + I*a) - (-I*b^3*c)^(1/3)*(-I*sqrt(3)
)*d*x + d*x)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) - 1)
)*e^(1/2*(-I*b^3*c)^(1/3)*(-I*sqrt(3) + 1) - I*a) + 12*c*cos((d*x + c)^(1/3)
)*b + a)/(c*x)
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + b\sqrt[3]{c + dx}\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)**(1/3))/x**2,x)
```

```
[Out] Integral(cos(a + b*(c + d*x)**(1/3))/x**2, x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos((d*x + c)^(1/3)*b + a)/x^2, x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a + b(c + dx)^{1/3}\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*(c + d*x)^(1/3))/x^2,x)
```

```
[Out] int(cos(a + b*(c + d*x)^(1/3))/x^2, x)
```



# Chapter 4

## Appendix

### Local contents

4.1	Download section . . . . .	440
4.2	Listing of Grading functions . . . . .	440

## 4.1 Download section

The following zip files contain the raw integrals used in this test.

**Mathematica format** Mathematica\_syntax.zip

**Maple and Mupad format** Maple\_syntax.zip

**Sympy format** SYMPY\_syntax.zip

**Sage math format** SAGE\_syntax.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*           Small rewrite of logic in main function to make it*)
(*           match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*           is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*           antiderivative*)
(* "A" if result can be considered optimal*)
```

```

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A","none"}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
        ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
      ,(*ELSE*)(*result does not contains complex*)
      If[leafCountResult<=2*leafCountOptimal,
        finalresult={"A","none"}
        ,(*ELSE*)
        finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
      ]
    ]
    ,(*ELSE*)(*expnResult>expnOptimal*)
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      finalresult={"C","Result contains higher order function than in optimal. Order "<
    ,
    finalresult={"F","Contains unresolved integral."}
  ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)

```

(\*9 = unknown function\*)

```
ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]==Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]==Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]==Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
        Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
  If[Head[expn]==Plus || Head[expn]==Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]==RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]==Integrate || Head[expn]==Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
  },func]
```

```
SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```

ExpIntegralE, ExpIntegralEi, LogIntegral,
SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
Gamma, LogGamma, PolyGamma,
Zeta, PolyLog, ProductLog,
EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]

```

## 4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B","result has leaf size over 500,000. Avoiding possible recursion issues";
  fi;

  leaf_count_optimal := leafcount(optimal);
  ExpnType_result := ExpnType(result);
  ExpnType_optimal := ExpnType(optimal);

```

```

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#   is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
#   antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of r
                    convert(leaf_count_result,string)," vs. $2 (" ,
                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co

        end if
    else #result contains complex but optimal is not
        if debug then
            print("result contains complex but optimal is not");
        fi;
        return "C","Result contains complex when optimal does not.";
    fi;
else # result do not contain complex

```

```

    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well")
    fi;
    if leaf_count_result<=2*leaf_count_optimal then
        if debug then
            print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A","";
    else
        if debug then
            print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of o
                        convert(leaf_count_result,string)," $ vs. $2(",
                        convert(leaf_count_optimal,string),")=",convert(2*leaf_cou

    fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                  convert(ExpnType_result,string)," vs. order ",
                  convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function

```

```

# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+`) or type(expn,'*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,

```



```

    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

```

```

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type(expn,'*')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:

```

```

    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)-str(leaf_count_optimal)
            else:
                grade = "C"
                grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)-str(ExpnType_optimal)

```

```

# print("Before returning. grade=", grade, " grade_annotation=", grade_annotation)

return grade, grade_annotation

```

#### 4.2.4 SageMath grading function

```

# Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fricas, Giac and Maxima results.
# Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'
# June 4, 2022 Made default grade_annotation "none" instead of "" due
#       issue later when reading the file.
# July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    # print("Enter tree_size, expr is ", expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1] == 1/2: # expr.args[1] == Rational(1,2):
            if debug: print("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

```

```

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)

```

```

    return 1
  else:
    return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
  else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or instan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```



```

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_resu

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```